

# The Welfare Implications of Massive Money Injection: The Japanese Experience from 2013 to 2020

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# The Welfare Implications of Massive Money Injection: The Japanese Experience from 2013 to 2020

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## Abstract

This paper derives a money demand function that explicitly takes the costs of storing money into account. This function is then used to examine the consequences of the large-scale money injection conducted by the Bank of Japan since April 2013. The main findings are as follows. First, the opportunity cost of holding money calculated using 1-year government bond yields has been negative since the fourth quarter of 2014, and most recently (2020:Q2) was  $-0.2\%$ . Second, the marginal cost of storing money, which was  $0.3\%$  in the most recent quarter, exceeds the marginal utility of money, which was  $0.1\%$ . Third, the optimum quantity of money, measured by the ratio of M1 to nominal GDP, is 1.2. In contrast, the actual money-income ratio in the most recent quarter was 1.8. The welfare loss relative to the maximum welfare obtained under the optimum quantity of money in the most recent quarter was  $0.2\%$  of nominal GDP. The findings imply that the Bank of Japan needs to reduce M1 by more than 30%, for example through measures that impose a penalty on holding money.

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# 1 Introduction

Seven years have passed since the Bank of Japan (BOJ) welcomed Haruhiko Kuroda as its new Governor and started a new regime of monetary easing, which was nicknamed the “Kuroda bazooka.” The policy goal that the BOJ set itself was to overcome deflation. At the time, the year-on-year rate of change in Japan’s consumer price index (CPI) was -0.9% and had been below zero for a long time. The measure the BOJ chose to escape deflation was to print lots of money. That is, the BOJ thought that it would be possible to overcome deflation by increasing the quantity of money. Specifically, in April 2013, the BOJ announced that it would double the monetary base within two years and thereby raise the CPI inflation rate to 2%. However, currently, CPI inflation remains stuck at 0.3%. The BOJ has not achieved its target of 2%, and there is little prospect that it will be achieved in the near future. While it is true that inflation currently is heavily affected by the sharp fall in aggregate demand since the outbreak of the COVID crisis in February 2020, which is putting downward pressure on prices, even before the crisis CPI inflation was only between 0.2 and 0.8% and therefore below the BOJ’s target.

Why was the Kuroda bazooka unsuccessful? The reason is unforeseen developments – in two senses. First, things expected by the BOJ and academic researchers, including the author, in 2013 did not come to pass. That is, things did not go according to the scenario originally envisaged. Second, things that nobody expected in 2013 happened. While the COVID pandemic is the first thing that springs to mind, unforeseen developments are not limited to such exogenous shocks. The Kuroda bazooka itself also has had a negative impact on the economy through a channel that no one had anticipated in 2013. This study seeks to examine the economic effects of the Kuroda bazooka and, by doing so, clarify where the BOJ and the Japanese economy stand now. Further, based on this, it discusses where the BOJ and its monetary policy are headed and where they should be headed.

The remainder of the study is organized as follows. Section 2 provides a brief overview of the BOJ’s monetary easing policies since 2013 as well as their consequences. Section 3 derives a money demand function that explicitly takes the cost of storing money into account. Section 4 then presents the empirical results. Finally, Section 5 discusses the policy implications.

## 2 Consequences of Massive Money Injection in 2013-2020

### 2.1 The BOJ's monetary easing

The BOJ has tried various policies to overcome deflation. Specifically, from 1999 to 2000, the BOJ adopted a “zero interest rate policy,” in which it lowered the policy interest rate to zero. This was followed by “quantitative easing,” which started in 2001 and ended in 2006. However, immediately after that, the global financial crisis occurred, as a result of which Japan's economy fell back into deflation. Responding to this, in January 2013, the BOJ adopted a “price stability target” with the aim of raising the annual rate of increase in the CPI to 2%.

In March 2013, Haruhiko Kuroda was appointed by Prime Minister Abe as the new BOJ Governor. One of his first steps in this role was to introduce a new monetary easing regime in April 2013, which was nicknamed the “Kuroda bazooka.” Specifically, the BOJ announced that it was aiming to achieve the 2% inflation target within two years and, that in order to achieve this objective, the BOJ would introduce “quantitative and qualitative easing (QQE),” which sought to double the amount of base money within two years. Further, in February 2016, the BOJ introduced a “negative interest rate policy,” under which the BOJ applies a negative interest rate of minus 0.1% to current account deposits held by private banks at the BOJ. This was followed in September 2016 by the introduction of “yield curve control (YCC),” in which the BOJ conducts operations using Japanese government bonds (JGB) so as to keep the 10-year JGB yield at 0%. YCC was modified in July 2018 by allowing 10-year JGB yield to fluctuate within a band of -0.2 to +0.2%. Most recently, in April 2020, the BOJ decided to enhance monetary easing in response to the COVID crisis by (1) increasing purchases of commercial paper (CP) and corporate bonds up to 20 trillion yen; (2) strengthening of the loan facility for financial institutions that it had created in March to help them lend to firms; and (3) conducting active purchases of JGBs and T-bills. Table 1 provides an overview of major policy decisions by the BOJ since 2013.

Despite these efforts by the BOJ to escape deflation, CPI inflation currently stands at only 0.3%, and there is little prospect that the BOJ's 2% inflation target will be achieved in the near future. Why was the Kuroda bazooka unsuccessful? The reason are unforeseen

developments – in two senses. On the one hand, things expected by the BOJ and academic researchers in 2013 did not happen. On the other hand, things that nobody expected in 2013 happened.

## 2.2 Things that were expected to happen but did not happen

The first thing that many had expected in 2013 but that did not happen in practice is a rise in household inflation expectations. The BOJ made a major switch in its approach and started to actively communicate to households that it would raise the inflation rate in the future. Its calculation was that this would lower real interest rates and stimulate aggregate demand, so that prices would rise. However, various surveys on inflation expectations, including one conducted by the BOJ itself, indicate that while inflation expectations rose slightly in the early days of the Kuroda bazooka regime, this did not last, and expectations have now returned more or less to the level before the Kuroda bazooka.

Various hypotheses have been put forward to explain why inflation expectations did not rise. IMF (2018) focuses on Diamond et al.'s (2020) hypothesis that people's inflation expectations depend on the inflation they have experienced during their lifetime. For example, those in their 50s experienced the inflation during the two oil crises of the 1970s. Further, even older cohorts experienced the hyperinflation Japan suffered shortly after World War II. In fact, according to a survey conducted by Diamond et al. (2020), inflation expectations among these cohorts rose steadily in response to the BOJ's announcements. In contrast, inflation expectations did not rise among those born in the 1980s and 1990s (i.e., those who have experienced only deflation).<sup>1</sup>

Diamond et al. (2020) argue that from the mid-1990s, the BOJ neglected deflation for a long period of time, resulting in the emergence of a generation that is not used to inflation,

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<sup>1</sup>It would be inappropriate to explain the fact that younger cohorts' inflation expectations did not rise solely by the role played by inflation experience. Cohorts differ not only in terms of their experience but also their age, and the low inflation expectations of the cohort born in the 1980s and 1990s may reflect age-based differences rather than cohort-based differences. Specifically, the prices of the types of products younger people buy may not have risen as much as the types of products older people buy. Using scanner data to investigate how consumption baskets differ by age, Diamond et al. (2020) measured the inflation rate by age group. They showed that although the measured inflation rates of items purchased increase with age, the difference in inflation expectations could only partly be accounted for by the difference in inflation of the products purchased, and that the low inflation expectations of the younger cohorts were largely caused not by age differences but by differences across cohorts (i.e., differences in inflation experience).

and that especially among this generation inflation expectations are low. At the start of the Kuroda bazooka, no one had imagined that the experience of deflation would have had such a serious impact on inflation expectations.

The second thing that was expected to happen but did not happen was a change in firms' pricing behavior. At the start of the Kuroda bazooka, many thought that if the money supply was increased, the yen would depreciate, the prices of imported raw materials would rise, and firms would pass on the increased costs to product prices. The Kuroda bazooka was successful in bringing about a large depreciation of the yen. However, the number of firms that passed on increased costs to product prices was limited. Watanabe and Watanabe (2018) highlighted that the reason was that firms did not have sufficiently strong pricing power. Specifically, examining the shape of the distribution of price changes by item in eight countries, including Japan, Watanabe and Watanabe (2018) showed that the mode of Japan's distribution was near zero, while the mode for other countries such as the United States was around 2%. In other words, whereas in the United States and the other countries, firms' "default" was to raise prices at a rate of 2% each year, the "default" for Japanese firms was to leave prices unchanged. Watanabe and Watanabe (2018) argued that in Japan the long period of near-zero inflation since the mid-1990s had resulted in a notable change in social norms with regard to pricing, giving rise to a situation in which firms could not pass on cost increases to prices.

