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Strategic Ambiguity in Global Games*

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Abstract

In incomplete information games with ambiguous information, rational behavior depends on fundamental ambiguity (ambiguity about states) and strategic ambiguity (ambiguity about others' actions). We study the impact of strategic ambiguity in global games, which is evident when one of the actions yields a constant payoff. Ambiguous-quality information makes more players choose this action, whereas (unambiguous) low-quality information makes more players choose an ex-ante best response to the uniform belief over the opponents' actions. If the ex-ante best-response action yields a constant payoff, sufficiently ambiguous-quality information makes most players choose this action, thus inducing a unique equilibrium, whereas sufficiently low-quality information generates multiple equilibria. In applications to financial crises, we demonstrate that news of more ambiguous quality triggers a debt rollover crisis, whereas news of less ambiguous quality triggers a currency crisis.

JEL classification numbers: C72, D81, D82.

Keywords: strategic ambiguity; global game; currency crisis; debt rollover crisis; incomplete information; multiple priors.

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1 Introduction

Consider an incomplete information game with players who have ambiguous beliefs about a payoff-relevant state.[1] Players receive signals about a state, but they do not exactly know the true joint distribution of signals and a state. In this game, players' beliefs about the opponents' actions are also ambiguous even if players know the opponents' strategies, which assign an action to each signal, because their beliefs about the opponents' signals are ambiguous. Thus, rational behavior depends not only on fundamental ambiguity (ambiguity about states) but also on strategic ambiguity (ambiguity about others' actions).

Strategic ambiguity can have a substantial impact on equilibrium outcomes. To illustrate it, consider a continuum of players who decide whether or not to invest in a project. The payoff to "investing" is 2 if the project succeeds and -1 if the project fails, while the payoff to "not investing" is 0. The project's success depends upon the proportion of players to invest and a state, which is either g (good) or b (bad) with an equal probability $1/2$. The project succeeds if and only if the state is g and more than two thirds of the players invest. If players have no additional information about the state, this game has two symmetric pure-strategy equilibria, where all players invest, and no players invest, respectively.

We now assume that each player receives a private signal about the state: he receives G (resp. B) with probability $p \geq 1/2$ when the state is g (resp. b), and signals are conditionally independent across players. We also assume that players do not know the true value of p and make a decision on the basis of the most pessimistic assessment of p conforming to maxmin expected utility (MEU) preferences (Gilboa and Schmeidler, 1989) and prior-by-prior (full Bayesian) updating (Fagin and Halpern, 1990; Jaffray, 1992; Pires, 2002). Then, this game has a unique equilibrium, where no players invest. To see why, consider first a player who receives signal B. The conditional probability of state b is p by Bayes rule, so the worst-case scenario for investing is that the state is bad with probability 1. Facing such fundamental ambiguity, this player does not invest as if the signal were precise. Consider next a player who receives signal G. The expected proportion of the opponents with signal B is $2p(1-p) \leq 1/2$ by Bayes rule and the law of large numbers, so the worst-case scenario for investing is that half of the opponents receive signal B and do not invest. Facing such strategic ambiguity, this player does not invest as if the signal were imprecise. To summarize, ambiguous signals with unknown

[1]See Gilboa and Marinacci (2013) and Machina and Siniscalchi (2014) for a survey of the motivation and history of ambiguity and ambiguity aversion.

$p \in [1/2, 1]$ give rise to a unique rationalizable strategy,[2] wheres uninformative signals with known $p = 1/2$ result in multiple equilibria.

This paper studies the impact of strategic ambiguity in binary-action supermodular games of incomplete information in which players receive noisy private signals about a state, i.e., global games (Carlsson and van Damme, 1993).[3] Our model is a global game equipped with multiple priors, where players make a decision conforming to MEU preferences and prior-by-prior updating. In contrast to the above example, only a tiny portion of players have a dominant action facing fundamental ambiguity. However, strategic ambiguity amplifies the effect of fundamental ambiguity through ambiguous belief hierarchies on the opponents' dominant actions. As a result, the role of ambiguous-quality information can be quite different from that of low-quality information.

We first show that our model is also a supermodular game in terms of MEU preferences and develop a tractable procedure to analyze it. Because players evaluate each action by the most pessimistic beliefs, they behave as if they adopted different priors to evaluate different actions. Thus, we can analyze the model using a fictitious game with a pair of priors, each of which is in the set of priors and separately assigned to each action. We show that if every fictitious game admits a unique equilibrium, then our model also admits a unique equilibrium that survives iterated deletion of strictly interim-dominated strategies in terms of MEU preferences.

A fictitious game with a pair of priors is reduced to a single-prior game if one of the actions yields a constant payoff, which is referred to as a safe action (Morris and Shin, 2002). Our model with a safe action has a unique equilibrium not only when every single-prior game has a unique equilibrium but also when some have multiple equilibria. The latter is the case if the following holds: information quality is sufficiently ambiguous, and a safe action is an ex-ante best response (a best response before receiving a signal) to the uniform belief over the opponents' actions, which is referred to as an ex-ante Laplacian action. The unique equilibrium coincides with the equilibrium of the single prior game that maximizes the range of signals to which the safe action is assigned, where the maximum is taken over the set of all equilibria and the set of priors.

In summary, we find two effects of ambiguous information on equilibrium outcomes when one action is a safe action. First, ambiguous information enlarges the range of signals assigned

[2] In this example, players have a set of priors given by $p \in [1/2, 1]$, but we can obtain the same conclusion by replacing $[1/2, 1]$ with $[\underline{p}, \overline{p}]$, where $1/2 \leq \underline{p} < 1/2 + \sqrt{3}/6$ and $2/3 < \overline{p} \leq 1$.

[3] See the surveys in Morris and Shin (2002) and Angeletos and Lian (2016).

to a safe action, whereas it is known that low-quality information enlarges the range of signals assigned to an ex-ante Laplacian action. Thus, if a safe action is not ex-ante Laplacian, the effect of ambiguous information is opposite to that of low-quality information.[4] Even if a safe action is ex-ante Laplacian, the former is also distinct from the latter in another sense. In this case, sufficiently ambiguous information induces a unique equilibrium, whereas it is known that sufficiently low-quality information induces multiple equilibria. These effects of ambiguous information are attributed to ambiguous belief hierarchies and MEU preferences, which make a safe action more survivable in iterated deletion of strictly interim-dominated strategies. In contrast, a safe action does not play such a special role in standard global games because two actions' payoff differential determines best responses.

Our findings have the following implications for the question of whether ambiguous information contributes to financial crises originating from coordination failures, which can be modeled as global games of regime change. In the model of a currency crisis (Obstfeld, 1996; Morris and Shin, 1998), speculators must decide whether to attack a currency, where a crisis occurs if sufficiently many speculators attack the currency. We find that news of more ambiguous quality decreases the likelihood of the crisis because not to attack is a safe action. In the model of a debt rollover crisis (Calvo, 1988; Morris and Shin, 2004), creditors must decide whether to roll over a loan, where a crisis occurs if sufficiently many creditors do not roll over the loan. We find that news of more ambiguous quality increases the likelihood of the crisis because not to roll over is a safe action. These results complement and contrast with the findings of Iachan and Nenov (2015) on the effects of low-quality information in standard global games of regime change. They show that news of lower quality increases the likelihood of a currency crisis, but it does not influence that of a debt rollover crisis,[5] which is in sharp contrast to the above effects of ambiguous information.

The rest of this paper is organized as follows. Section 2 illustrates our findings using an example. In Section 3, we set up our model and report the main results. In Section 4, we discuss applications to financial crises. Section 5 provides several extensions. The last section concludes the paper.

[4] Kawagoe and Ui (2013) conducted a laboratory experiment using a two-player global game and obtained data supporting this result.

[5] This is true in the model of Morris and Shin (2004). Iachan and Nenov (2015) study another model of a debt rollover crisis, where news of lower quality influences the likelihood of the crisis.

1.1 Related literature

This paper joins a growing literature on the theory and applications of incomplete information games with MEU players. There are many applications to auctions and mechanism design. Earlier papers examined equilibria of first price sealed bid auctions with MEU agents (Salo and Weber, 1995; Lo, 1998). More recent papers study optimal mechanism design (Bose et al., 2006; Bose and Daripa, 2009; Bodoh-Creed, 2012) and implementability of social choice functions (Wolitzky, 2016; Song, 2018; De Castro and Yannelis, 2018; Guo and Yannelis, 2021). Ambiguous beliefs are exogenously given in these papers, whereas Bose and Renou (2014) and Di Tillio et al. (2017) allow a mechanism designer to endogenously engineer ambiguity. Using ambiguous mechanisms, Bose and Renou (2014) characterize implementable social choice functions, and Di Tillio et al. (2017) solve revenue maximization problems. Other recent applications include strategic voting (Ellis, 2016; Fabrizi et al., 2019; Pan, 2019; Ryan, 2021).

Because our focus is on global games with applications to financial crises, this paper is also related to a theoretical literature on financial crises associated with ambiguity. Caballero and Krishnamurthy (2008) explain flight to quality episodes using a variant of the Diamond-Dybvig model (Diamond and Dybvig, 1983) with agents in the face of ambiguity about liquidity shocks. Dicks and Fulghieri (2019) propose a theory of systemic risk using another variant of the Diamond-Dybvig model with two banks investing in ambiguous assets. Our work contributes to the literature[6] by presenting the global game approach to financial crises under ambiguity, which can also be applied to other global game models of financial crises.[7]

This paper builds on the following foundational studies. Epstein and Wang (1996) construct hierarchies of general preferences, thus providing a foundation for rationalizability and iterated deletion of strictly interim-dominated strategies in terms of MEU preferences (cf. Epstein, 1997). Ahn (2007) constructs a universal type space with multiple beliefs analogous to Mertens and Zamir (1985). Kajii and Ui (2005) introduce a general class of incomplete information games (Harsanyi, 1967–1968) with MEU players and two interim equilibrium concepts;[8] Kajii and Ui (2009) and Martins-da-Rocha (2010) study the corresponding "agreement theorem" (Aumann, 1976) and its converse. Our solution concept corresponds to rationalizability in Epstein and Wang (1996) and one of the solution concepts in Kajii and Ui (2005).

[6] Other papers include Uhlig (2010) and Routledge and Zin (2009).

[7] See Angeletos and Lian (2016, Section 5) for a survey.

[8] Incomplete information games with more general preferences are studied by Azrieli and Teper (2011), Grant et al. (2016), and Hanany et al. (2020).

2 A linear example

This section illustrates our main findings using a canonical linear-normal global game (Morris and Shin, 2001). A continuum of players have two actions, action 0 and action 1, which are interpreted as not investing and investing, respectively. The payoff to action 1 is $\theta+l-1$, where $\theta \in \mathbb{R}$ is normally distributed with mean $y \in (0,1)$ and precision $\eta > 0$ (i.e. variance $1/\eta$), and $l \in [0,1]$ is the proportion of the opponents choosing action 1. The payoff to action 0 is a constant 0, so we say that action 0 is a safe action. In summary, each player's payoff function is

$$u(a,l,\theta) = \begin{cases} 0 & \text{if } a = 0 \text{ (not investing)}, \\ \theta + l - 1 & \text{if } a = 1 \text{ (investing)}. \end{cases}$$

Player i observes a private signal $x_i = \theta + \varepsilon_i$, where a noise term ε_i is independently normally distributed with mean 0 and precision $\xi > 0$ (i.e. variance $1/\xi$), which can be regarded as a measure of information quality.

If it is common knowledge that $0 < \theta < 1$ or if players receive no signals about θ, this game has two symmetric pure-strategy equilibria. If the interim expected value of θ is strictly greater than 1, action 1 is a dominant action; if that of θ is strictly less than 0, action 0 is a dominant action. When a player has the uniform belief over the opponents' actions (i.e. the expected value of l is $1/2$), the ex-ante best response (the best response before receiving a signal) is action 1 if $y \geq 1/2$ and action 0 if $y \leq 1/2$. We call it an ex-ante Laplacian action.[9]

We assume that information quality is ambiguous. Players have a set of priors indexed by a measure of information quality $\xi \in \Xi \equiv [\underline{\xi}, \bar{\xi}] \subset \mathbb{R}_{++}$, and make a decision based on the most pessimistic assessment conforming to maxmin expected utility (MEU) preferences (Gilboa and Schmeidler, 1989) and prior-by-prior updating (Fagin and Halpern, 1990; Jaffray, 1992; Pires, 2002). Thus, after receiving a private signal, each player has a set of interim beliefs indexed by $\xi \in \Xi$ and evaluates each action in terms of the minimum expected payoff, where the minimum is taken over Ξ.[10]

We denote this game by (u, Ξ). Consider a monotone strategy, where a player chooses action 1 if and only if a private signal is above a cutoff point $\kappa \in \mathbb{R} \cup \{-\infty, \infty\}$. This strategy is

[9] The notion of an ex-ante Laplacian action is different from that of a Laplacian action in Morris and Shin (2002), which is the best response to the uniform belief over the opponents' actions when a player knows θ. A Laplacian is action 1 if $\theta \geq 1/2$ and action 0 if $\theta \leq 1/2$.