Several hypotheses have been put forward to explain why Japanese firms have not been passing on cost increases. Aoki et al. (2019), for example, highlight that low household inflation expectations made it difficult for firms to pass on cost increases.<sup>2</sup> Focusing on an economy with incomplete information based on the Lucas islands model, they examined what would happen in a situation in which prices remained unchanged for a long time. Their first finding is that firms would face a kinked demand curve. If prices remain unchanged for a long time, consumers strongly believe that the aggregate price level will also remain unchanged in the future. Under these circumstances, if consumers see that a store raised its

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<sup>2</sup>In contrast with Aoki et al. (2019), who focus on low household inflation expectations, Taylor (2000) highlights the role played by low corporate inflation expectations. According to Taylor (2000), when firms' inflation expectations are low, they expect that, since trend inflation is low, exchange rate depreciation and wage increases are unlikely to persist, and will come to an end before long. Since the exchange rate depreciation and wage increases are only temporary, firms choose not to pass these on to goods prices.

prices, they will immediately stop buying from that store and switch to another store. The reason is that consumers believe that prices at other stores have remained unchanged. Such consumer behavior causes the demand curve to be kinked. As is well known, under a kinked demand curve, it is optimal for sellers not to adjust prices even when costs rise slightly, and, as a result, prices remain unchanged.<sup>3</sup> The second finding by Aoki et al. (2019) is that this situation weakens the impact of central bank communication. When consumers experience a long period without inflation, they start to think that the central bank may actually be pursuing low inflation intentionally. Therefore, they come to believe that the central bank has lowered its inflation target. More importantly, they have less incentive to collect information regarding the central bank's inflation target. And once such a situation has arisen, consumers will not pay attention when the central bank announces an increase in the inflation target.

### **2.3 Things that no one anticipated to happen but that did happen**

While what was supposed to happen did not happen, things also happened that no one had anticipated. In this regard, what this study particularly focuses on are the consequences of the large-scale increase in the quantity of money. In order for the BOJ to increase the money supply, it needs to purchase assets issued by the government and private firms. Potential negative effects of the BOJ's large-scale asset purchases on the economy were extensively discussed from quite an early stage. For example, it was highlighted that the large-scale purchases of government bonds by the BOJ could loosen the government's fiscal discipline. On the other hand, the BOJ's large-scale purchases of exchange traded funds (ETFs) might give rise to distortions in the governance of private companies. It is possible that the Kuroda bazooka may have had these side effects. However, interestingly, the debate about the side effects of the Kuroda bazooka focused only on the BOJ's asset purchases, and, to the best of the author's knowledge, there was no debate about the possible side effects of the increase in the money supply itself. However, as detailed in Sections 3 and 4, it is highly likely that the large quantity of money supplied through the Kuroda bazooka itself may have had an adverse effect on the economy.

When the Kuroda bazooka program started, researchers, including the author, had no

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<sup>3</sup>The first to point out a kinked demand curve as a reason for price rigidity is Negishi (1979). See Dupraz (2017) for a recent study on kinked demand curves and the associated price rigidity.

idea that the large scale money injection itself would adversely affect the economy. It is only natural that the BOJ, which believes that by increasing the money supply it is possible to overcome deflation, thought that the more the money supply is increased, the better this would be for the economy. However, why did researchers – including those critical of the Kuroda bazooka – not realize that the large-scale money injection itself could have an adverse effect on the economy?

The reason is that the academic debate on the liquidity trap and the zero lower bound (ZLB) on nominal interest rates is based on the assumption that there is a finite satiation level for real money balances. At the time, the depiction of Japan's economy that attracted the most attention was the study by Krugman (1998). Krugman (1998) made the famous argument that deflation could be overcome if it was possible to instill in people the expectation that there would be sufficiently high inflation in the future. However, Krugman (1998) makes important points not only about inflation expectations but also about money. In his model based on the cash-in-advance constraint, when deciding whether to hold money or bonds, people weigh up the convenience of money, which can be used to pay for purchases, and the interest income earned from bonds. When the money supply becomes extremely large, interest rates will drop to zero; at the same time, a situation arises in which holding more money does not increase convenience. That is, a point is reached at which people's demand for money is satiated. Krugman's analysis is based on the assumption that the demand for money is satiated in an economy with zero interest rates like Japan's, and his policy recommendations crucially depend on this. The study which most explicitly models satiation in real money balances is that by Eggertsson and Woodford (2003). Using a money-in-the-utility-function model, they argue that in a situation where interest rates have reached the ZLB, the marginal utility of money falls to zero as a result of the large supply of money.

What should be emphasized here is that both studies imply that when interest rates are zero, an increase in the money supply has no effect on the economy, since money balances have reached their satiation level when interest rates are zero. Eggertsson and Woodford (2003) presented this as their "irrelevance proposition." Their message that increasing the money supply beyond this satiation level has no effect on the economy is often interpreted as simply meaning that any further increases in the money supply will have no positive impact such

as raising the rate of inflation. However, this can also be interpreted as meaning that even if large quantities of money are supplied, nothing bad will happen. While it is unclear how the irrelevance proposition affected the BOJ's policy making, what can be said for certain is that mainstream researchers, including the author, on the basis of this proposition thought that increases in the money supply beyond the satiation level were neither "medicine" nor "poison" and therefore did not stop the BOJ in turning toward massive money injection. In other words, researchers took the position of being "not dissatisfied" with the Kuroda bazooka. On the other hand, the BOJ may not have been concerned that it could be "poison" and thought instead that if all went well it might be "medicine," so that it was worth trying.

But what actually happened as a result of the large-scale money injection? Since even with the large increases in the money supply deflation was not overcome, the BOJ's claim at the time that prices would rise if the money supply was increased has been refuted. This is often regarded as suggesting that the "money satiation argument" has won. However, this does not at all mean that the large-scale money injection did not have any impact. As will be shown in Section 4, the large supply of money did have an adverse effect on the economy. In that sense, the money satiation argument that the increase in money supply neither had a positive nor a negative effect is also shown to be incorrect.

Many studies on the ZLB, such as Eggertsson and Woodford (2003), use the relationship that the marginal utility of money and the opportunity cost of holding money are equal. Money here refers to the money held by households and non-financial firms, specifically M1. The opportunity cost of holding money is equal to the return on interest bearing financial assets less the return on money (for example, interest on money held in savings accounts, which is included in M1). The marginal utility of money decreases as the amount of money held increases, but it never becomes negative. An important implication of this is that the opportunity cost should be non-negative as well. However, when calculating the opportunity cost using Japanese data, it often takes a negative value. For example, the opportunity cost calculated using 1-year government bond yields has been negative since the fourth quarter of 2014, and in the fourth quarter of 2016 fell as low as  $-0.3\%$ . As of the second quarter of 2020, it was  $-0.2\%$ . The assumption that the opportunity cost of holding money equals the marginal utility of money does not seem to hold in the data.

Why does the opportunity cost take a negative value? Recent studies on negative interest rate policies, such as Rognlie (2016) and Eggertsson et al. (2019), argue that while holding money provides utility, its storage involves costs. Specifically, they suggest that the cost of storing money depends on the amount of money held and that the marginal cost of storage is an increasing function of the amount of money held (that is, the cost of storing money is a convex function of the amount of money held).<sup>4</sup> Under these circumstances, what people compare the opportunity cost with is the marginal utility of money minus the marginal cost of storing money. The marginal utility of money is non-negative, but the marginal cost of storage is positive, so the difference between the two can be negative.<sup>5</sup> Therefore, the opportunity cost can also be negative. In fact, according to the estimates in Section 4, the marginal utility of money in the second quarter of 2020 was 0.1%, but the marginal cost of holding money was 0.3%. In other words, a marginal increase in money holdings leads to a loss for money holders of 0.2%. This corresponds to an opportunity cost of  $-0.2\%$ .

The presence of non-negligible storage costs of money has important welfare implications for the Kuroda bazooka. As the money supply increases, the marginal utility of money decreases and approaches zero. On the other hand, the marginal cost of storing money increases as the money supply increases. Therefore, the marginal utility and marginal cost coincide at a certain finite value of the money supply. Economic welfare increases as the amount of money increases as long as the amount of money is below the satiation level. However, once the quantity of money exceeds the satiation level, increases in the quantity of money deteriorate economic welfare. In other words, supplying money above the satiation level is a “poison”

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<sup>4</sup>Thornton (1999) pointed out that currency carries the risk of being stolen or lost, and that physically moving currency is expensive, so that the costs of holding currency are not negligible. Focusing on 13 countries in the euro zone, Schmiedel et al. (2012) examined how much banks and retailers spent on various payment instruments and showed that for cash, the costs came to 2.3 cents per euro. These costs include costs such as those related to machinery and equipment to support the use of cash – such as automatic teller machines provided by banks and point of sale terminals at retailers – as well as labor costs. Since the overall costs include the costs borne by banks and retailers, not all of the costs fall on households that use cash. However, when households use cash, they may potentially be charged by banks and retailers for these costs in the form of cash handling fees.

<sup>5</sup>Hicks (1937: 154-155) described Keynes’s liquidity trap as follows: “If the costs of holding money can be neglected, it will always be profitable to hold money rather than lend it out, if the rate of interest is not greater than zero. Consequently, the rate of interest must always be positive.” However, based on Japan’s experience under the Kuroda bazooka, the assumption that “the costs of holding money can be neglected” is not an appropriate approximation of reality. Friedman (1969) did not consider a situation in which the money supply exceeds the optimum quantity of money, since he believed that money storage costs could be ignored.

that has a negative impact on the economy.

Friedman (1969) highlighted that the money supply is at the optimal level when the marginal utility of money is zero and called this the optimal quantity of money. Rognlie (2016) extends Friedman’s (1969) argument to a situation in which storage costs are not negligible and shows that the quantity of money is at the optimal (i.e., satiation) level when the marginal utility of money equals the marginal cost of money storage. According to the estimates in Section 4, for Japan, the quantity of money is optimal when the ratio of M1 to nominal GDP is 1.2. In contrast, the actual ratio of M1 to GDP in the second quarter of 2020 was 1.8, meaning that the quantity of money in Japan is excessive. The opportunity cost of holding money is too low and, as a result, there is excessive demand for money. Measuring economic welfare following Bailey (1956) as the sum of the consumer surplus and seigniorage, the welfare loss relative to the maximum welfare obtained under the optimum quantity of money currently (as of 2020: Q2 ) corresponds to 0.2% of nominal GDP.

### **3 Demand for Money in an Economy with Non-Negligible Storage Costs of Money**

This section sets up the theoretical framework for the empirical analysis conducted in Section 4. Specifically, a money demand function explicitly taking the storage costs of holding money into account is derived. Next, Bailey’s (1956) measure for the welfare cost of inflation is calculated to discuss how economic welfare depends on the quantity of money held by households and non-financial firms. Finally, Friedman’s (1969) optimum quantity of money in an economy with money storage costs is discussed.

#### **3.1 The case of no money storage costs**

Let us start with a version of Sidrauski’s (1967) model. The representative household maximizes the present value of the sum of utilities,

$$U_t = \sum_{T=t}^{\infty} \beta^{T-t} U(c_T, z_T)$$

where  $c$  and  $z$  denote consumption and real money balances. Following Lucas (2000), it is assumed that the current period utility function is given by

$$U(c, z) = \frac{1}{1 - \sigma} \left[ c \varphi \left( \frac{z}{c} \right) \right]^{1 - \sigma} \quad (1)$$

where  $\sigma > 0$  and  $\sigma \neq 1$ , and  $\varphi(\cdot)$  is a strictly increasing and concave function.  $\varphi(\cdot)$  will be specified later. The household faces the following flow budget constraint:

$$M_t + B_t = (1 + i_{t-1}^m)M_{t-1} + (1 + i_{t-1})B_{t-1} + P_t y_t - P_t c_t \quad (2)$$

where  $B_t$ ,  $i_t$ ,  $i_t^m$ ,  $P_t$ , and  $y_t$  respectively denote the amount of a one-period risk-free bond held by the household, the nominal interest rate associated with the bond, the nominal interest rate on money (e.g., the interest rate on saving deposits), the price level, and income. The first order conditions for utility maximization imply that the optimal holding of money has to satisfy

$$\frac{U_z}{U_c} = \frac{\varphi' \left( \frac{z}{c} \right)}{\varphi \left( \frac{z}{c} \right) - \frac{z}{c} \varphi' \left( \frac{z}{c} \right)} = x \quad (3)$$

where  $x$  is the opportunity cost of holding money, which is defined as  $x \equiv i - i^m$ .

Following Lucas (2000), the set-up considers an endowment economy characterized by a balanced growth equilibrium path on which the money growth rate is constant and maintained by a constant ratio of transfers to income. In this set-up, the money-income ratio, given by  $m = z/y$ , is also constant. Eq. (3) can then be rewritten as

$$\frac{\varphi'(m)}{\varphi(m) - m\varphi'(m)} = x$$

This implies that, if money demand is of log-log form, i.e.,

$$\begin{aligned} m(x) &= Ax^\alpha && \text{for } x > 0 \\ \text{or } \ln m &= \ln A + \alpha \ln(x) \end{aligned} \quad (4)$$

with  $A > 0$  and  $\alpha < 0$ , the function  $\varphi(\cdot)$  solves a differential equation of the form

$$\frac{\varphi'(m)}{\varphi(m)} = \frac{\psi(m)}{1 + m\psi(m)} = \frac{A^{-1/\alpha} m^{1/\alpha}}{1 + mA^{-1/\alpha} m^{1/\alpha}}$$

where  $\psi(\cdot)$  is the inverse money demand function (i.e.,  $(\psi(\cdot) \equiv m^{-1}(\cdot))$ ). The solution to this differential equation is given by<sup>6</sup>

$$\varphi(m) = \left(1 + A^{-\frac{1}{\alpha}} m^{\frac{1+\alpha}{\alpha}}\right)^{\frac{\alpha}{1+\alpha}}$$

Conversely, if the utility function (1) is specified as

$$U(c, z) = \frac{1}{1-\sigma} \left[ c \left(1 + A^{-\frac{1}{\alpha}} \left(\frac{c}{z}\right)^{\frac{1+\alpha}{\alpha}}\right)^{\frac{\alpha}{1+\alpha}} \right]^{1-\sigma} \quad (5)$$

the money demand function derived from utility maximization is of log-log form.<sup>7</sup>

### 3.2 Linear storage costs

Recent studies on negative interest rate policies such as Rognlie (2016) and Eggertsson et al. (2019) argue that the cost of holding cash is not negligible. Here, the storage cost of money is introduced into Sidrauski's (1967) model closely following Eggertsson et al. (2019). The flow budget constraint now changes to

$$M_t + B_t = (1 + i_{t-1}^m)M_{t-1} + (1 + i_{t-1})B_{t-1} + P_t y_t - P_t c_t - S(M_{t-1}) \quad (6)$$

where  $S(M_{t-1})$  denotes the storage cost of money. Note that  $S(M_{t-1})$  represents nominal storage costs and depends on nominal (rather than real) money balances. The first order conditions for utility maximization imply

$$\frac{U_z}{U_c} = \frac{\varphi'(\frac{z}{c})}{\varphi(\frac{z}{c}) - \frac{z}{c}\varphi'(\frac{z}{c})} = x + S'(M)$$

Following Eggertsson et al. (2019), it is assumed that the marginal storage cost is positive and constant, so that  $S'(M) = \theta > 0$ .

Starting with the utility function given by Eq. (5) yields a money demand function of the following form:

$$\begin{aligned} m &= A(x + \theta)^\alpha && \text{for } x > -\theta \\ \text{or} \quad \ln m &= \ln A + \alpha \ln(x + \theta) \end{aligned} \quad (7)$$

<sup>6</sup>Lucas (2000) shows that the solution is given by  $\varphi(m) = (1 + A^2/m)^{-1}$  when  $\alpha = -1/2$ .