[10] Players are assumed to have a common set of priors. See Kajii and Ui (2009) for an implication of this assumption.

referred to as a switching strategy (with cutoff κ) and denoted by $s[\kappa] : \mathbb{R} \to \{0, 1\}$, i.e.,

$$s[\kappa](x) \equiv \begin{cases} 1 & \text{if } x > \kappa, \\ 0 & \text{if } x \leq \kappa. \end{cases} \tag{1}$$

To characterize a best response to $s[\kappa]$, consider a player with a private signal x who believes that the opponents follow $s[\kappa]$. The expected payoff to action 1 with respect to ξ is calculated as

$$\pi^1_\xi(x, \kappa) \equiv E_\xi[\theta|x] + \text{Prob}_\xi[x' > \kappa|x] - 1 = \frac{\eta y + \xi x}{\eta + \xi} - \Phi\left(\sqrt{\frac{\xi(\eta + \xi)}{\eta + 2\xi}}\left(\kappa - \frac{\eta y + \xi x}{\eta + \xi}\right)\right), \tag{2}$$

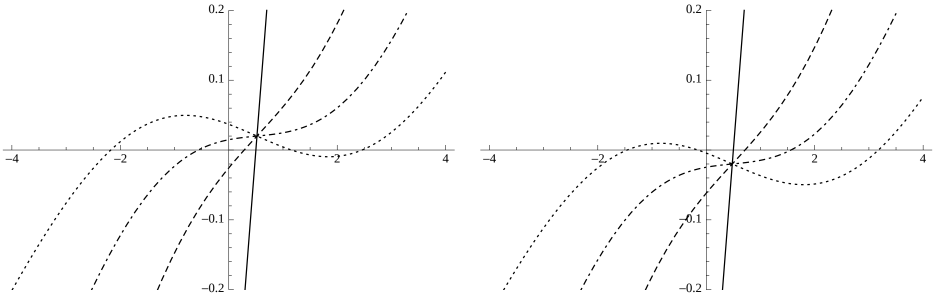
where $\text{Prob}_\xi[x' > \kappa|x]$ is the probability that an opponent receives a private signal greater than κ, and Φ is the cumulative distribution function of the standard normal distribution. Then, this player prefers action 1 to action 0 if and only if $\min_{\xi\in\Xi} \pi^1_\xi(x, \kappa) \geq 0$. Note that $\min_{\xi\in\Xi} \pi^1_\xi(x, \kappa)$ is strictly increasing in x and decreasing in κ. Thus, $s[\kappa']$ is the best response to $s[\kappa]$ if $\min_{\xi\in\Xi} \pi^1_\xi(\kappa', \kappa) = 0$. In particular, a strategy profile where all players follow $s[\kappa]$ is an equilibrium if κ is a solution to

$$\min_{\xi\in\Xi} \pi^1_\xi(\kappa, \kappa) = 0, \tag{3}$$

which is referred to as a switching equilibrium (with cutoff κ). Consequently, a unique switching equilibrium exists if (3) has a unique solution, and its strategy is shown to be a unique strategy surviving iterated deletion of strictly interim-dominated strategies with respect to MEU preferences by an argument similar to that in the standard global game analysis.

An equilibrium of (u, Ξ) is related to an equilibrium of a single-prior game $(u, \{\xi\})$ with $\xi \in \Xi$ in several senses. Most importantly, the maximum equilibrium cutoff of (u, Ξ) coincides with the maximum of the maximum equilibrium cutoffs of all the single-prior games, which is denoted by $k(\Xi) \equiv \max_{\xi\in\Xi} \max\{\kappa \mid \pi^1_\xi(\kappa, \kappa) = 0\}$. This is because $\pi^1_\xi(\kappa, \kappa) > 0$ for all $\kappa > k(\Xi)$ and $\xi \in \Xi$. It immediately implies that $\min_{\xi\in\Xi} \pi^1_\xi(\kappa, \kappa) > 0$ for all $\kappa > k(\Xi)$ and $0 \leq \min_{\xi\in\Xi} \pi^1_\xi(k(\Xi), k(\Xi)) \leq \pi^1_{\hat{\xi}}(k(\Xi), k(\Xi)) = 0$, where $\hat{\xi} \in \arg\max_{\xi\in\Xi} \max\{\kappa \mid \pi^1_\xi(\kappa, \kappa) = 0\}$.

Especially, if (u, Ξ) has a unique switching equilibrium, its cutoff must be $k(\Xi)$. For example, if a single-prior game $(u, \{\xi\})$ has a unique equilibrium for each $\xi \in \Xi$, $s[k(\Xi)]$ is the unique equilibrium strategy in (u, Ξ) because $k(\Xi)$ is the unique solution to $\pi^1_{\hat{\xi}}(\kappa, \kappa) = 0$, where $\hat{\xi}$ is given above, and $\min_{\xi\in\Xi} \pi^1_\xi(\kappa, \kappa) \leq \pi^1_{\hat{\xi}}(\kappa, \kappa) < 0$ for all $\kappa < k(\Xi)$. As shown by Morris and



(a) $y = 0.52 > 1/2$ (b) $y = 0.48 < 1/2$

Figure 1: Graphs of $\pi^1_\xi(\kappa,\kappa)$ with $\eta = 2$, $\xi = 10^6$ in the solid line, $\xi = 1.1$ in the dashed line, $\xi = 0.56$ in the dash-dot line, and $\xi = 0.37$ in the dotted line. Cutoffs are given by κ-intercepts.

Shin (2001), this is the case if each $\xi \in \Xi$ is strictly greater than a threshold value given by $\xi^* \equiv \eta\left(\eta - 2\pi + (\eta^2 + 12\pi\eta + 4\pi^2)^{1/2}\right)/(8\pi)$.

Claim 1. *If the minimum precision in Ξ is strictly greater than ξ^*, there exists a unique switching equilibrium with cutoff $k(\Xi)$.*

In Figure 1, a solution to $\pi^1_\xi(\kappa,\kappa) = 0$ corresponds to a horizontal intercept in the graph of $\pi^1_\xi(\kappa,\kappa)$, and it is unique for each $\xi \neq 0.37$. For example, the graph of $\min_{\xi\in\Xi}\pi^1_\xi(\kappa,\kappa)$ with $\Xi = [0.56, 1.1]$ is the lower envelope of the dashed line and the dash-dot line, which has a unique horizontal intercept. The unique equilibrium cutoff $k(\Xi)$ is the horizontal intercept of the dashed line in Figure 1a if $y = 0.52$, and that of the dash-dot line in Figure 1b if $y = 0.48$.

Even if the minimum precision is less than ξ^*, (u, Ξ) can admit a unique equilibrium. This is the case if action 0 is ex-ante Laplacian (i.e., $y < 1/2$) and the width of Ξ is sufficiently large; that is, information quality is ambiguous enough.

We first give an intuition by informally discussing the limiting case as $\underline{\xi} \to 0$ (we assume $\underline{\xi} > 0$ in our formal analysis). Let $\Xi = [0, \bar{\xi}]$ with $\bar{\xi} > 0$. Then, $s[-\infty]$ (always investing) is not an equilibrium strategy because action 0 is a best response action to $s[-\infty]$ for a private signal $x < -\eta y/\bar{\xi}$, i.e., $\min_{\xi\in\Xi}\pi^1_\xi(x,-\infty) < \pi^1_{\bar{\xi}}(-\eta y/\bar{\xi}, -\infty) = 0$. In this case, the worst-case scenario is that the precision is highest, under which a player chooses a safe action (i.e., action 0) believing that the state is bad. Furthermore, $s[\kappa]$ with finite κ is not an equilibrium strategy because action 0 is a best response action to $s[\kappa]$ for any private signal x, i.e., $\min_{\xi\in\Xi}\pi^1_\xi(x,\kappa) \leq \pi^1_0(x,\kappa) = y - 1/2 < 0$ for every x. In this case, the worst-case scenario is that the precision is lowest, under which a player chooses an ex-ante Laplacian action (i.e., action 0) believing that the opponents' signals are uniformly distributed, and thus so are their signals.[11] Consequently, $s[\infty]$ is the unique equilibrium strategy.

[11] Note that $\text{Prob}_\xi[x' > \kappa | x] \to 1/2$ as $\xi \to 0$ because a private signal is uniformly distributed in the limit.

More generally, we have the following claim.

Claim 2. *Suppose that $y < 1/2$. There exists a unique switching equilibrium with cutoff $k(\Xi)$ if either of the following conditions holds.*

(i) *For any minimum precision $\underline{\xi} > 0$, the maximum precision $\overline{\xi} > 0$ is sufficiently high.*

(ii) *For any maximum precision $\overline{\xi} > 0$, the minimum precision $\underline{\xi} > 0$ is sufficiently low.*

Moreover, $\lim_{\underline{\xi}\to 0} k(\Xi) = \infty$ *for all* $\overline{\xi} > 0$.

This claim is based upon the following two observations. First, (3) has no solution less than $-\eta y/\overline{\xi}$ because $\min_{\xi\in\Xi} \pi^1_\xi(\kappa,\kappa) < \pi^1_{\overline{\xi}}(-\eta y/\overline{\xi}, -\infty) = 0$ for $\kappa < -\eta y/\overline{\xi}$. Second, (3) has exactly one solution greater than y because $\pi^1_\xi(y,y) = y - 1/2 < 0$ for all $\xi \in \Xi$, $\min_{\xi\in\Xi} \pi^1_\xi(\kappa,\kappa) > 0$ for sufficiently large κ, and $d^2\pi^1_\xi(\kappa,\kappa)/d\kappa^2$ is strictly positive for all $\kappa > y$ and $\xi \in \Xi$. Therefore, Claim 2 is true if each of (i) and (ii) implies that (3) has no solution on the interval $[-\eta y/\overline{\xi}, y]$, or equivalently,

$$\min_{\xi\in\Xi} \pi^1_\xi(\kappa,\kappa) < 0 \text{ for all } \kappa \in [-\eta y/\overline{\xi}, y]. \tag{4}$$

The condition (i) implies (4) if $\overline{\xi} > \xi^*$. In this case, $\min_{\xi\in\Xi} \pi^1_\xi(\kappa,\kappa) \le \pi^1_{\overline{\xi}}(\kappa,\kappa) < 0$ for all $\kappa \le y$ because $\pi^1_{\overline{\xi}}(y,y) < 0$ and $\pi^1_{\overline{\xi}}(\kappa,\kappa) = 0$ has a unique solution, which is greater than y.[12] The condition (ii) implies (4) if $\underline{\xi}$ satisfies $\pi^1_{\underline{\xi}}(y, -\eta y/\overline{\xi}) < 0$ (such $\underline{\xi}$ exists because $\lim_{\xi\to 0} \pi^1_\xi(y, -\eta y/\overline{\xi}) = y - 1/2 < 0$). In this case, $\min_{\xi\in\Xi} \pi^1_\xi(\kappa,\kappa) \le \pi^1_{\underline{\xi}}(\kappa,\kappa) \le \pi^1_{\underline{\xi}}(y, -\eta y/\overline{\xi}) < 0$ for all $\kappa \in [-\eta y/\overline{\xi}, y]$. Moreover,

$$\lim_{\underline{\xi}\to 0} k(\Xi) \ge \lim_{\xi\to 0} \max\{\kappa \mid \pi^1_\xi(\kappa,\kappa) = 0\} \ge \lim_{\xi\to 0} \sup\{\kappa \mid \pi^1_\xi(\kappa,\kappa) < 0\} = \infty$$

because $\lim_{\xi\to 0} \pi^1_\xi(\kappa,\kappa) = y - 1/2 < 0$. As illustrated in Figure 2, the unique horizontal intercept can be very large if $\underline{\xi}$ is very small.

We contrast the above effects of ambiguous-quality information with those of low-quality information using a benchmark game $(u, \{\xi^0\})$ with $\xi^0 > \xi^*$, a game with low-quality information $(u, \{\xi\})$ with $\xi^0 > \xi$, and a game with ambiguous-quality information (u, Ξ) with $\overline{\xi} > \xi^0 > \underline{\xi}$. Ambiguous-quality information and low-quality information have different effects in the following two respects.

[12] We can illustrate this case using Figure 1b. The graph of $\min_{\xi\in\Xi} \pi^1_\xi(\kappa,\kappa)$ with $\Xi = [0.37, 0.56]$ is the lower envelope of the dash-dot line and the dotted line, and the unique horizontal intercept is the rightmost horizontal intercept of the dotted line.