<sup>7</sup>Note that the log-log money demand function can be derived using McCallum and Goodfriend's (1989) transaction time model. The inventory theory approach for the demand for money proposed by Baumol (1952) and Tobin (1956) also implies a log-log form.

which is close to log-log form but differs from it in that a constant term,  $\theta$ , is added to  $x$  before taking the logarithm. Note that  $m$  takes a finite value when  $x = 0$ , which is an important difference from the case of no storage costs.

### 3.3 Convex storage costs

The assumption that the marginal cost of storage is constant is a good approximation as long as the quantity of money held is not that large. However, if the opportunity cost of holding money is so low that the demand for money is extremely large, it is inappropriate to assume that the marginal cost of storage is constant irrespective of the quantity of money held. Instead, it would be better to assume that the marginal cost of storing money increases with the quantity of money held (i.e., the cost of storage is convex). In this case, as  $x$  decreases, the quantity of money held increases, so that  $\theta$  in (7) increases. Taking this into consideration, Eq. (7) is modified as follows:

$$\ln m = \ln A + \alpha h(x) \tag{8}$$

where  $h(\cdot)$  is the log-like transformation proposed by Ravallion (2017) and is defined as

$$h(x) \equiv \begin{cases} \sinh^{-1}(\mu x) - \ln(2\mu) & x > 0 \\ \sinh(\mu x) - \ln(2\mu) & \text{otherwise} \end{cases} \tag{9}$$

where  $\sinh(\cdot)$  is the hyperbolic sine transformation given by  $\sinh(k) \equiv 1/2 [\exp(k) - \exp(-k)]$ , and  $\sinh^{-1}(\cdot)$  is the inverse hyperbolic transformation given by  $\sinh^{-1}(k) \equiv \ln(k + \sqrt{k^2 + 1})$ .  $\mu$  is a positive parameter. Note that, for  $k > 0$ , this transformation is “log-like” in that  $h(k)$  becomes more like  $\ln(2k)$  as  $k$  rises (i.e.,  $\lim_{k \rightarrow \infty} [h(k) - \ln(2k)] = 0$ ). However,  $h(k)$  deviates from the log when  $k$  is close to zero or negative. Also, note that  $h'(k) > 0$  and  $h''(k) < 0$ , so that  $h(\cdot)$  is a concave transformation.

The money demand function represented by Eq. (8) has the following properties. First, when  $x$  is sufficiently large,  $h(\cdot)$  is close to the log, so that the demand for money represented by Eq. (8) behaves like money demand functions of log-log form, such as Eqs. (4) and (7). Second, as  $h'(\cdot) > 0$  and  $h''(\cdot) < 0$ , a marginal decline in the opportunity cost of money leads to a greater increase in the demand for money when the opportunity cost of money is at a lower level, as is the case with money demand functions of log-log and semi-log form. Third,

the responsiveness of the demand for money with respect to the opportunity cost of money, namely  $dm/dx$ , can never be infinitely large, unlike in money demand functions of log-log form such as Eqs. (4) and (7). An important implication of the second and third properties is that there is no lower bound for the opportunity cost of money. A decline in the opportunity cost of money increases the demand for money, but with strictly convex storage costs, this will be associated with a higher marginal storage cost. As a result, the demand for money never becomes infinite even when the opportunity cost of money is near zero or below zero.

### 3.4 Bailey's (1956) measure for the welfare loss

Bailey (1956) defines the welfare cost of inflation in terms of how much lower welfare is in an economy with a positive opportunity cost of money than in an economy with a zero opportunity cost. Figure 1 illustrates Bailey's measure for the case with no storage cost of money. The vertical and horizontal axes represent, respectively, the opportunity cost of holding money and the money-income ratio. The demand for money is represented by the black line. When the opportunity cost of money is positive and at  $x_0$ , the demand for money is given by  $m_0$ , so that the seigniorage is measured by the square highlighted in red, and the consumer surplus is measured by the area highlighted in blue. If the opportunity cost falls from  $x_0$ , welfare, which is measured by the sum of the red and blue areas, monotonically increases and finally reaches its maximum when the opportunity cost is zero (i.e., the Friedman rule). Put differently, when the opportunity cost is at  $x_0$ , there exists a welfare loss, which is represented by the area highlighted in green. This is what Bailey (1956) refers to as the welfare cost of inflation. A high opportunity cost of money typically arises from a high nominal interest rate on bonds, which is normally accompanied by a high inflation rate. High inflation, if not anticipated, erodes the real value of money, so that it can be regarded as a tax on money. Bailey (1956) shows that this inflation tax is costly since it creates a dead-weight loss like in the case of an excise tax on a commodity.

When the money demand function is given by Eq. (4), Bailey's (1956) measure for the welfare gain achieved by lowering the opportunity cost from  $x$  to zero is given by

$$w(x) = \int_{m(x)}^{m(0)} \psi(k) dk = \int_0^x m(k) dk - xm(x) = A \left( -\frac{\alpha}{1+\alpha} \right) x^{1+\alpha} \quad (10)$$

This equation implies that  $w'(x) > 0$  and  $w''(x) < 0$  for  $x > 0$ ; that is, the marginal welfare gain of lowering the opportunity cost from  $x$  toward zero is positive and *increases* as  $x$  approaches zero. This property of the log-log money demand function has been extensively discussed by Lucas (2000) and Wolman (1997). It arises due to the following reasons. First, the marginal utility of money,  $U_z/U_c$ , stays strictly positive even when  $m$  takes an extremely large value, meaning that there exists no satiation of money if the money demand function is of the form given by Eq. (4), so that the zero lower bound is an asymptotic one. Therefore, lowering the interest rate from a positive level toward zero always leads to an increase in utility. Second, the extent to which money demand responds to a reduction in the opportunity cost gets bigger as  $x$  approaches zero.

In an economy with linear storage costs, the welfare gain of lowering the opportunity cost from  $x$  to zero is given by

$$\begin{aligned} w(x) &= \int_0^x m(k)dk - xm(x) \\ &= \frac{A}{1+\alpha} [(x+\theta)^{1+\alpha} - \theta^{1+\alpha}] - xA(x+\theta)^\alpha \end{aligned} \quad (11)$$

implying that  $w'(x) > 0$  for  $x > 0$  and  $w'(x) < 0$  for  $-\theta < x < 0$ , and that  $w'(0) = 0$  and  $w''(0) > 0$ . An important difference from the case without storage costs is that there exists a finite satiation level of money even for the log-log money demand function, at which the marginal utility of money coincides with the marginal storage cost of money, and that the satiation level is achieved by setting  $x = 0$  (i.e., the Friedman rule). Any deviation from the Friedman rule, whether  $x > 0$  or  $x < 0$ , results in a suboptimal outcome.

### 3.5 Numerical example

Figure 2 shows the relationship between the opportunity cost of money and the money-income ratio (i.e., money demand function) in the upper panel, the relationship between the money-income ratio and Bailey's measure for the welfare loss in the middle panel, and the relationship between the opportunity cost and Bailey's measure in the bottom panel. The blue line in each panel corresponds to the case of no storage costs, while the red and the green lines correspond respectively to linear and convex storage costs. The parameters are set as follows:  $\alpha = -0.5$ ;  $A = 1$ ;  $\theta = 0.01$ ; and  $\mu = 100$ .

Let us start with the case of no storage costs. The money demand function presented in the upper panel shows that  $m$  monotonically increases as  $x$  declines, and that  $m$  takes an infinitely large value when  $x$  is zero. Put differently, the marginal utility of money decreases as  $m$  increases, but it is always strictly positive and never reaches zero, meaning that money satiation does not occur at a finite value of  $m$  but occurs only asymptotically. In the case of no storage costs, households hold more and more money balances as the opportunity cost declines, so that their welfare monotonically improves (i.e., Bailey's measure for the welfare cost decreases), which is shown in the middle and bottom panels. Note that, as shown in the bottom panel, the marginal welfare gain of lowering the opportunity cost from  $x$  toward zero is positive and *increases* as  $x$  comes closer to  $x = 0$ .