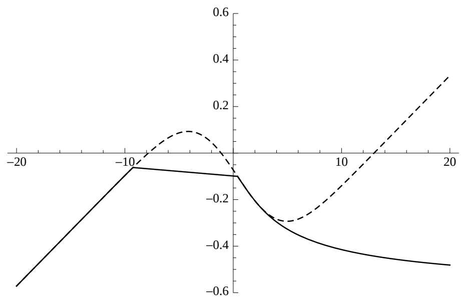
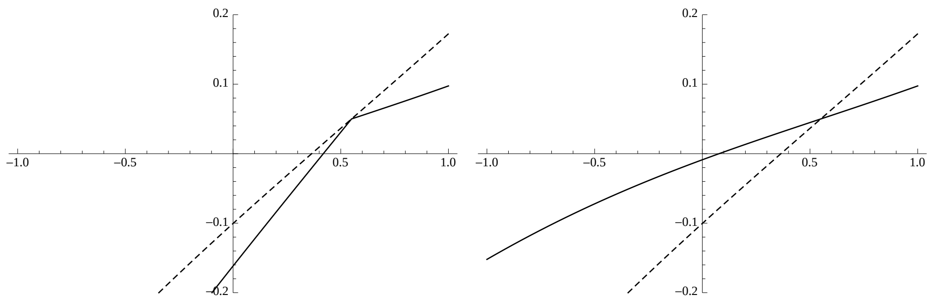


Figure 2: Graphs of $\min_{\xi \in \Xi} \pi^1_\xi(\kappa, \kappa)$ with $y = 0.4$, $\eta = 2$, $\Xi = [1/10^4, 1/10]$ in the solid line, and $\Xi = \{1/20\}$ in the dashed line.



(a) Ambiguous quality with $\Xi = [1, 3]$ (b) Low quality with $\Xi = \{1\}$

Figure 3: Graphs of $\min_{\xi \in \Xi} \pi^1_\xi(\kappa, \kappa)$ with $y = 0.55$, $\eta = 2$, and $\Xi = \{2\}$ in the dashed line.

First, assume that $y > 1/2$; that is, action 1 is ex-ante Laplacian. It is well known that if $\xi > \xi^*$, then the unique equilibrium cutoff $k(\{\xi\})$ in $(u, \{\xi\})$ increases with ξ (see Figure 1a) and converges to $1/2$ as ξ approaches infinity (Morris and Shin, 2001). Thus, low-quality information decreases the equilibrium cutoff, i.e., $k(\{\xi\}) < k(\{\xi^0\})$ (see Figure 3b), whereas ambiguous-quality information increases the equilibrium cutoff, i.e., $k(\{\xi^0\}) < k(\Xi)$ (see Figure 3a).

Second, assume that $y < 1/2$; that is, action 0 is ex-ante Laplacian. If $\xi > 0$ is sufficiently small, $(u, \{\xi\})$ has three equilibria (see the horizontal intercepts of the dotted line Figure 1b). Thus, low-quality information induces multiple equilibria, whereas ambiguous-quality information induces a unique equilibrium by Claim 2. To contrast the difference, consider the limiting case of $(u, \{\xi\})$ when $\xi = 0$ and that of (u, Ξ) when $\Xi = [0, \bar{\xi}]$ with $\bar{\xi} > 0$. In $(u, \{0\})$, a player regards his private signal as uninformative, and both $s[-\infty]$ and $s[\infty]$ are equilibrium strategies. In $(u, [0, \bar{\xi}])$, contrastingly, a player regards his private signal as either informative or uninformative depending upon the worst-case scenario. Thus, $s[-\infty]$ cannot be an equilibrium as discussed before, while $s[\infty]$ is an equilibrium strategy as in the case of $(u, \{0\})$.

3 Main results

In this section, we introduce a general model and present our main results. All proofs are relegated to the appendix.

There is a continuum of players indexed by $i \in [0, 1]$.[13] Each player has a binary action set $\{0, 1\}$ and a payoff function $u : \{0, 1\} \times [0, 1] \times \mathbb{R} \to \mathbb{R}$, where $u(a, l, \theta)$ is a payoff to action $a \in \{0, 1\}$ when a proportion $l \in [0, 1]$ of the opponents choose action 1 and a payoff state is $\theta \in \mathbb{R}$. An action $a \in \{0, 1\}$ is called a safe action if $u(a, l, \theta)$ is independent of (l, θ).

Player i observes a noisy private signal $x_i = \theta + \varepsilon_i$, where ε_i is an independent noise term. Each player with each private signal has MEU preferences determined by a payoff function u and a set of beliefs indexed by a set of parameters Ξ. For each $\xi \in \Xi$, let $p_\xi(\theta)$, $q_\xi(\varepsilon_i)$, and $p_\xi(\theta|x_i)$ denote the probability density function of θ, that of ε_i, and the conditional probability density function of θ given x_i, respectively.[14] We assume that Ξ is a compact and connected set.

A game is denoted by a pair (u, Ξ). A strategy in (u, Ξ) is a measurable mapping $\sigma : \mathbb{R} \to \{0, 1\}$, which assigns an action to each private signal. To formally define an equilibrium concept, consider a player with a private signal x who believes that the opponents follow σ. The expected payoff to action $a \in \{0, 1\}$ with respect to ξ is

$$E_\xi[u(a, E_\xi[\sigma|\theta], \theta)|x] \equiv \int u(a, E_\xi[\sigma|\theta], \theta) p_\xi(\theta|x) d\theta,$$

where $E_\xi[\sigma|\theta] \equiv \int \sigma(x) q_\xi(x - \theta) dx$ is the proportion of the opponents taking action 1 when the state is θ. Thus, this player prefers action a to action a' if and only if

$$\min_{\xi \in \Xi} E_\xi[u(a, E_\xi[\sigma|\theta], \theta)|x] \geq \min_{\xi \in \Xi} E_\xi[u(a', E_\xi[\sigma|\theta], \theta)|x]. \tag{5}$$

A strategy profile in which all players follow σ is an equilibrium if $\sigma(x) = a$ implies (5) for $a' \neq a$. We focus on a pure-strategy equilibrium, which may not satisfy the requirement of a mixed-strategy equilibrium in incomplete information games with MEU players, as discussed in Kajii and Ui (2005). We discuss this issue in Section 5.2.

To characterize a switching equilibrium, where all players follow a switching strategy $s[\kappa]$

[13]We consider a continuum of players because many applications of global games assume so. It is straightforward to translate our results for a symmetric two-player case as in Carlsson and van Damme (1993).

[14]Thus, $p_\xi(\theta|x_i) = p_\xi(\theta) q_\xi(x_i - \theta) / \int p_\xi(\theta) q_\xi(x_i - \theta) dx_i$. However, our discussion goes through even if $p_\xi(\theta|x_i)$ is not a conditional probability density function under a common prior.

given by (1), let

$$\pi_\xi^a(x,\kappa) \equiv E_\xi[u(a, E_\xi[s[\kappa]|\theta], \theta)|x]$$

denote the expected payoff to action $a \in \{0,1\}$ with respect to ξ when a player receives a private signal x and the opponents follow $s[\kappa]$. Then, $s[\kappa]$ is an equilibrium strategy if and only if

$$\min_{\xi\in\Xi} \pi_\xi^1(x,\kappa) - \min_{\xi\in\Xi} \pi_\xi^0(x,\kappa) \begin{cases} \geq 0 & \text{if } x > \kappa, \\ \leq 0 & \text{if } x \leq \kappa. \end{cases} \tag{6}$$

If the left-hand side of (6) is continuous and increasing in x, (6) is reduced to

$$\min_{\xi\in\Xi} \pi_\xi^1(\kappa,\kappa) - \min_{\xi\in\Xi} \pi_\xi^0(\kappa,\kappa) = 0. \tag{7}$$

We assume the following conditions on (u, Ξ).

A1 (Action Monotonicity) For each θ, $u(1,l,\theta)$ is increasing in l; $u(0,l,\theta)$ is decreasing in l.

A2 (State Monotonicity) For each l, $u(1,l,\theta)$ is increasing in θ; $u(0,l,\theta)$ is decreasing in θ.

A3 (Stochastic Dominance) If $x > x'$, $p_\xi(\theta|x)$ first-order stochastically dominates $p_\xi(\theta|x')$ for each $\xi \in \Xi$.

A4 (Continuity) $\pi_\xi^a(x,\kappa)$ exists for all $(x,\kappa,\xi) \in \mathbb{R} \times \mathbb{R} \cup \{-\infty,\infty\} \times \Xi$, and the function $(x,\kappa,\xi) \mapsto \pi_\xi^a(x,\kappa)$ is continuous for each $a \in \{0,1\}$.

A5 (Limit Dominance) There exist $\underline{\theta}, \overline{\theta} \in \mathbb{R}$ satisfying

$$u(1,1,\underline{\theta}) - u(0,1,\underline{\theta}) < 0, \ \lim_{x\to-\infty} \int_{-\infty}^{\underline{\theta}} p_\xi(\theta|x)d\theta = 1 \text{ for each } \xi,$$
$$u(1,0,\overline{\theta}) - u(0,0,\overline{\theta}) > 0, \ \lim_{x\to\infty} \int_{\overline{\theta}}^{\infty} p_\xi(\theta|x)d\theta = 1 \text{ for each } \xi.$$

By A1 and A2, the payoff to action 1 (resp. action 0) is increasing (resp. decreasing) in the proportion of the opponents choosing action 1 and the state. In a single-prior game, it is enough to assume monotonicity of the payoff differential $u(1,l,\theta) - u(0,l,\theta)$ (cf. Morris and Shin, 2002). In a multiple-priors game, however, we need monotonicity of $u(1,l,\theta)$ and $u(0,l,\theta)$ separately because worst-case beliefs depend upon actions. A3 implies that high signals convey good news for θ. If q_ξ satisfies the monotone likelihood ratio property, A3 holds (Milgrom, 1981). Many probability distributions including the normal, the exponential, and the uniform distributions satisfy this property. A4 is a technical assumption, and it is satisfied if a payoff function is continuous and bounded. In some applications of global games, a payoff function is discontinuous or unbounded, yet A4 is satisfied in most cases. A5 requires that action 1 (resp. action 0) be a dominant strategy for sufficiently high (resp. low) signals.

Under these assumptions, $s[\kappa]$ is an equilibrium strategy if and only if κ is a solution to (7) because the following lemma implies the equivalence of (6) and (7).

Lemma 1. *The function* $(x,\kappa) \mapsto \min_{\xi\in\Xi} \pi^1_\xi(x,\kappa) - \min_{\xi\in\Xi} \pi^0_\xi(x,\kappa)$ *is continuous, increasing in x, and decreasing in* κ.

Moreover, (u,Ξ) exhibits strategic complementarities in the following sense.

Lemma 2. *If* $\sigma(x) \geq \sigma'(x)$ *for all* $x \in \mathbb{R}$*, then*

$$\min_{\xi\in\Xi} E_\xi[u(1, E_\xi[\sigma|\theta],\theta)|x] - \min_{\xi\in\Xi} E_\xi[u(0, E_\xi[\sigma|\theta],\theta)|x]$$
$$\geq \min_{\xi\in\Xi} E_\xi[u(1, E_\xi[\sigma'|\theta],\theta)|x] - \min_{\xi\in\Xi} E_\xi[u(0, E_\xi[\sigma'|\theta],\theta)|x] \text{ for all } x\in\mathbb{R}.$$

Therefore, the set of strategies surviving iterated deletion of strictly interim-dominated strategies includes the maximum and minimum elements, which are the maximum and minimum equilibrium strategies (Milgrom and Roberts, 1990; Vives, 1990; Van Zandt and Vives, 2007). In particular, this set is a singleton if and only if (7) has a unique solution.

Proposition 1. *Let* $\underline{\kappa},\overline{\kappa} \in \mathbb{R}$ *be the minimum and maximum solutions to* (7)*, respectively. Then,* $s[\underline{\kappa}]$ *and* $s[\overline{\kappa}]$ *are equilibrium strategies in* (u,Ξ)*, and* $s[\overline{\kappa}](x) \leq \sigma(x) \leq s[\underline{\kappa}](x)$ *for all* $x\in\mathbb{R}$ *if a strategy* σ *survives iterated deletion of strictly interim-dominated strategies. In particular,* $s[\kappa^*]$ *is the (essentially)*[15] *unique strategy surviving the iterated deletion if and only if* $\kappa^* = \underline{\kappa} = \overline{\kappa}$.

To study a unique equilibrium in (u,Ξ), we consider a fictitious game with a pair of priors indexed by $(\xi_0,\xi_1) \in \Xi\times\Xi$, where players evaluate actions 0 and 1 using ξ_0 and ξ_1, respectively. If each fictitious game has a unique switching equilibrium, (u,Ξ) also has a unique switching equilibrium, and the equilibrium cutoff equals the "maxmin" of the equilibrium cutoffs in the fictitious games, as shown by the next proposition, which generalizes Claim 1.

[15] An action of a player with a private signal κ^* can be any action in an equilibrium strategy surviving iterated deletion, as in the standard global game analysis.