Turning to the case of linear storage costs, the demand for money monotonically increases as the opportunity cost declines, as in the case of no storage costs. An important difference from the previous case is that the demand for money takes a finite value even when the opportunity cost is zero. This means that the demand for money is satiated at that level. The middle panel shows that welfare deteriorates (i.e., Bailey's measure for the welfare cost increases) as  $m$  deviates from the satiation level. The bottom panel shows that welfare deteriorates as  $x$  deviates from zero, regardless of whether  $x > 0$  or  $x < 0$ , but the welfare deterioration is asymmetrically larger when  $x$  falls from zero than when it rises from zero.

Finally, the demand for money in the case of convex storage costs is also downward sloping, as in the previous two cases, but unlike in the previous cases,  $m$  does not approach a certain value as  $x$  declines, so that  $m$  continues to be finite even when  $x$  takes a large negative value. This means that there is no lower bound on the opportunity cost, unlike in the previous two cases.

The three cases shown in the figure have different implications for the conduct of monetary policy near the ZLB. If money storage costs are negligible, money satiation never occurs, so that central banks need not worry about the possibility of injecting too much money into the economy. All they should worry about is a shortage of money supply rather than excess supply of money. Given this, the strategy central banks should take is to inject as much money as possible. However, in the case of non-negligible storage costs, be they linear or convex, the demand for money can be satiated at a finite level of money, so that central

banks need to pay attention to the risk of injecting too much money.<sup>8</sup>

## 4 Empirical Results

### 4.1 Money demand functions

Figure 3 shows developments in the money-income ratio and the opportunity cost of holding money over the last 40 years. The money-income ratio is defined as M1 divided by nominal GDP. The opportunity cost of money is defined as the difference between 1-year JGB yields and the interest rate on saving deposits. More specifically, M1, in addition to cash, contains saving deposits, i.e., interest-bearing deposits held for settlement purposes by households and firms (especially small firms) at commercial banks, which makes up about 64% of M1. The opportunity cost is calculated by subtracting the interest rate on saving deposits multiplied by their share in M1 from 1-year JGB yields.

The opportunity cost thus calculated shows a rapid decline in the first half of the 1990s and reaches somewhere around zero in the mid-1990s. It has essentially remained near zero since then. However, this does not necessarily mean that there were no significant fluctuations in the opportunity cost. In fact, there were important changes in the opportunity cost even during this period. For example, the opportunity cost rose in the second quarter of 2006, deviating from zero. As explained in Section 2, the BOJ ended quantitative easing in March 2006 and started to raise the policy rate in July 2006, resulting in an increase in 1-year JGB yields. Another episode during which the opportunity cost deviated from zero occurred in 2014:4Q, when it became negative. The opportunity cost has been below zero since then.

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<sup>8</sup>Using a semi-log money demand function, Rognlie (2016) shows that money satiation occurs at a finite level of money supply. Specifically, if the demand for money is given by  $\ln m = \ln B + \beta r$ ,  $m$  takes a finite value even when  $x = 0$ . An important difference from the result here is that in Rognlie (2016) it is the functional form of the money demand function that yields money satiation at a finite level. In contrast, the analysis here starts from a log-log money demand function and shows that money satiation occurs at a finite level only when money storage costs are non-negligible. As for the functional form of money demand functions, previous studies based on US data seem to suggest that the log-log form performs better than the semi-log form. Specifically, using US long-term annual data covering the period from 1900 to 1994, Lucas (2000) finds that the log-log form fits better than the semi-log form. More recently, Watanabe and Yabu (2018), using recent US quarterly data that include observations from the period of near-zero interest rates following the global financial crisis, showed that the log-log form performs better than the semi-log form. As for studies based on Japanese data, Nakashima and Saito (2012) and Miyao (2002) show that semi-log specifications fit well. However, using recent data containing more observations with near-zero interest rates, Watanabe and Yabu (2019) show that the log-log form also fits better to Japanese data than the semi-log form.

This can be regarded as the result of the large-scale money injection through the Kuroda bazooka program, as well as the negative interest rate policy introduced in January 2016.

One might say that these fluctuations in the opportunity cost since the mid-1990s still simply represent tiny changes around zero, as shown in the figure. However, the money demand functions derived in Section 3, such as Eqs. (4), (7), and (8), indicate that what matters is not the level of the opportunity cost but its log. Small fluctuations in  $x$ , especially those around zero, may not necessarily be that small in  $\ln x$ .

Turning to the money-income ratio, this was stable and remained somewhere between 0.3 and 0.4 up until the mid-1990s but has been on a significant upward trend since then. The flip side of this is that investments by households in interest bearing assets, such as bonds, have been declining since the mid-1990s. Watanabe and Yabu (2019) examine this by decomposing the decline in households' investment in interest bearing assets into the extensive margin (i.e., the fraction of households with interest-bearing assets) and the intensive margin (i.e., the amount of interest-bearing assets per household for households with interest-bearing assets). They show that about two-thirds of the decline in interest-bearing assets per household during this period is accounted for by changes in the extensive margin. In response to the secular decline in interest rates during this period, the number of households with no financial assets outside M1 (i.e., cash and saving deposits) has been increasing, which has contributed to the surge in the money-income ratio. Note that the outbreak of the COVID crisis accelerated the increase in the demand for M1, so that as of 2020:2Q the money-income ratio had risen to 1.76.

Figure 4 employs the same data as Figure 3, but the two variables are now in log form. The vertical axis represents the log of the money-income ratio, while the horizontal axis represents the log of the opportunity cost plus 0.005. This scatter plot can be regarded as the money demand function given by Eq. (7) with the marginal cost of storing money,  $\theta$ , set to  $\theta = 0.0005$ . Examining the cointegrating relationship between the money-income ratio and the interest rate on short-term certificates of deposit (CD), Watanabe and Yabu (2019) show that there exists a structural break in 2006, when the CD rate rose in response to an increase in the policy rate, but the demand for money did not decline very much, resulting in an upward shift of the money demand schedule. A similar structural break can be seen in

Figure 4 in that the stable relationship between the two variables, which is represented by the blue dots, collapsed in 2006.<sup>9</sup> However, the figure also shows that a downward-sloping money demand schedule reemerged from 2009:Q1 onward, when the BOJ restarted monetary easing in response to the onset of the global financial crisis.

Next, to examine how the demand for money responded to changes in the opportunity cost, money demand functions are estimated using data from 2006:Q2 onward. Specifically, based on Eq. (7), the log of the money-income ratio is regressed on the log of the opportunity cost plus  $\theta$ . For example, when  $\theta$  is set to  $\theta = 0.005$ , the following result is obtained:

$$\ln m = -1.5382 - 0.3162 \times \ln(x + 0.005) \quad (12)$$

The coefficient on  $\ln(x + \theta)$  represents the responsiveness of  $m$  with respect to  $x$ , although it cannot be interpreted as the elasticity unless  $\theta$  is zero. The estimated coefficient, which is -0.3, is slightly smaller than but close to the estimate of the interest elasticity of -0.5 obtained by Lucas (2000) employing US data containing positive interest rate observations only. Using Japanese data with positive interest rate observations only, Watanabe and Yabu (2019) show that the interest elasticity is somewhere around -0.1. Another important thing implied by the regression result is that the estimate for Friedman's (1969) optimum quantity of money, which is obtained by substituting  $x = 0$  into (12), is 1.147. Given that the money-income ratio in 2020:2Q was 1.764, the current level exceeds the optimum quantity of money by more than 50%. This issue will be discussed in more detail in the next subsection.

Similar regressions are conducted using different values for  $\theta$  ( $\theta = 0.01, 0.02$ ). Also, based on the assumption of convex storage costs, the log of the money-income ratio is regressed on  $h(x)$  with  $\mu$  set to  $\mu = 100$  to obtain the following:

$$\ln m = -2.9417 - 0.5863 \times h(x) \quad (13)$$

The optimum quantity of money implied from this regression result is 1.179, which is quite close to the estimate obtained earlier. Figure 5 shows the fitted lines obtained for the four

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<sup>9</sup>Watanabe and Yabu (2019) highlight that high switching costs between money and interest-bearing assets were one potential factor underlying the upward shift of the money demand schedule in 2006. Specifically, they argue that some Japanese households kept almost their entire wealth in the form of money (i.e., cash and saving deposits) over the two-decade-long near-zero interest period and consequently failed to learn about new financial technology making it possible to invest in interest-bearing assets.

different specifications, indicating that the linear storage specification with  $\theta = 0.005$  fits better than the other specifications. However, the most recent observation (i.e., the observation for 2020:2Q), which is highlighted in red, lies far from even the specification with  $\theta = 0.005$ , suggesting that something very different from the past has occurred in the demand for money due to the COVID crisis.

## 4.2 Bailey's welfare measure

This subsection conducts welfare analysis regarding the quantity of money making use of the money demand functions estimated in the previous subsection. The first exercise consists of estimating the marginal utility of money. As shown in Sections 3.1 and 3.2, the marginal utility of money,  $U_z/U_c$ , is a function of  $m$ , so that the marginal utility for a particular value of  $m$  can be calculated using the estimates for  $A$  and  $\alpha$ .<sup>10</sup> Moreover, Sections 3.1 and 3.2 showed that the marginal utility of money equals the sum of the opportunity cost of holding money and the marginal cost of storing money (i.e.,  $U_z/U_c = x + \theta$ ). Using this relationship, and given the estimate for the marginal utility for a particular quarter and the actual value for the opportunity cost in that quarter, the marginal cost of storing money for that quarter can be estimated.

Figure 6 shows the estimation results for the marginal utility and the marginal cost. The figure shows that the marginal utility follows a downward trend during the entire observation period. The reason is the secular increase in  $m$  during that period, resulting from the monetary easing conducted by the BOJ since 2009:Q1, especially from the Kuroda bazooka since 2013. Interestingly, it seems that the marginal utility and the opportunity cost move more or less parallel to each other. In contrast, the estimated marginal storage cost of money has remained almost constant over time, albeit with some temporary ups and downs. The lack of a secular increase in the marginal cost suggests that money storage cost is not a convex but a linear function of  $m$ .

The second exercise based on the money demand functions estimated consists of calculating the extent to which the money-income ratio deviates from the optimum quantity of money. The results are presented in the upper panel of Table 2. Comparing the money-income

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<sup>10</sup>The calculation in this subsection is based on the estimation results for the specification with  $\theta = 0.005$ :  $A = \exp(-1.5382)$  and  $\alpha = -0.3162$ .

ratio with the optimum quantity of money in the most recent three quarters shows that, in all four specifications, the money-income ratio in the most recent quarter exceeds the optimum quantity of money by more than 50%. Even before the COVID crisis (i.e., 2019:Q4), it exceeded the optimal level by 30%.

The third exercise consists of calculating Bailey’s welfare loss. Specifically, Bailey’s welfare loss is calculated using Eq. (11) for the case of linear storage costs, with three different values used for  $\theta$  ( $\theta = 0.005, 0.01, 0.02$ ). For the case of convex storage costs, the corresponding integral value is numerically calculated using Eq. (8). Figure 7 presents the results. It indicates that Baily’s welfare loss was more than 0.8% of nominal GDP in the 1980s and the first half of the 1990s, when the money-income ratio was stable and remained somewhere between 0.3 and 0.4 (see Figure 1). This is of the same order of magnitude as the estimate by Lucas (2000) for the US economy, which shows that a nominal interest rate of 2% leads to a welfare loss of 0.8% of nominal GDP. Turning to the most recent quarter, Table 2 shows that the estimated welfare loss ranges from 0.20% to 0.23% of nominal GDP. Even before the [outbreak of the] COVID crisis (i.e., 2019:4Q), the welfare loss was not negligible but much smaller, ranging from 0.07% to 0.08%.

## 5 Summary and Policy Implications

Using a model that explicitly considers the cost of storing money, this study examined the effect of the “Kuroda bazooka” launched by the BOJ in 2013 under the new Governor Haruhiko Kuroda. The main results of this study can be summarized as follows. First, the opportunity cost of holding money, calculated using 1-year government bond yields and the interest rate on money held in savings accounts, has been negative since the fourth quarter of 2014 and most recently (2020:Q2) was  $-0.2\%$ . Second, the marginal utility of money in the second quarter of 2020 was  $0.4\%$ , while the marginal cost of holding money was  $0.6\%$ . The difference between the two corresponds to the opportunity cost of holding money, which was  $-0.2\%$  in the same quarter. Third, the optimum quantity of money, at which the marginal utility of money equals the marginal cost of storing money, is 1.2. In contrast, the actual ratio of M1 to GDP in the second quarter of 2020 was 1.7, meaning that the quantity of money in Japan is excessive. Fourth, measuring welfare following Bailey (1956) as the sum of the consumer

surplus and seigniorage, the welfare loss relative to the maximum welfare obtained under the optimum quantity of money currently (2020:Q2) corresponds to 0.2% of nominal GDP.

Considering the BOJ's future policies based on these results, the BOJ should not continue to increase the supply of money but should start reducing it instead. Specifically, the ratio of M1 to nominal GDP needs to be reduced by more than 30% from the current 1.8 to 1.2. To do so, the opportunity cost of holding money needs to be raised. One conceivable way to achieve this would be to boost CPI inflation by about 0.2 percentage points and raise nominal interest rates by the same margin, thereby reducing the relative attractiveness of money. In practice, however, the experience with monetary easing since 2013 shows that boosting CPI inflation would not be that easy. One possible alternative therefore would be to penalize the holding of money to make it less attractive. Specifically, as proposed by Agarwal and Kimball (2019) and others, it is worth considering the introduction of a mechanism that allows applying negative interest to money, such as the introduction of central bank digital currency, and setting the interest rate on money to about  $-0.3\%$ .

## References

- [1] Agarwal, Ruchir, and Miles S. Kimball. 2019. "Enabling Deep Negative Rates to Fight Recessions: A Guide." IMF Working Paper WP/19/84.
- [2] Aoki, Kosuke, Hibiki Ichiue, and Tatsushi Okuda. 2019. "Consumers' Price Beliefs, Central Bank Communication, and Inflation Dynamics." Bank of Japan Working Paper Series No. 19-E-14.
- [3] Bailey, Martin J. 1956. "The Welfare Cost of Inflationary Finance." *Journal of Political Economy*, 64(2): 93-110.
- [4] Baumol, William J. 1952. "The Transactions Demand for Cash: An Inventory Theoretic Approach." *Quarterly Journal of Economics*, 66(4): 545-56.
- [5] Brunnermeier, Markus K., and Yann Koby. 2018. "The Reversal Interest Rate." *NBER Working Paper* No. 25406.

- [6] Diamond, Jess, Kota Watanabe, and Tsutomu Watanabe. 2020. “The Formation of Consumer Inflation Expectations: New Evidence From Japan’s Deflation Experience.” *International Economic Review* 61(1), 241-281.
- [7] Dupraz, Stephane. 2017. “A Kinked-Demand Theory of Price Rigidity.” Banque de France Working Paper #656.
- [8] Eggertsson, Gauti B., Ragnar E. Juelsrud, Laurance H. Summers, and Ella Getz Wold. 2019. “Negative Nominal Interest Rates and the Bank Lending Channel.” *NBER Working Paper* No. 25416.
- [9] Eggertsson, Gauti, and Michael Woodford. 2003. “The Zero Bound on Interest Rates and Optimal Monetary Policy.” *Brookings Papers on Economic Activity* 34(1), 139-235.
- [10] Friedman, Milton. 1969. “The Optimum Quantity of Money.” In Milton Friedman, *The Optimum Quantity of Money and Other Essays*, 1-50. Chicago: Aldine Publishing Company.
- [11] Hicks, John R. 1937. “Mr. Keynes and the ‘Classics’: A Suggested Interpretation.” *Econometrica*, 5(2): 147-59.
- [12] International Monetary Fund. 2018. “JAPAN: Staff Report for the 2018 Article IV Consultation.” November 6, 2018.
- [13] Krugman, Paul R. 1998. “It’s Baaack : Japan’s Slump and the Return of the Liquidity Trap.” *Brookings Papers on Economic Activity*, 1998(2), 137-205.
- [14] Lucas, Robert E., Jr. 2000. “Inflation and Welfare.” *Econometrica*, 68(2): 247-74.
- [15] McCallum, Bennett T., and Marvin S. Goodfriend. 1989. “Demand for Money: Theoretical Studies.” In John Eatwell, Murray Milgate, and Peter Newman, eds., *Money*, 117-30. London: Palgrave Macmillan.
- [16] Miyao, Ryuzo. 2002. “Liquidity Traps and the Stability of Money Demand: Is Japan Really Trapped at the Zero Bound?” *RIEB Discussion Paper* No. 127.