Proposition 2. *Suppose that there exists a unique value $\kappa = k(\xi_0, \xi_1)$ solving*

$$\pi^1_{\xi_1}(\kappa, \kappa) - \pi^0_{\xi_0}(\kappa, \kappa) = 0 \tag{8}$$

for each $(\xi_0, \xi_1) \in \Xi \times \Xi$, and that $k : \Xi \times \Xi \to \mathbb{R}$ is bounded. Then, $s[\kappa^*]$ is the (essentially) unique strategy surviving iterated deletion of strictly interim-dominated strategies, where

$$\kappa^* = \min_{\xi_0 \in \Xi} \max_{\xi_1 \in \Xi} k(\xi_0, \xi_1) = \max_{\xi_1 \in \Xi} \min_{\xi_0 \in \Xi} k(\xi_0, \xi_1). \tag{9}$$

For example, suppose that $\pi^0_\xi(\kappa, \kappa)$ is independent of ξ, which is typically the case when action 0 is a safe action.[16] Then, we can use a single-prior game as a fictitious game because $\pi^1_{\xi_1}(\kappa, \kappa) - \pi^0_{\xi_0}(\kappa, \kappa) = \pi^1_{\xi_1}(\kappa, \kappa) - \pi^0_{\xi_1}(\kappa, \kappa)$. Hence, if each single-prior game with $\xi \in \Xi$ has a unique equilibrium, (u, Ξ) also has a unique equilibrium, whose cutoff coincides with the maximum of the equilibrium cutoffs of the single-prior games with $\xi \in \Xi$ by (9). As argued by Carlsson and van Damme (1993) and Morris and Shin (2002), each single-prior game has a unique equilibrium in the following two typical cases: the variance of ε_i is sufficiently small, which is assumed in Claim 1, and the variance of θ is sufficiently large, which will be assumed in Section 4.

More generally, when (u, Ξ) has one or more equilibria and $\pi^0_\xi(\kappa, \kappa)$ is independent of ξ, the maximum equilibrium cutoff of (u, Ξ) equals the maximum of the *maximum* equilibrium cutoffs of the single-prior games, as shown by the next proposition. Note that if no other equilibrium cutoff is less than the maximum one, (u, Ξ) obviously has a unique equilibrium, which can be the case even if some single-prior game with $\xi \in \Xi$ has multiple equilibria. A similar result holds when $\pi^1_\xi(\kappa, \kappa)$ is independent of ξ, although we formally state it in the case when $\pi^0_\xi(\kappa, \kappa)$ is independent of ξ.

Proposition 3. *Suppose that $c^0(\kappa) = \pi^0_\xi(\kappa, \kappa)$ is independent of ξ. Then, $\kappa^0 \equiv \max_{\xi \in \Xi} \max\{\kappa \mid \pi^1_\xi(\kappa, \kappa) = c^0(\kappa)\}$ is the maximum equilibrium cutoff in (u, Ξ). If $\min_{\xi \in \Xi} \pi^1_\xi(\kappa, \kappa) < c^0(\kappa)$ for all $\kappa < \kappa^0$, then $s[\kappa^0]$ is the (essentially) unique strategy surviving iterated deletion of strictly interim-dominated strategies.*

One of the implications is the following corollary, which generalizes the first half of Claim 2.

[16] Even if no action is a safe action, $\pi^0_\xi(\kappa, \kappa)$ can be independent of ξ, which is the case in the bank run model of Goldstein and Pauzner (2005). See Section 5.1.

Corollary 4. *Suppose that* $c^0(\kappa) = \pi^0_\xi(\kappa,\kappa)$ *is independent of* ξ *and that* $(u, \{\xi^0\})$ *has a unique switching equilibrium for some* $\xi^0 \in \arg\max_{\xi\in\Xi} \max\{\kappa \mid \pi^1_\xi(\kappa,\kappa) = c^0(\kappa)\}$*. Then,* $s[\kappa^0]$ *is the (essentially) unique strategy surviving iterated deletion of strictly interim-dominated strategies.*

This corollary focuses on a single-prior game $(u, \{\xi^0\})$ with $\xi^0 \in \Xi$ such that its maximum equilibrium cutoff equals κ^0, i.e., the maximum of the maximum equilibrium cutoffs of the single-prior games. It says that if $(u, \{\xi^0\})$ has a unique equilibrium, then (u, Ξ) also has a unique equilibrium with the same cutoff κ^0. This is an immediate consequence of Proposition 3 because a solution to $\pi^1_{\xi^0}(\kappa,\kappa) - \pi^0_{\xi^0}(\kappa,\kappa) = 0$ is unique, and thus $\min_{\xi\in\Xi} \pi^1_\xi(\kappa,\kappa) - \pi^0_\xi(\kappa,\kappa) \le \pi^1_{\xi^0}(\kappa,\kappa) - \pi^0_{\xi^0}(\kappa,\kappa) < 0$ for all $\kappa < \kappa^0$.

Recall that, in the example of Section 2, $(u, [\underline{\xi}, \overline{\xi}])$ has a unique equilibrium for arbitrary $\underline{\xi}$ if $y < 1/2$ and $\overline{\xi} > \xi^*$. Corollary 4 explains why: a single-prior game $(u, \{\overline{\xi}\})$ has a unique equilibrium if $\overline{\xi} > \xi^*$, and its cutoff equals the maximum of the maximum equilibrium cutoffs of the single-prior games if $y < 1/2$, which implies that $(u, [\underline{\xi}, \overline{\xi}])$ has a unique equilibrium by Corollary 4.

We can also generalize the second half of Claim 2. Let $\Xi \equiv [\underline{\xi}, \overline{\xi}] \subset \mathbb{R}_{++}$, where $\xi \in \Xi$ is the precision of a noise term in a private signal. Assume that $q_\xi(\varepsilon_i)$ has a support $\mathbb{R}$ and satisfies $\int \varepsilon_i q_\xi(\varepsilon_i) d\varepsilon_i = 0$, $\int \varepsilon_i^2 q_\xi(\varepsilon_i) d\varepsilon_i = 1/\xi^2$, $q_\xi(\varepsilon_i) = q_\xi(-\varepsilon_i)$, and $q_\xi(\varepsilon_i) \le q_\xi(\varepsilon_i')$ if $\varepsilon_i \le \varepsilon_i' < 0$. Let action 0 be a safe and ex-ante Laplacian action; that is, $u(1, l, \theta) = c^0 \in \mathbb{R}$ for each l and θ and $c^0 > \int_{-\infty}^{\infty} u(1, 1/2, \theta) p(\theta) d\theta$, where $\int_{-\infty}^{\infty} u(1, 1/2, \theta) p(\theta) d\theta$ is the ex-ante expected payoff to action 1 when a player has the uniform belief over the opponents' actions. The next proposition shows that the minimum equilibrium cutoff can be arbitrarily large if the minimum precision $\underline{\xi}$ is sufficiently small, and there exists a unique equilibrium if a single-prior game with sufficiently small ξ has at most one switching equilibrium with a sufficiently large cutoff.

Proposition 5. *Consider* $(u, [\underline{\xi}, \overline{\xi}])$ *described above with arbitrarily fixed* $\overline{\xi} > 0$*. Suppose that action* 0 *is a safe and ex-ante Laplacian action. Then, for every* $\overline{\kappa}$*, there exists* $\delta(\overline{\kappa}) > 0$ *such that, if* $\underline{\xi} \le \delta(\overline{\kappa})$*, then the minimum equilibrium cutoff in* $(u, [\underline{\xi}, \overline{\xi}])$ *is greater than* $\overline{\kappa}$*. In addition, suppose that there exists* $\overline{\kappa}$ *such that* $(u, \{\xi\})$ *has at most one switching equilibrium with a cutoff greater than* $\overline{\kappa}$ *for every* $\xi \le \delta(\overline{\kappa})$*. Then, there exists* $\delta' \in (0, \delta(\overline{\kappa}))$ *such that, if* $\underline{\xi} \le \delta'$*, then* $s[\kappa^0]$ *is the (essentially) unique strategy surviving iterated deletion of strictly interim-dominated strategies in* $(u, [\underline{\xi}, \overline{\xi}])$*, where* κ^0 *is given in Proposition 3.*

Recall that, in the example of Section 2, $(u, [\underline{\xi}, \overline{\xi}])$ has a unique equilibrium for arbitrary $\overline{\xi}$ if $y < 1/2$ and $\underline{\xi}$ is sufficiently small. Proposition 5 explains why: if $y < 1/2$, then action 0 is a safe and ex-ante Laplacian action, and $(u, \{\xi\})$ has exactly one switching equilibrium with a cutoff greater than y for every $\xi > 0$, which implies that $(u, [\underline{\xi}, \overline{\xi}])$ has a unique equilibrium by Proposition 5.

4 Applications to regime change

We apply our main results to global games of regime change (Angeletos et al., 2007), which have applications to financial crises such as currency crises (Morris and Shin, 1998), debt rollover crises (Morris and Shin, 2004), and bank runs (Goldstein and Pauzner, 2005) among others. We study the effect of ambiguous-quality information on the likelihood of currency crises and debt rollover crises and compare it with that of low-quality information studied by Iachan and Nenov (2015).

In a game of regime change, one of two regimes, $R \in \{0, 1\}$, is realized depending upon the proportion of players choosing action 0 and a state θ. More specifically, regime 0 occurs if and only if the proportion of players choosing action 0 is greater than θ, and the payoff differential $u(1, l, \theta) - u(0, l, \theta)$ equals $d_R(\theta)$ if regime $R \in \{0, 1\}$ occurs:

$$u(1, l, \theta) - u(0, l, \theta) = \begin{cases} d_0(\theta) & \text{if } 1 - l \geq \theta, \\ d_1(\theta) & \text{if } 1 - l < \theta, \end{cases}$$

where $d_0(\theta)$ and $d_1(\theta)$ are non-decreasing in θ, bounded, and satisfies $d_0(\theta) < 0 < d_1(\theta)$. Later, we will assume that either action 0 or 1 is a safe action.

A state θ is drawn from the improper uniform distribution over the real line, and a noise term ε_i is drawn from a normal distribution with mean zero and precision ξ. Then, $p_\xi(\theta|x) = \sqrt{\xi}\phi(\sqrt{\xi}(\theta - x))$ and $q_\xi(\varepsilon) = \sqrt{\xi}\phi(\sqrt{\xi}\varepsilon)$, where ϕ is the probability density function of the standard normal distribution. When the state is θ and each player follows $s[\kappa]$, the proportion of players choosing action 0 is $\Phi(\sqrt{\xi}(\kappa - \theta))$, where Φ is the cumulative distribution function of the standard normal distribution. Let $\theta = \theta^*(\kappa, \xi) \in (0, 1)$ be the unique solution to

$$\Phi(\sqrt{\xi}(\kappa - \theta)) = \theta. \tag{10}$$

Then, regime 0 occurs if and only if $\theta \leq \theta^*(\kappa, \xi)$, which is referred to as a regime-change cutoff. We can readily show that $\theta^*(\kappa, \xi)$ is strictly increasing in κ because regime 0 is more likely to occur when more players choose action 0.

A single-prior game $(u, \{\xi\})$ conforms to the global game of regime change in Iachan and Nenov (2015), and it admits a unique switching equilibrium with cutoff $\kappa = k(\xi)$, which is the unique solution to

$$\pi^1_\xi(\kappa,\kappa) - \pi^0_\xi(\kappa,\kappa) = \int_{-\infty}^{\theta^*(\kappa,\xi)} d_0(\theta)\sqrt{\xi}\phi(\sqrt{\xi}(\theta-\kappa))d\theta + \int_{\theta^*(\kappa,\xi)}^{\infty} d_1(\theta)\sqrt{\xi}\phi(\sqrt{\xi}(\theta-\kappa))d\theta = 0. \quad (11)$$

As a special case, assume that $d_0(\theta) = d_0$ and $d_1(\theta) = d_1$ are constant. Then, (11) is reduced to

$$d_0\Phi(\sqrt{\xi}(\theta^*(\kappa,\xi) - \kappa)) + d_1(1 - \Phi(\sqrt{\xi}(\theta^*(\kappa,\xi) - \kappa))) = d_0 + (d_1 - d_0)\theta^*(\kappa,\xi) = 0 \quad (12)$$

by (10), which implies that the regime-change cutoff is a constant $d_0/(d_0 - d_1)$, and $k(\xi) = d_0/(d_0 - d_1) + \Phi^{-1}(d_0/(d_0 - d_1))/\sqrt{\xi}$.

We consider (u, Ξ) with $\Xi = [\underline{\xi}, \bar{\xi}] \subset \mathbb{R}_{++}$ assuming that one of the actions is a safe action.[17] It is readily shown that (u, Ξ) satisfies the condition in Proposition 2. Thus, (u, Ξ) has a unique switching equilibrium with the following cutoff.

$$\kappa^* = \begin{cases} \max_{\xi\in\Xi} k(\xi) & \text{if action 0 is a safe action,} \\ \min_{\xi\in\Xi} k(\xi) & \text{if action 1 is a safe action.} \end{cases} \quad (13)$$

Therefore, if action 0 is a safe action, ambiguous-quality information increases the regime-change cutoff and makes regime 0 more likely because

$$\theta^*(\kappa^*, \xi) = \theta^*(\max_{\xi'\in\Xi} k(\xi'), \xi) = \max_{\xi'\in\Xi} \theta^*(k(\xi'), \xi) \geq \theta^*(k(\xi), \xi) \text{ for each } \xi \in \Xi. \quad (14)$$

Similarly, if action 1 is a safe action, ambiguous-quality information decreases the regime-change cutoff and makes regime 1 more likely because

$$\theta^*(\kappa^*, \xi) = \theta^*(\min_{\xi'\in\Xi} k(\xi'), \xi) = \min_{\xi'\in\Xi} \theta^*(k(\xi'), \xi) \leq \theta^*(k(\xi), \xi) \text{ for each } \xi \in \Xi. \quad (15)$$

Note that low-quality information does not always have a similar effect in a single-prior game. For example, if $d_0(\theta)$ and $d_1(\theta)$ are constant, the regime-change cutoff is also constant, so information quality has no influence.

[17] We can relax this assumption. See Section 5.1.

To elaborate on the difference between ambiguous quality and low quality, we use the result of Iachan and Nenov (2015). They find that the effect of low-quality information in $(u, \{\xi\})$ is determined by the sensitivity of $d_0(\theta)$ and $d_1(\theta)$ with respect to θ and obtain the following result.[18]

Proposition 6. *If $d_0(\theta)$ is constant, $\theta^*(k(\xi), \xi)$ is increasing in ξ. If $d_1(\theta)$ is constant, $\theta^*(k(\xi), \xi)$ is decreasing in ξ. If $d_0(\theta)$ and $d_1(\theta)$ are constant, $\theta^*(k(\xi), \xi)$ is independent of ξ.*

Suppose that one of the regimes has a constant payoff differential and one of the actions is a safe action. Then, we can determine and compare the effects of low-quality information and ambiguous-quality information. For example, if $d_1(\theta)$ is constant, then low-quality information increases the regime-change cutoff. Thus, if action 0 is a safe action, ambiguous-quality information and low-quality information have a similar effect. However, if action 1 is a safe action, they have opposite effects, which is the case in the model of currency crises, as is discussed next.

Currency crises

We study the effect of ambiguous-quality information on the likelihood of currency crises using the currency attack model of Morris and Shin (1998). Speculators decide whether to attack the currency by selling it short. The current value of the currency is e^*. If an attack is successful, the currency collapses and floats to the shadow rate $f(\theta) < e^*$, where θ is the state of fundamentals and $f(\theta)$ is increasing in θ. There is a fixed transaction cost $t \in (0, e^* - f(\theta))$ of attacking. Thus, the net payoff to a successful attack is $e^* - f(\theta) - t$, while that to an unsuccessful attack is $-t$. An attack is successful if and only if the proportion of speculators attacking the currency is greater than θ. Writing 1 for the action not to attack (and 0 for that to attack), we have the following payoff function:

$$u(a, l, \theta) = \begin{cases} 0 & \text{if } a = 1, \\ e^* - f(\theta) - t & \text{if } a = 0 \text{ and } 1 - l \geq \theta, \\ -t & \text{if } a = 0 \text{ and } 1 - l < \theta, \end{cases} \tag{16}$$

where l is the proportion of the opponents not attacking the currency.

[18] Iachan and Nenov (2015) obtain more general results without assuming that either $d_0(\theta)$ or $d_1(\theta)$ is constant.

Consider a game of regime change with (16), where $d_0(\theta) = -(e^* - f(\theta) - t)$, $d_1(\theta) = t$, and a currency crisis corresponds to regime 0. Because action 1 is a safe action, the regime-change cutoff is given by (15). Thus, ambiguous-quality information decreases the regime-change cutoff and makes a currency crisis less likely. Contrastingly, by Proposition 6, low-quality information increases the regime-change cutoff and makes a currency crisis more likely because $d_1(\theta)$ is constant.

Debt rollover crises

We study the effect of ambiguous-quality information on the likelihood of debt rollover crises using the creditor coordination model of Morris and Shin (2004). Creditors hold a loan secured on collateral and decide whether to roll over the loan. A creditor who rolls over the loan receives 1 if an underlying investment project succeeds, and receives 0 if the project fails. A creditor who does not roll over the loan receives the value of the collateral $\lambda \in (0, 1)$. The project succeeds if and only if the proportion of creditors not rolling over the loan is less than $\theta \in \mathbb{R}$. Writing 1 for the action to roll over (and 0 for that not to roll over), we have the following payoff function:

$$u(a, l, \theta) = \begin{cases} \lambda & \text{if } a = 0, \\ 1 & \text{if } a = 1 \text{ and } 1 - l < \theta, \\ 0 & \text{if } a = 1 \text{ and } 1 - l \geq \theta, \end{cases} \tag{17}$$

where l is the proportion of creditors rolling over the loan.

Consider a game of regime change with (17), where $d_0(\theta) = -\lambda$, $d_1(\theta) = 1 - \lambda$, and a debt rollover crisis corresponds to regime 0. Because action 0 is a safe action, the regime-change cutoff is given by (14). Thus, ambiguous-quality information increases the regime-change cutoff and makes a debt rollover crisis more likely. Contrastingly, low-quality information has no influence on the regime-change cutoff because $d_0(\theta)$ and $d_1(\theta)$ are constant and thus the regime-change cutoff in a single-prior game $(u, \{\xi\})$ is $\theta^*(k(\xi), \xi) = \lambda$ by (12).

We can obtain the regime-change cutoff in (u, Ξ) using (12), (13), and (14):[19]

$$\max_{\xi' \in \Xi} \theta^*(k(\xi'), \xi) = \max_{\xi' \in \Xi} \theta^*(\lambda + \Phi^{-1}(\lambda)/\xi'^{1/2}, \xi) = \begin{cases} \theta^*(\lambda + \Phi^{-1}(\lambda)/\overline{\xi}^{1/2}, \xi) & \text{if } \lambda < 1/2, \\ \theta^*(\lambda + \Phi^{-1}(\lambda)/\underline{\xi}^{1/2}, \xi) & \text{if } \lambda > 1/2 \end{cases}$$

because $\Phi^{-1}(\lambda) \gtrless 0$ if and only if $\lambda \gtrless 1/2$, where we interpret $\xi \in \Xi$ as the true precision. We can also evaluate the range of the regime-change cutoff over all $\Xi \subset \mathbb{R}_{++}$ and $\xi \in \Xi$, which is useful to study under what conditions ambiguous information triggers the crisis. Denote the infimum and the supremum of the regime-change cutoff by $\underline{\theta}^*$ and $\overline{\theta}^*$, respectively. Then, $\underline{\theta}^* = \theta^*(k(\xi), \xi) = \lambda$ by (14), which is the minimum achieved at $\Xi = \{\xi\}$. If $\lambda < 1/2$,

$$\overline{\theta}^* = \sup_{\overline{\xi} \geq \xi \geq \underline{\xi} > 0} \theta^*(\lambda + \Phi^{-1}(\lambda)/\overline{\xi}^{1/2}, \xi) = \sup_{\xi > 0} \lim_{\overline{\xi} \to \infty} \theta^*(\lambda + \Phi^{-1}(\lambda)/\overline{\xi}^{1/2}, \xi) = \theta^*(\lambda, 0) = 1/2$$

since $\lambda < \theta^*(\lambda, \xi) < 1/2$ for all $\xi > 0$.[20] Thus, the regime-change cutoff is close to the supremum if the maximum of Ξ is very large and both the minimum of Ξ and the true precision is very small. If $\lambda > 1/2$,

$$\overline{\theta}^* = \sup_{\overline{\xi} \geq \xi \geq \underline{\xi} > 0} \theta^*(\lambda + \Phi^{-1}(\lambda)/\underline{\xi}^{1/2}, \xi) = \lim_{\underline{\xi} \to 0} \theta^*(\lambda + \Phi^{-1}(\lambda)/\underline{\xi}^{1/2}, \xi) = 1$$

since $\theta^*(\kappa, \xi) < 1$ for all $\kappa \in \mathbb{R}$ and $\xi > 0$. Thus, the regime-change cutoff is close to the supremum if the minimum of Ξ is very small. In summary, $\overline{\theta}^* = 1/2$ if $\lambda < 1/2$, and $\overline{\theta}^* = 1$ if $\lambda > 1/2$.[21]

We can partition the space of fundamentals into three intervals: $(-\infty, \underline{\theta}^*]$, $(\underline{\theta}^*, \overline{\theta}^*)$, and $[\overline{\theta}^*, \infty)$. If $\theta \in (-\infty, \underline{\theta}^*]$, the crisis occurs; if $\theta \in [\overline{\theta}^*, \infty)$, it does not occur. Suppose that $\theta \in (\underline{\theta}^*, \overline{\theta}^*)$. Then, the crisis does not occur if the width of Ξ is sufficiently small; it occurs either (i) if $\lambda > 1/2$ and the minimum of Ξ is very small, or (ii) if $\lambda < 1/2$, the maximum of Ξ is very large, and both the minimum of Ξ and the true precision is very small.

[19] We assume $\lambda \neq 1/2$ to simplify our discussion.

[20] If $\theta \geq 1/2 > \kappa$, the left-hand side of (10) is strictly less than $1/2$, and if $\theta \leq \kappa < 1/2$, it is greater than or equal to $1/2$, each of which is a contradiction.

[21] This discontinuity is analogous to Dow and Werlang (1992): they study portfolio decisions of an MEU agent and show the existence of an interval of prices within which the agent neither buys nor sells short the asset.

5 Discussion

In this section, we discuss some extensions of our analysis.

5.1 An action yielding a sate-independent payoff and a bank run

In Section 4, we assume that one of the actions is a safe action and θ is drawn from the improper uniform distribution. We can obtain the same results under a slightly weaker requirement. Suppose that action $a \in \{0, 1\}$ yields a state-independent payoff; that is, $u(a, l, \theta) = f(l)$, where $f : [0, 1] \to \mathbb{R}$. Then, $\pi^a_\xi(\kappa, \kappa)$ is independent of ξ and κ because

$$\pi^a_\xi(\kappa, \kappa) = \int_{-\infty}^{\infty} u(a, 1 - Q_\xi(\kappa - \theta), \theta) q_\xi(\kappa - \theta) d\theta = \int_0^1 u(a, l, \kappa - Q_\xi^{-1}(1 - l)) dl = \int_0^1 f(l) dl,$$

where $Q_\xi(\kappa - \theta) = \Phi(\sqrt{\xi}(\kappa - \theta))$ and $q_\xi(\kappa - \theta) = \sqrt{\xi}\phi(\sqrt{\xi}(\kappa - \theta))$. Thus, the discussion in Section 4 remains valid even if a safe action is replaced with an action with a state-independent payoff.

An example with such an action is the global game model of bank runs studied by Goldstein and Pauzner (2005), which is a variant of the Diamond-Dybvig model with noisy private signals. Depositors must decide whether to withdraw money from a bank in period 1 or wait until period 2. If a depositor withdraws money in period 1, the bank pays a fixed amount of money to him until it runs out of money, following a sequential-service constraint. Thus, the payoff to early withdrawal is determined solely by depositors' decisions and independent of a state. An argument similar to that in Section 4 shows that ambiguous-quality information makes a bank run more likely. This result also complements that of Iachan and Nenov (2015), who show that low-quality information makes a bank run less likely. These results together imply that a bank run is less likely under unambiguous low-quality information.

5.2 Mixed-strategy equilibria

Players are assumed to use pure strategies, which is common in the ambiguity literature. However, we can also consider mixed strategies. Let $\bar{\sigma} : \mathbb{R} \to [0, 1]$ denote a mixed strategy that assigns action 1 to a private signal x with probability $\bar{\sigma}(x)$. Kajii and Ui (2005) consider two equilibrium concepts, a mixed-strategy equilibrium and an equilibrium in belief. A strategy profile in which all players follow $\bar{\sigma}$ is a mixed-strategy equilibrium if the following condition is satisfied: when a player receives a private signal x and the opponents follow $\bar{\sigma}$, the minimum expected payoff to the mixed action $\bar{\sigma}(x)$ is greater than or equal to that to any other mixed action. The same strategy profile is an equilibrium in belief if the following condition is satisfied: when a player receives a private signal x and the opponents follow $\bar{\sigma}$, if $\bar{\sigma}(x) > 0$, then the minimum expected payoff to action 1 is greater than or equal to that to action 0, and if $\bar{\sigma}(x) < 1$, then the minimum expected payoff to action 0 is greater than or equal to that to action 1.