- [17] Nakashima, Kiyotaka, and Makoto Saito. 2012. "On the Comparison of Alternative Specifications for Money Demand: The Case of Extremely Low Interest Rate Regimes in Japan." *Journal of the Japanese and International Economies*, 26(3): 454-471.
- [18] Negishi, Takashi. 1979. *Microeconomic Foundations of Keynesian Macroeconomics*. Vol. 27. Amsterdam; New York: North-Holland Publishing Company.
- [19] Ravallion, Martin. 2017. "A Concave Log-Like Transformation Allowing Non-Positive Values." *Economics Letters*, 161, 130-132.
- [20] Rognlie, Matthew. 2016. "What Lower Bound? Monetary Policy with Negative Interest Rates." [http://mattrognlie.com/negative\\_rates.pdf](http://mattrognlie.com/negative_rates.pdf) (accessed September 6, 2019).
- [21] Schmiedel, Heiko, Gergana L. Kostova, and Wiebe Ruttenberg. 2012. "The Social and Private Costs of Retail Payment Instruments: A European Perspective." ECB Occasional Paper 137.
- [22] Sidrauski, Miguel. 1967. "Rational Choice and Patterns of Growth in a Monetary Economy." *American Economic Review*, 57(2): 534-544.
- [23] Taylor, John B. 2000. "Low Inflation, Pass-Through, and the Pricing Power of Firms." *European Economic Review*, 44(7), 1389-1408.
- [24] Thornton, Daniel L. 1999. "Nominal Interest Rates: Less Than Zero?" Federal Reserve Bank of St. Louis, *Monetary Trends*, January 1999.
- [25] Tobin, James. 1956. "The Interest-Elasticity of Transactions Demand for Cash." *Review of Economics and Statistics*, 38(3): 241-47.
- [26] Watanabe, Kota, and Tsutomu Watanabe. 2018. "Why has Japan Failed to Escape from Deflation?" *Asian Economic Policy Review* 13(1), 23-41.
- [27] Watanabe, Tsutomu, and Tomoyoshi Yabu. 2018. "The Demand for Money at the Zero Interest Rate Bound." *CARF Working Paper Series*, CARF-F-444, September 2018.

- [28] Watanabe, Tsutomu, and Tomoyoshi Yabu. 2019. “How Large is the Demand for Money at the ZLB? Evidence from Japan.” *CARF Working Paper Series*, CARF-F-465, September 2019.
- [29] Wolman, Alexander L. 1997. “Zero Inflation and the Friedman Rule: A Welfare Comparison.” *Federal Reserve Bank of Richmond Economic Quarterly*, 83(4): 1-20.

Table 1: Monetary Policy Decisions Made by the Bank of Japan in 2013-2020

Jan 22, 2013	Joint statement by the government and the BOJ on overcoming deflation and achieving sustainable economic growth. BOJ set an inflation target of 2%.
Apr 4, 2013	BOJ introduced Quantitative and Qualitative Monetary Easing (QQE). BOJ decided to double the monetary base to achieve the 2% inflation target with a time horizon of about two years.
Oct 31, 2014	BOJ expanded QQE by increasing its annual net purchases of JGBs from 50 trillion yen to 80 trillion yen.
Jan 29, 2016	BOJ decided to apply a negative interest rate of minus 0.1% to part of BOJ current account balances.
Sep 21, 2016	BOJ decided to employ yield curve control (YCC) with the target level of 10-year JGB yields set to zero.
July 31, 2018	BOJ decided to modify YCC by allowing 10-year JGB yields to fluctuate within a band of -0.2 to 0.2%
April 27, 2020	BOJ decided to enhance monetary easing in response to the outbreak of COVID-19 by (1) increasing purchases of CP and corporate bonds up to 20 trillion yen; (2) strengthening the loan facility for financial institutions to help them lend to firms; and (3) conducting active purchases of JGBs and T-bills.

Table 2: Money-Income Ratio Before and During the Corona Crisis

	Linear storage costs			Convex storage costs
	$\theta = 0.005$	$\theta = 0.010$	$\theta = 0.020$	$\mu = 100$
	<i>Deviation from Optimum</i>			<i>Quantity of Money</i>
2019:Q4	30.1%	28.1%	27.2%	26.6%
2020:Q1	33.2%	31.1%	30.2%	29.6%
2020:Q2	53.7%	51.4%	50.3%	49.6%
	<i>Bailey's Measure for the Welfare Loss</i>			
2019:Q4	0.07%	0.07%	0.08%	0.08%
2020:Q1	0.08%	0.08%	0.09%	0.09%
2020:Q2	0.18%	0.20%	0.22%	0.23%

**Note:** The estimate for the optimum quantity of money is 1.147 for linear storage costs with  $\theta = 0.005$ , 1.165 for  $\theta = 0.01$ , 1.173 for  $\theta = 0.02$ , and 1.179 for convex storage costs with  $\mu = 100$ . The money-income ratio is 1.493 in 2019:Q4, 1.528 in 2020:Q1, and 1.764 in 2020:Q2.