A pure-strategy equilibrium in this paper satisfies the condition of an equilibrium in belief, but it may not satisfy that of a mixed-strategy equilibrium because ambiguity-averse players may prefer objective randomization.[22] However, if one of the actions is a safe action, a pure-strategy equilibrium is also a mixed strategy-equilibrium. To see this, suppose that action 0 is a safe action with $u(0, l, \theta) = c_0 \in \mathbb{R}$. If $\sigma : \mathbb{R} \to \{0, 1\}$ is a pure-strategy equilibrium, then $\min_{\xi\in\Xi} E_\xi[u(\sigma(x), E_\xi[\sigma|\theta], \theta)|x] \geq \min_{\xi\in\Xi} pE_\xi[u(1, E_\xi[\sigma|\theta], \theta)|x] + (1 - p)c_0$ for all $x \in \mathbb{R}$ and $p \in [0, 1]$, so σ is also a mixed-strategy equilibrium. This implies that the discussions in Sections 2 and 4 and the propositions except Proposition 1 remain valid even if we adopt a mixed-strategy equilibrium as an equilibrium concept.

5.3 Heterogeneous information quality

In Sections 2 and 4, each player is assumed to believe that the probability distribution of a private signal is the same for all the players. Laskar (2014) makes this observation based upon an earlier version of this paper (Ui, 2009)[23] and studies a linear-normal global game with two MEU players whose private signals may follow different probability distributions, where the expected value of a private signal is ambiguous.

On the other hand, the general model in Section 3 allows the probability distributions to be different across players, for which the main results remain valid. For example, we can consider a game of regime change (u, Ξ) with $\Xi = \Xi_1 \times \Xi_2$, where $\xi_1 \in \Xi_1$ and $\xi_2 \in \Xi_2$ are the precision of a noise term in a player's private signal and that in his opponents' private signals, respectively. When the state is θ and each opponent follows $s[\kappa]$ in $(u, \{\xi\})$ with $\xi = (\xi_1, \xi_2)$, the proportion of the opponents choosing action 0 is $\Phi(\sqrt{\xi_2}(\kappa - \theta))$, so the regime-change cutoff in $(u, \{\xi\})$ is

[22] See Section 4.3 of Kajii and Ui (2005). This property plays an essential in Ellis (2016).

[23] This contains the linear example and the models of financial crises together with some of the main results.

$\theta^*(\kappa, \xi_2)$ by (10). Thus, the equilibrium cutoff in $(u, \{\xi\})$ solves

$$\pi^1_\xi(\kappa,\kappa) - \pi^0_\xi(\kappa,\kappa) = \int_{-\infty}^{\theta^*(\kappa,\xi_2)} d_0(\theta)\sqrt{\xi_1}\phi(\sqrt{\xi_1}(\theta-\kappa))d\theta + \int_{\theta^*(\kappa,\xi_2)}^{\infty} d_1(\theta)\sqrt{\xi_1}\phi(\sqrt{\xi_1}(\theta-\kappa))d\theta = 0$$

because $p_\xi(\theta|x) = \sqrt{\xi_1}\phi(\sqrt{\xi_1}(\theta - x))$. Then, the equilibrium cutoff in (u, Ξ) is given by (13), and the same discussion goes through even with heterogeneous information quality.

5.4 Monotone payoff differentials

The assumptions A1 and A2 require that $u(1, l, \theta)$ and $u(0, l, \theta)$ be increasing and decreasing in (l, θ), respectively. They guarantee the monotone comparative statics in Lemmas 1 and 2, while monotonicity of $u(1, l, \theta) - u(0, l, \theta)$ suffices in single-prior games. This is a limitation of our analysis because we cannot apply the main results to the class of global games with monotone payoff differentials that do not satisfy A1 and A2. We will be able to overcome this limitation by using recent ideas of Dziewulski and Quah (2021). Dziewulski and Quah (2021) introduce the notion of first-order stochastic dominance for sets of priors. If the set of interim beliefs with a higher private signal first-order stochastically dominates that with a lower private signal in the sense of Dziewulski and Quah (2021), then monotonicity of payoff differentials guarantees the monotone comparative statics in Lemmas 1 and 2. Such an extension of our analysis is left as a promising direction for future research.

5.5 Smooth ambiguity preferences

As a model with more general ambiguity-averse preferences, we can consider global games with smooth ambiguity preferences (Klibanoff et al., 2005). To briefly illustrate such a model, let $\Xi = [\underline{\xi}, \overline{\xi}]$ be a closed and bounded interval. Assume that when the opponents follow $s[\kappa]$, a player with a private signal x prefers action a to action a' if and only if

$$\frac{1}{\overline{\xi} - \underline{\xi}} \int_{\underline{\xi}}^{\overline{\xi}} \phi_\alpha\left(\pi^a_\xi(x,\kappa)\right) d\xi \geq \frac{1}{\overline{\xi} - \underline{\xi}} \int_{\underline{\xi}}^{\overline{\xi}} \phi_\alpha(\pi^{a'}_\xi(x,\kappa))d\xi,$$

where $\phi_\alpha(x) \equiv -\frac{1}{\alpha}e^{-\alpha x}$ with $\alpha > 0$. The constant α is the coefficient of ambiguity aversion, and it is a measure of ambiguity attitude, while Ξ is a measure of ambiguity perception. This model converges to the MEU model as α approaches infinity. Thus, the effects of ambiguous information studied in this paper are understood as those of ambiguity perception when the coefficient of ambiguity aversion is extremely large. Even if α is finite, we can readily show this model exhibits strategic complementarities in terms of smooth ambiguity preferences, and we can conduct a similar analysis, where the effects of ambiguous information should be weaker than those in the MEU model. We leave a further study of this model as a future research topic, where this paper's results serve as a benchmark.

6 Conclusion

In our analysis, we exploit both similarities and differences between a multiple-priors game and a single-prior game. A multiple-priors game inherits strategic complementarities from a single-prior game, but the payoff differential alone does not determine the best responses in a multiple-priors game. Notably, a safe action plays a special role, e.g., if action 0 is a safe action, the maximum equilibrium cutoff coincides with the maximum of the maximum equilibrium cutoffs of all the fictitious single-prior games. This result is a consequence of the inherited strategic complementarities, which leads us to the following findings. Ambiguous-quality information and low-quality information can have opposite effects on the equilibrium outcomes, the number of equilibria, and the likelihood of financial crises.

We have focused on exogenously fixed ambiguity. However, we can extend our framework to study a social planner's decision problem who is interested in implementing desirable outcomes by engineering ambiguity, analogous to Bose and Renou (2014) and Di Tillio et al. (2017). Such a problem can be referred to as ambiguous information design,[24] a many player generalization of ambiguous persuasion described by Beauchêne et al. (2019). Beauchêne et al. (2019) study a Bayesian persuasion problem (Kamenica and Gentzkow, 2011) with an MEU receiver, where a sender can choose an ambiguous communication device. One of the most successful results in information design is obtained in binary-action supermodular games (e.g., Inostroza and Pavan, 2020; Li et al., 2020; Morris et al., 2020). Thus, its ambiguous version should be an important topic for future research.

Appendix A Proofs of Proposition 1

We first prove Lemmas 1 and 2.

[24] See Bergemann and Morris (2019) for a survey on information design.

Proof of Lemma 1. This function is continuous by Continuity, increasing in x by Action Monotonicity, State Monotonicity, and Stochastic Dominance, and decreasing in κ by Action Monotonicity. □

Proof of Lemma 2. Because $E_\xi[\sigma|\theta] \geq E_\xi[\sigma'|\theta]$, it holds that $u(1, E_\xi[\sigma|\theta], \theta) \geq u(1, E_\xi[\sigma'|\theta], \theta)$ and $u(0, E_\xi[\sigma|\theta], \theta) \leq u(0, E_\xi[\sigma'|\theta], \theta)$ by Action Monotonicity, which implies this lemma. □

We also use the following lemma, which ensures that action 0 is a dominant action when a private signal is sufficiently low, and action 1 is a dominant action when a private signal is sufficiently high.

Lemma A. *There exist* $\underline{x}, \overline{x} \in \mathbb{R}$ *such that* $\min_{\xi\in\Xi} \pi^1_\xi(x,\kappa) - \min_{\xi\in\Xi} \pi^0_\xi(x,\kappa) < 0$ *for all* $x \leq \underline{x}$ *and* $\kappa \in \mathbb{R}$ *and* $\min_{\xi\in\Xi} \pi^1_\xi(x,\kappa) - \min_{\xi\in\Xi} \pi^0_\xi(x,\kappa) > 0$ *for all* $x \geq \overline{x}$ *and* $\kappa \in \mathbb{R}$.

Proof. We prove the existence of $\underline{x}$ (we can prove that of $\overline{x}$ similarly). For $\xi^0 \in \arg\min_{\xi\in\Xi} \pi^0_\xi(x,\kappa)$, which exists by Continuity and the compactness of Ξ,

$$\min_{\xi\in\Xi} \pi^1_\xi(x,\kappa) - \min_{\xi\in\Xi} \pi^0_\xi(x,\kappa) \leq \pi^1_{\xi^0}(x,\kappa) - \pi^0_{\xi^0}(x,\kappa) \leq \max_{\xi\in\Xi} E_\xi[u(1,1,\theta) - u(0,1,\theta)|x]$$

by Action Monotonicity. Thus, it is enough to show that $\lim_{x\to-\infty} \max_{\xi\in\Xi} E_\xi[u(1,1,\theta)-u(0,1,\theta)|x] < 0$, which is true if

$$\lim_{x\to-\infty} E_\xi[u(1,1,\theta) - u(0,1,\theta)|x] < 0 \tag{A.1}$$

for each ξ by Dini's theorem because $E_\xi[u(1,1,\theta) - u(0,1,\theta)|x]$ is increasing in x by State Monotonicity and Stochastic Dominance, and $\xi \mapsto E_\xi[u(1,1,\theta) - u(0,1,\theta)|x]$ is continuous on a compact set Ξ.

Let $\varepsilon = -(u(1,1,\underline{\theta}) - u(0,1,\underline{\theta})) > 0$, where $\underline{\theta} \in \mathbb{R}$ is given in Limit Dominance. Note that $u(1,1,\theta) - u(0,1,\theta) \leq -\varepsilon$ for all $\theta \leq \underline{\theta}$ by State Monotonicity, and thus

$$E_\xi[u(1,1,\theta) - u(0,1,\theta)|x] \leq -\varepsilon \int_{-\infty}^{\underline{\theta}} p_\xi(\theta|x)d\theta + \int_{\underline{\theta}}^{\infty} (u(1,1,\theta) - u(0,1,\theta))p_\xi(\theta|x)d\theta. \tag{A.2}$$

Then, (A.1) holds for the following reason. First, by Limit Dominance,

$$\lim_{x\to-\infty} \left(-\varepsilon \int_{-\infty}^{\underline{\theta}} p_\xi(\theta|x)d\theta\right) = -\varepsilon. \tag{A.3}$$

Next, for arbitrary $x' \in \mathbb{R}$, there exists $\hat{\theta} > \underline{\theta}$ such that $\int_{\hat{\theta}}^{\infty} (u(1,1,\theta) - u(0,1,\theta))p_\xi(\theta|x')d\theta < \varepsilon/2$, which implies that

$$\lim_{x\to-\infty}\int_{\hat\theta}^{\infty}(u(1,1,\theta)-u(0,1,\theta))p_\xi(\theta|x)d\theta \le \int_{\hat\theta}^{\infty}(u(1,1,\theta)-u(0,1,\theta))p_\xi(\theta|x')d\theta < \varepsilon/2$$

by State Monotonicity and Stochastic Dominance. Because

$$\lim_{x\to-\infty}\left|\int_{\underline{\theta}}^{\hat\theta}(u(1,1,\theta)-u(0,1,\theta))p_\xi(\theta|x)d\theta\right| \le c\lim_{x\to-\infty}\int_{\underline{\theta}}^{\hat\theta}p_\xi(\theta|x)d\theta = 0$$

by Limit Dominance, where $c = \max_{\theta\in[\underline{\theta},\hat\theta]}|u(1,1,\theta)-u(0,1,\theta)|$, we have

$$\lim_{x\to-\infty}\int_{\underline{\theta}}^{\infty}(u(1,1,\theta)-u(0,1,\theta))p_\xi(\theta|x)d\theta < \varepsilon/2. \tag{A.4}$$

Therefore, (A.1) holds by (A.2), (A.3), and (A.4). □

Using the above three lemmas, we can prove Proposition 1 by a standard argument (e.g., Morris and Shin, 2002), so we provide only a sketch of the proof (the full proof is available upon request). Let $\Sigma_n \equiv \{\sigma \mid s[\overline{\kappa}_n](x) \le \sigma(x) \le s[\underline{\kappa}_n](x)\}$, where $\underline{\kappa}_0 = -\infty$ and $\overline{\kappa}_0 = \infty$, and $\underline{\kappa}_n$ and $\overline{\kappa}_n$ are defined inductively by

$$\underline{\kappa}_{n+1} = \min\{x\in\mathbb{R} \mid \min_{\xi\in\Xi}\pi^1_\xi(x,\underline{\kappa}_n) - \min_{\xi\in\Xi}\pi^0_\xi(x,\underline{\kappa}_n) = 0\},$$
$$\overline{\kappa}_{n+1} = \max\{x\in\mathbb{R} \mid \min_{\xi\in\Xi}\pi^1_\xi(x,\overline{\kappa}_n) - \min_{\xi\in\Xi}\pi^0_\xi(x,\overline{\kappa}_n) = 0\}.$$

Then, we can show by induction that Σ_n is the set of strategies surviving n rounds of iterated deletion of strictly interim-dominated strategies. Furthermore, we can show that $\underline{\kappa} = \lim_{n\to\infty}\underline{\kappa}_n$ and $\overline{\kappa} = \lim_{n\to\infty}\overline{\kappa}_n$, which implies Proposition 1.

Appendix B Proof of Proposition 2

We use the following lemma.

Lemma B. *Let $f : \mathbb{R}\times\Xi \to \mathbb{R}$ be a continuous function, where Ξ is a compact set. Assume that, for each $\xi\in\Xi$, $f(x,\xi) = 0$ has a unique solution $x^*(\xi)$ such that $f(x,\xi) < 0$ if and only if $x < x^*(\xi)$, and $x^*(\xi)$ is bounded over Ξ. Then, $\max_{\xi\in\Xi} x^*(\xi)$ and $\min_{\xi\in\Xi} x^*(\xi)$ exist, and they are unique solutions of $\min_{\xi\in\Xi} f(x,\xi) = 0$ and $\max_{\xi\in\Xi} f(x,\xi) = 0$, respectively.*

Proof. If $x^*(\xi)$ is continuous, $\max_{\xi\in\Xi} x^*(\xi)$ and $\min_{\xi\in\Xi} x^*(\xi)$ exist because Ξ is compact. So we first show that $x^*(\xi)$ is continuous. Note that, for any convergent sequence $\{\xi_k\}_{k=1}^{\infty}$ with a limit $\bar{\xi}$, there exists a subsequence $\{\xi_{k_l}\}_{l=1}^{\infty}$ such that $\lim_{l\to\infty} x^*(\xi_{k_l}) = \bar{x}$ for some $\bar{x}$ because $\{x^*(\xi_k)\}_{k=1}^{\infty}$ is a bounded sequence. Then, $\lim_{l\to\infty} f(x^*(\xi_{k_l}), \xi_{k_l}) = f(\bar{x}, \bar{\xi}) = 0$, and thus $\bar{x} = x^*(\bar{\xi})$. That is, the limit of any convergent subsequence of $\{x^*(\xi_k)\}_{k=1}^{\infty}$ is $\bar{x}$, which implies that $\lim_{k\to\infty} x^*(\xi_k) = \bar{x}$.

We show that $\overline{x}^* \equiv \max_{\xi\in\Xi} x^*(\xi)$ is a unique solution to $\min_{\xi\in\Xi} f(x,\xi) = 0$. Note that $f(x,\xi) > 0$ for all $x > \overline{x}^* \geq x(\xi)$ and $\xi \in \Xi$, which implies that $\min_{\xi\in\Xi} f(x,\xi) > 0$ for all $x > \overline{x}^*$ because $f(x,\xi)$ is continuous in ξ and Ξ is compact. Note also that $f(x,\xi^*) < 0$ for all $x < \overline{x}^*$ if $x^*(\xi^*) = \overline{x}^*$, which implies that $\min_{\xi\in\Xi} f(x,\xi) \leq f(x,\xi^*) < 0$ for all $x < \overline{x}^*$. Hence, $\overline{x}^*$ is the unique solution to $\min_{\xi\in\Xi} f(x,\xi) = 0$ because $\min_{\xi\in\Xi} f(x,\xi)$ is continuous in x (by the maximum theorem). We can similarly show that $\min_{\xi\in\Xi} x^*(\xi)$ is a unique solution to $\max_{\xi\in\Xi} f(x,\xi) = 0$. □

Proof of Proposition 2. By Proposition 1, it suffices to prove that (9) is a unique solution to (7). Let $f(\kappa,\xi_0,\xi_1) \equiv \pi^1_{\xi_1}(\kappa,\kappa) - \pi^0_{\xi_0}(\kappa,\kappa)$. Then, (7) is written as $\min_{\xi_1}\max_{\xi_0} f(\kappa,\xi_0,\xi_1) = \max_{\xi_0}\min_{\xi_1} f(\kappa,\xi_0,\xi_1) = 0$.

We first show that if $\kappa < k(\xi_0,\xi_1)$, then $f(\kappa,\xi_0,\xi_1) < 0$. Let $\kappa' < \min\{\underline{x}, \inf_{\xi_0,\xi_1} k(\xi_0,\xi_1)\}$, where $\underline{x}$ is given in Lemma A. Then, it suffices to show $f(\kappa',\xi_0,\xi_1) < 0$ because $k(\xi_0,\xi_1)$ is a unique solution to $f(\kappa,\xi_0,\xi_1) = 0$. Note that $\max_{\xi_0}\min_{\xi_1} f(\kappa',\xi_0,\xi_1) < 0$ by Lemma A, so there exist $\xi_0', \xi_1' \in \Xi$ such that $f(\kappa',\xi_0',\xi_1') < 0$. Note also that $f(\kappa',\xi_0,\xi_1) \neq 0$ for all $\xi_0,\xi_1 \in \Xi$ because $\kappa' < \inf_{\xi_0,\xi_1} k(\xi_0,\xi_1)$. Hence, we must have $f(\kappa',\xi_0,\xi_1) < 0$ for all $\xi_0,\xi_1 \in \Xi$; otherwise, there would exist $\xi_0,\xi_1 \in \Xi$ such that $f(\kappa',\xi_0,\xi_1) = 0$ by the intermediate value theorem. Similarly, we can show that if $\kappa > k(\xi_0,\xi_1)$, then $f(\kappa,\xi_0,\xi_1) > 0$.

By the above argument, we can apply Lemma B to the function $(\kappa,\xi_0) \mapsto f(\kappa,\xi_0,\xi_1)$ for each ξ_1; that is, $\min_{\xi_0} k(\xi_0,\xi_1)$ is the unique solution of $\max_{\xi_0} f(\kappa,\xi_0,\xi_1) = 0$. Note that $\max_{\xi_0} f(\kappa,\xi_0,\xi_1) < 0$ if and only if $\kappa < \min_{\xi_0} k(\xi_0,\xi_1)$ (see the proof of Lemma B). Thus, we can again apply Lemma B to the function $(\kappa,\xi_1) \mapsto \max_{\xi_0} f(\kappa,\xi_0,\xi_1)$; that is, $\max_{\xi_1}\min_{\xi_0} k(\xi_0,\xi_1)$ is the unique solution of $\min_{\xi_1}\max_{\xi_0} f(\kappa,\xi_0,\xi_1) = 0$. Similarly, $\min_{\xi_0}\max_{\xi_1} k(\xi_0,\xi_1)$ is the unique solution of $\max_{\xi_0}\min_{\xi_1} f(\kappa,\xi_0,\xi_1) = \min_{\xi_1}\max_{\xi_0} f(\kappa,\xi_0,\xi_1) = 0$. □

Appendix C Proof of Proposition 3

Proof of Proposition 3. If $\kappa > \kappa^0$, then $\pi^1_\xi(\kappa, \kappa) - c^0(\kappa) > 0$ for all $\xi \in \Xi$ because $\pi^1_\xi(\kappa, \kappa) - c^0(\kappa) \neq 0$ and $\pi^1_\xi(x, x) - c^0(x) > 0$ for all $x > \overline{x}$, where $\overline{x}$ is given in Lemma A. Thus, if $\kappa > \kappa^0$, then $\min_\xi \pi^1_\xi(\kappa, \kappa) - c^0(\kappa) > 0$ by Continuity and compactness of Ξ. On the other hand, $\min_\xi \pi^1_\xi(\kappa^0, \kappa^0) - c^0(\kappa^0) \leq \pi^1_{\xi^0}(\kappa^0, \kappa^0) - c^0(\kappa^0) = 0$ for $\xi^0 \in \arg\max_{\xi \in \Xi} \max\{\kappa \mid \pi^1_\xi(\kappa, \kappa) = c^0(\kappa)\}$. Because $\min_\xi \pi^1_\xi(\kappa, \kappa) - c^0(\kappa)$ is continuous in κ (by Continuity and the maximum theorem), κ^0 is the maximum solution to $\min_\xi \pi^1_\xi(\kappa, \kappa) - c^0(\kappa) = 0$, i.e., the maximum equilibrium cutoff in (u, Ξ). If $\min_\xi \pi^1_\xi(\kappa, \kappa) < c^0(\kappa)$ for all $\kappa < \kappa^0$, κ^0 is a unique solution to $\min_\xi \pi^1_\xi(\kappa, \kappa) - c^0(\kappa) = 0$, so $s[\kappa^0]$ is the unique strategy surviving iterated deletion of strictly interim-dominated strategies by Proposition 1. □

Appendix D Proof of Proposition 5

We write $q_\xi(\varepsilon_i) = \sqrt{\xi} q(\sqrt{\xi}\varepsilon_i)$, where q is a probability density function satisfying the following conditions: $\int \varepsilon_i q(\varepsilon_i) d\varepsilon_i = 0$, $\int \varepsilon_i^2 q(\varepsilon_i) d\varepsilon_i = 1$, $q(\varepsilon_i) = q(-\varepsilon_i)$, and $q(\varepsilon_i)$ is increasing in ε_i if $\varepsilon_i < 0$. To prove Proposition 5, we use the following lemma.

Lemma C. *For any $x, \kappa \in \mathbb{R}$, if action 0 is a safe and ex-ante Laplacian action, then*

$$\lim_{\xi \to 0} \pi^1_\xi(x, \kappa) = \lim_{\xi \to 0} \int u(1, E_\xi[s[\kappa] | \theta], \theta) p_\xi(\theta | x) d\theta = \int u(1, 1/2, \theta) p(\theta) d\theta < c^0. \tag{A.5}$$

Thus, for any $\kappa_0, \kappa_1 \in \mathbb{R}$ with $\kappa_0 \leq \kappa_1$, there exists $\delta > 0$ such that if $\xi < \delta$ then $\pi^1_\xi(\kappa, \kappa) \leq \pi^1_\xi(\kappa_1, \kappa_0) < c^0$ for all $\kappa \in [\kappa_0, \kappa_1]$.

Proof. It is enough to show the second equality in (A.5). Note that

$$\pi^1_\xi(x, \kappa) = \int u(1, E_\xi[s[\kappa] | \theta], \theta) p_\xi(\theta | x) d\theta = \frac{\int u(1, E_\xi[s[\kappa] | \theta], \theta) p(\theta) q(\sqrt{\xi}(x - \theta)) d\theta}{\int p(\theta) q(\sqrt{\xi}(x - \theta)) d\theta}.$$

Consider the limit of the denominator. By the monotone convergence theorem, $\int p(\theta) q(\sqrt{\xi}(x - \theta)) d\theta \to \int p(\theta) q(0) d\theta = q(0)$ as $\xi \to 0$ because $q(\sqrt{\xi}(\theta - x))$ is decreasing in ξ (recall the assumption that $q(\varepsilon) = q(-\varepsilon)$ and $q(\varepsilon)$ is increasing in ε if $\varepsilon < 0$). Next, consider the limit of the numerator. By the dominated convergence theorem,

$$\int u(1, E_\xi[s[\kappa] | \theta], \theta) p(\theta) q(\sqrt{\xi}(\theta - x)) d\theta \to \int u(1, 1/2, \theta) p(\theta) q(0) d\theta$$

as $\xi \to 0$ because $E_\xi[s[\kappa]|\theta] = \int_\kappa^\infty \sqrt{\xi} q(\sqrt{\xi}(x-\theta))dx = 1 - Q(\sqrt{\xi}(\kappa - \theta)) \to 1/2$, where Q is the cumulative distribution function, and

$$|u(1, E_\xi[s[\kappa]|\theta], \theta)p(\theta)q(\sqrt{\xi}(x-\theta))| \leq (|u(1,1,\theta)| + |u(1,0,\theta)|)p(\theta)q(0),$$

where the right-hand side is integrable. □

Proof of Proposition 5. Fix $\overline{\kappa}$. Let $\underline{x} < \overline{\kappa}$ be such that $\pi^1_{\overline{\xi}}(\kappa,\kappa) < c^0 = \pi^0_{\overline{\xi}}(\kappa,\kappa)$ for all $\kappa < \underline{x}$, which exists by Lemma A. Lemma C implies that there exists $\delta \in (0, \overline{\xi})$ such that if $\xi < \delta$ then $\pi^1_\xi(\kappa,\kappa) < c^0$ for all $\kappa \in [\underline{x}, \overline{\kappa}]$. Let $\underline{\xi} < \delta$. Then,

$$\min_{\xi \in [\underline{\xi}, \overline{\xi}]} \pi^1_\xi(\kappa,\kappa) \leq \begin{cases} \pi^1_{\overline{\xi}}(\kappa,\kappa) < c^0 & \text{if } \kappa < \underline{x}, \\ \pi^1_{\underline{\xi}}(\kappa,\kappa) < c^0 & \text{if } \kappa \in [\underline{x}, \overline{\kappa}]. \end{cases}$$

This implies that every equilibrium cutoff in $(u, [\underline{\xi}, \overline{\xi}])$ is greater than $\overline{\kappa}$.

Let δ be given above. Suppose that, for each $\xi < \delta$, $\pi^1_\xi(\kappa,\kappa) = c^0$ has at most one solution greater than $\overline{\kappa}$. Let $\overline{x} > \overline{\kappa}$ be such that $\min_{\xi \in [\delta, \overline{\xi}]} \pi^1_\xi(\kappa,\kappa) > c^0$ for all $\kappa > \overline{x}$, which exists by Lemma A. By Lemma C, there exists $\delta' \in (0, \delta)$ such that if $\xi < \delta'$ then $\pi^1_\xi(\kappa,\kappa) < c^0$ for all $\kappa \in [\underline{x}, \overline{x}]$. Let $\underline{\xi}' < \delta'$. Then,

$$\min_{\xi \in [\underline{\xi}', \overline{\xi}]} \pi^1_\xi(\kappa,\kappa) \leq \begin{cases} \pi^1_{\overline{\xi}}(\kappa,\kappa) < c^0 & \text{if } \kappa < \underline{x}, \\ \pi^1_{\underline{\xi}'}(\kappa,\kappa) < c^0 & \text{if } \kappa \in [\underline{x}, \overline{x}], \end{cases} \tag{A.6}$$

so $\kappa^0 = \max_{\xi \in [\underline{\xi}', \overline{\xi}]} \max\{\kappa \mid \pi^1_\xi(\kappa,\kappa) = c^0\} > \overline{x}$ because, by Proposition 3, κ^0 is the maximum solution to $\min_{\xi \in [\underline{\xi}', \overline{\xi}]} \pi^1_\xi(\kappa,\kappa) = c^0$. Let $\xi^0 \in \arg\max_{\xi \in [\underline{\xi}', \overline{\xi}]} \max\{\kappa \mid \pi^1_\xi(\kappa,\kappa) = c^0\}$; that is, $\pi^1_{\xi^0}(\kappa^0,\kappa^0) = c^0$. Then, we must have $\xi^0 < \delta$ because $\kappa^0 > \overline{x}$ implies $\min_{\xi \in [\delta, \overline{\xi}]} \pi^1_\xi(\kappa^0,\kappa^0) > c^0$. Thus, $\pi^1_{\xi^0}(\kappa,\kappa) < c^0$ for all $\kappa \in [\underline{x}, \overline{\kappa}]$ by the definition of δ. Note that $\pi^1_{\xi^0}(\kappa,\kappa) \neq c^0$ for all $\kappa \in (\overline{\kappa}, \kappa^0)$ by the assumption. Accordingly, $\min_{\xi \in [\underline{\xi}', \overline{\xi}]} \pi^1_\xi(\kappa,\kappa) \leq \pi^1_{\xi^0}(\kappa,\kappa) < c^0$ for all $\kappa \in (\overline{\kappa}, \kappa^0)$ by Continuity. This implies that $\min_{\xi \in [\underline{\xi}', \overline{\xi}]} \pi^1_\xi(\kappa,\kappa) < c^0$ for all $\kappa < \kappa^0$ by (A.6). Therefore, by Proposition 3, $s[\kappa^0]$ is the unique strategy surviving iterated deletion of strictly interim-dominated strategies in $(u, [\underline{\xi}', \overline{\xi}])$. □

References

Angeletos, G. M., Hellwig, C., Pavan, A., 2007. Dynamic global games of regime change: learning, multiplicity, and the timing of attacks. Econometrica 75, 711–756.

Angeletos, G. M., Lian, C., 2016. Incomplete information in macroeconomics: accommodating frictions in coordination. In: Taylor, J. B., Uhlig, H. (Eds.), Handbook of Macroeconomics, vol. 2B. Elsevier, pp. 1345-1425.

Ahn, D., 2007. Hierarchies of ambiguous beliefs. J. Econ. Theory 136, 286–301.

Azrieli, Y., Teper, R., 2011. Uncertainty aversion and equilibrium existence in games with incomplete information. Games Econ. Behav. 73, 310–317.

Aumann, R. J., 1976. Agreeing to disagree. Ann. Statist. 4, 1236–1239.

Beauchêne, D., Li, J., Li, M., 2019. Ambiguous persuasion. J. Econ. Theory 179, .312–365.

Bergemann, D., Morris, S., 2019. Information design: A unified perspective. J. Econ. Lit. 57, 44–95.

Bodoh-Creed, A. L., 2012. Ambiguous beliefs and mechanism design. Games Econ. Behav. 75 518–537.

Bose, S., Daripa, A., 2009. A dynamic mechanism and surplus extraction under ambiguity. J. Econ. Theory 144, 2084–2114.

Bose, S., Ozdenoren, E., Pape, A., 2006. Optimal auctions with ambiguity. Theoretical Econ. 1, 411–438.

Bose, S., Renou, L., 2014. Mechanism design with ambiguous communication devices. Econometrica 82, 1853–1872.

Caballero, R. J., Krishnamurthy, A., 2008. Collective risk management in a flight to quality episode. J. Finance 63, 2195–2230.

Calvo, G. A., 1988. Servicing the public debt: the role of expectations. Am. Econ. Rev. 78, 647–661.

Carlsson, H., van Damme, E., 1993. Global games and equilibrium selection. Econometrica 61, 989–1018.

De Castro, L., Yannelis, N. C., 2018. Uncertainty, efficiency and incentive compatibility: ambiguity solves the conflict between efficiency and incentive compatibility. J. Econ. Theory 177, 678–707.

Diamond, D., Dybvig, P., 1983. Bank runs, deposit insurance, and liquidity. J. Polit. Economy 91, 401–419.

Di Tillio, A., Kos, N., Messner, M., 2017. The design of ambiguous mechanisms. Rev. Econ. Stud. 84, 237–276.

Dicks, D., Fulghieri, P., 2019. Uncertainty aversion and systemic risk. J. Political Econ. 127, 1118–1155.

Dow, J., Werlang, S., 1992. Uncertainty aversion, risk aversion, and the optimal choice of portfolio. Econometrica 60, 197–204.

Dziewulski, P., Quah, J. K.-H., 2021. Comparative statics with linear objectives: normal demand, monotone marginal costs, and ranking multi-prior beliefs. Working paper.

Ellis, A., 2016. Condorcet meets Ellsberg. Theoretical Econ. 11, 865–895.

Epstein, L. G., 1997. Preference, rationalizability and equilibrium. J. Econ. Theory 73, 1–29.

Epstein, L. G., Wang, T., 1996. “Beliefs about beliefs” without probabilities. Econometrica 64, 1343–1373.

Fabrizi, S., Lippert, S., Pan, A., Ryan, M., 2019. Unanimous jury voting with an ambiguous likelihood. Working paper.

Fagin, R., Halpern, J., 1990. A new approach to updating beliefs. In: Bonissone, P. P., Henrion, M., Kanal, L. N., Lemmer, J. F. (Eds.), Uncertainty in Artificial Intelligence 6. Elsevier, pp. 317–325.

Gilboa, I., Marinacci, M., 2013. Ambiguity and the Bayesian paradigm. In: Acemoglu, D., Arellano, M., Dekel, E. (Eds.), Advances in Economics and Econometrics: Tenth World Congress vol. 1. Cambridge Univ. Press, pp. 179–242.

Gilboa, I., Schmeidler, D., 1989. Maxmin expected utility with a non-unique prior. J. Math. Econ. 18, 141–153.

Goldstein, I., Pauzner, A., 2005. Demand-deposit contracts and the probability of bank runs. J. Finance 60, 1293–1327.

Grant, S., Meneghel, I., Tourky, R., 2015. Savage games. Theoretical Econ. 11, 641–682.

Guo, H., Yannelis, N. C., 2021. Full implementation under ambiguity. Am. Econ. J. Microecon. 13, 148–178.

Hanany, E., Klibanoff, P., Mukerji, S., 2015. Incomplete information games with ambiguity averse players. Amer. Econ. J. Microecon. 12, 135–187.

Harsanyi, J. C., 1967–1968. Games with incomplete information played by Bayesian players. Manage. Sci. 14, 159–182, 320–334, 486–502.

Iachan, F., Nenov, P., 2015. Information quality and crises in regime-change games. J. Econ. Theory 158, 739–768.

Inostroza, N., Pavan, A., 2020. Persuasion in global games with application to stress testing. Working paper.

Jaffray, J.-Y., 1992. Bayesian updating and belief functions. IEEE Trans. Syst. Man Cybern. 22, 1144–1152.

Kajii, A., Ui, T., 2005. Incomplete information games with multiple priors. Japanese Econ. Rev. 56, 332–351.

Kajii, A., Ui, T., 2009. Interim efficient allocations under uncertainty. J. Econ. Theory 144, 337–353.

Kamenica, E., Gentzkow, M., 2011. Bayesian persuasion. Amer. Econ. Rev. 101, 2590–2615.

Kawagoe, T., Ui, T., 2013. Global games and ambiguous information: an experimental study. Working paper.

Klibanoff, P., Marinacci, M., Mukerji, S., 2005. A smooth model of decision making under ambiguity. Econometrica 73, 1849–1892.

Laskar, D., 2014. Ambiguity and perceived coordination in a global game. Econ. Letters 122, 317–320.

Li, F., Song, Y., Zhao, M., 2020. Global manipulation by local obfuscation. Working paper.

Lo, K. C., 1998. Sealed bid auctions with uncertainty averse bidders. Econ. Theory 12, 1–20.

Machina, M., Siniscalchi, M., 2014. Ambiguity and ambiguity Aversion. In: Machina, M., Viscusi, K. (Eds.), Handbook of the Economics of Risk and Uncertainty, vol. 1. Elsevier, pp. 729–807.

Martins-da-Rocha, V. F., 2010. Interim efficiency with MEU-preferences. J. Econ. Theory 145, 1987–2017.

Mertens, J.-F., Zamir, S., 1985. Formulation of Bayesian analysis for games with incomplete information. Int. J. Game Theory 14, 1–29.

Milgrom, P., 1981. Good news and bad news: representation theorems and applications. Bell J. Econ. 12, 380–391.

Milgrom, P., Roberts, J., 1990. Rationalizability, learning, and equilibrium in games with strategic complementarities. Econometrica 58, 1255–1277.

Morris, S., Shin, H. S., 1998. Unique equilibrium in a model of self-fulfilling currency attacks. Am. Econ. Rev. 88, 587–597.

Morris, S., Shin, H. S., 2001. Rethinking multiple equilibria in macroeconomic modeling. NBER Macroeconomics Annual 2000, 139–161.

Morris, S., Shin, H. S., 2002. Global games: theory and applications. In: Dewatripont, M., Hansen, L., Turnovsky, S. (Eds.), Advances in Economics and Econometrics: Theory and Applications, Proceedings of the Eighth World Congress of the Econometric Society. Cambridge Univ. Press, pp. 56–114.

Morris, S., Shin, H. S., 2004. Coordination risk and the price of debt. Eur. Econ. Rev. 48, 133–153.

Morris, S., Oyama, D., Takahashi, S., 2020. Implementation via information design in binary-action supermodular games. Working paper.

Obstfeld, M., 1986. Models of currency crises with self-fulfilling features. Eur. Econ. Rev. 40, 1037–1047.

Pan, A., 2019. A note on pivotality. Games 10, 24.

Pires, C. P., 2002. A rule for updating ambiguous beliefs. Theory Dec. 53, 1573–7187.

Routledge, B, Zin, S., 2009. Model uncertainty and liquidity. Rev. Econ. Dynamics 12, 543

Ryan, M., 2021. Feddersen and Pesendorfer meet Ellsberg. Theory Dec. doi: 10.1007/s11238-020-09797-7

Salo, A., Weber, M., 1995. Ambiguity aversion in first-price sealed-bid auctions. J. Risk Uncertainty 11, 123–137.

Song, Y., 2018. Efficient implementation with interdependent valuations and maxmin agents. J. Econ. Theory 176, 693–726.

Uhlig, H., 2010. A model of a systemic bank run. J. Monetary Econ. 57, 78–96.

Ui, T., 2009. Ambiguity and risk in global games. Working paper.

Van Zandt, T., Vives, X., 2007. Monotone equilibria in Bayesian games of strategic complementarities. J. Econ. Theory 134, 339–360.

Vives, X., 1990. Nash equilibrium with strategic complementarities. J. Math. Econ. 19, 305–321.

Wolitzky, A., 2016. Mechanism design with maxmin agents: theory and an application to bilateral trade. Theor. Econ. 11, 971–1004.