Figure 1: Bailey's Measure for the Welfare Loss

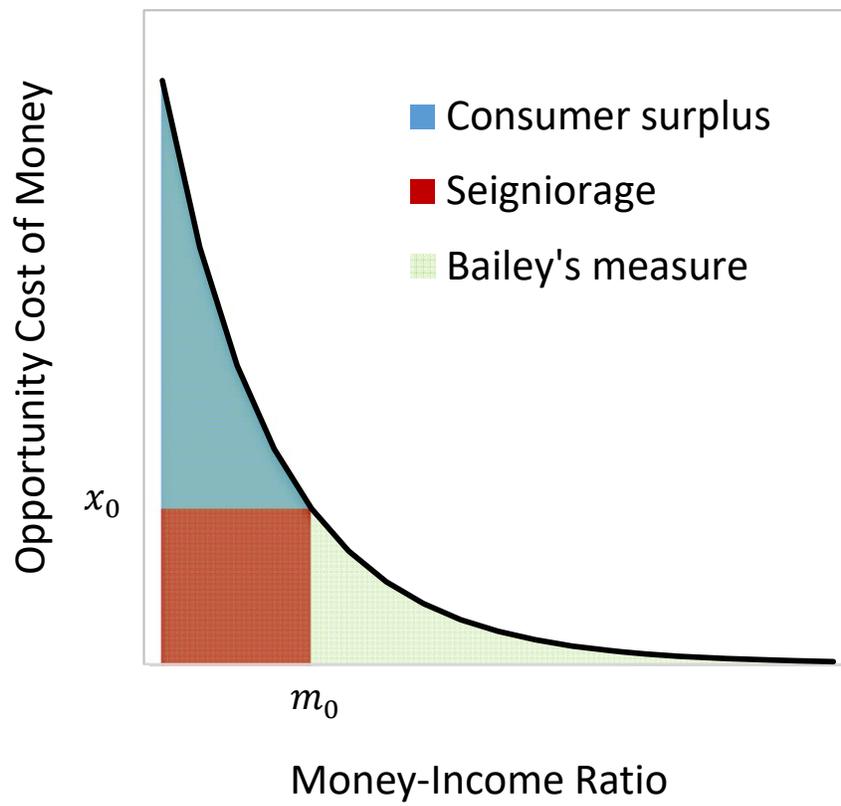
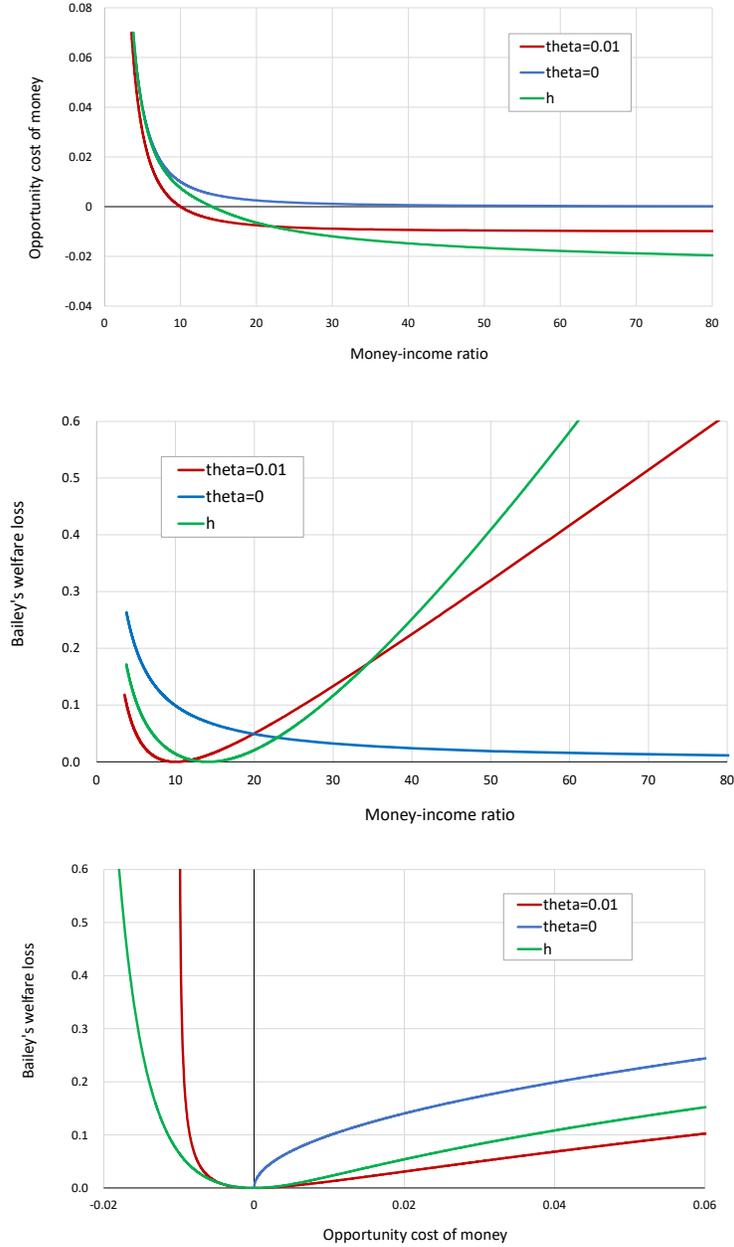
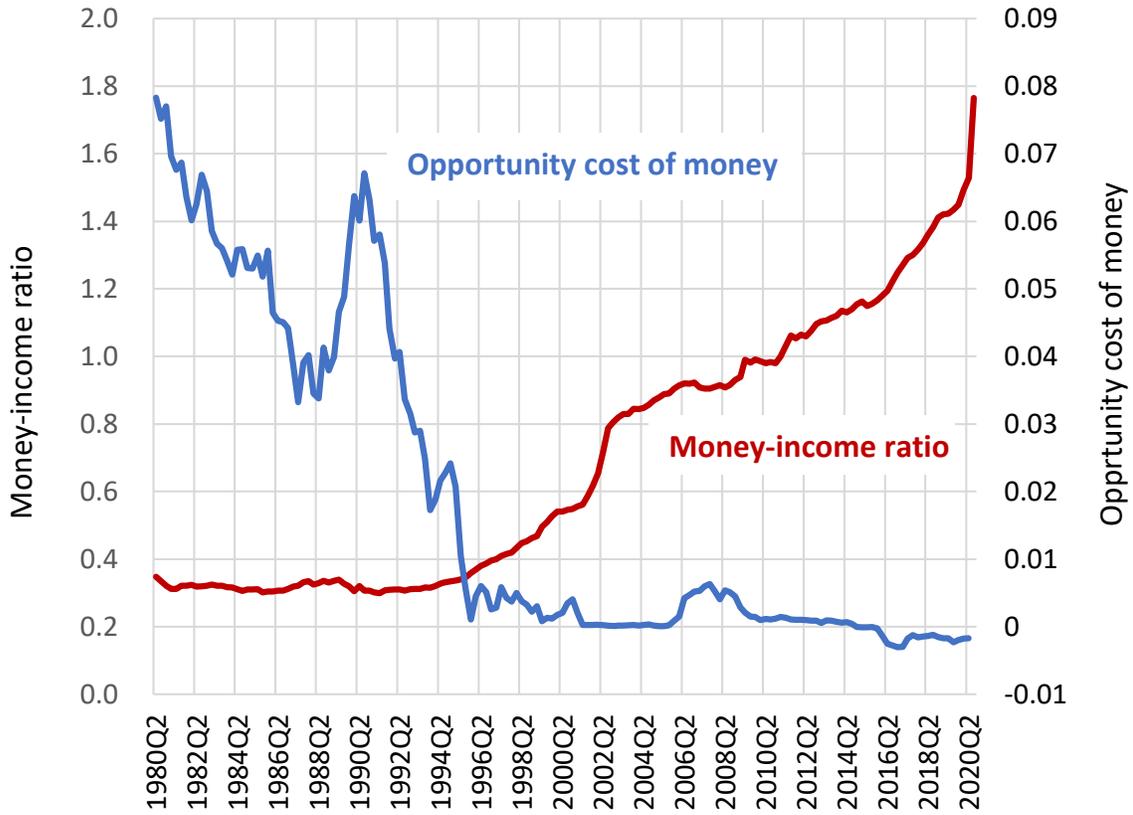


Figure 2: Numerical Example



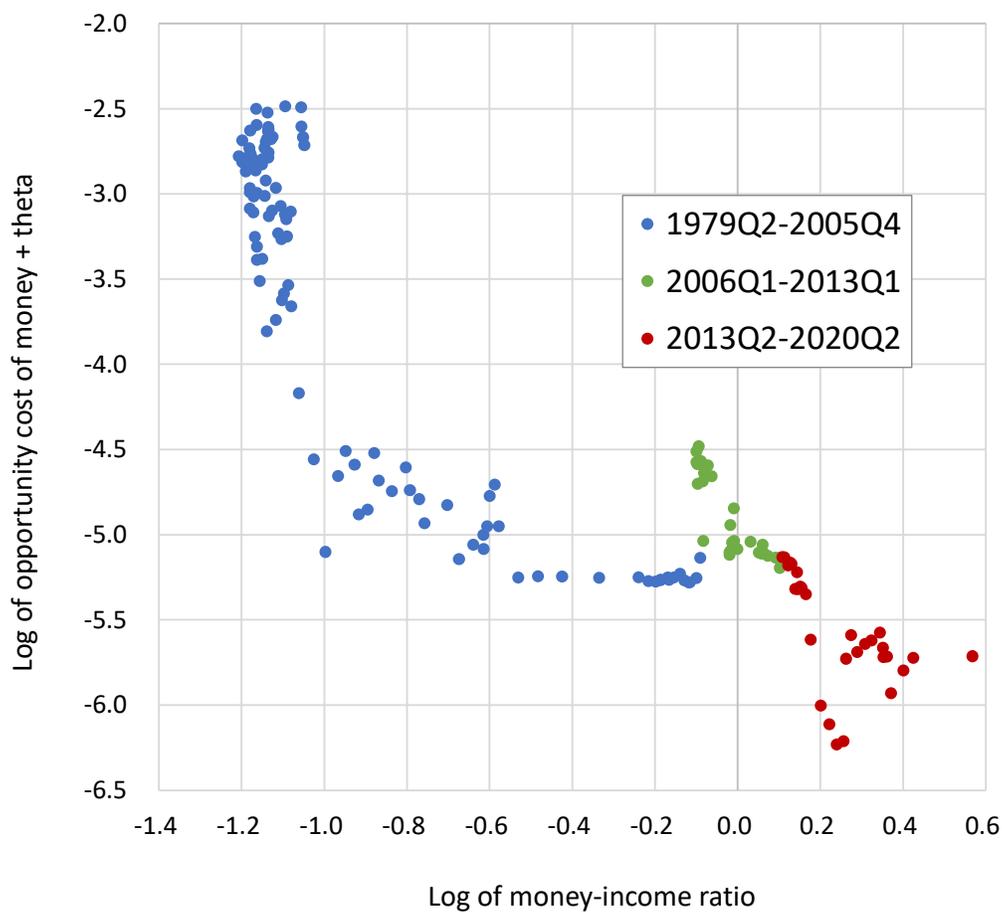
**Note:**  $m$  and  $w(x)$  are calculated using Eqs. (4), (7), (9), (10) and (11) with  $A$  and  $\alpha$  set to  $A = 1$  and  $\alpha = -0.5$ . The blue line corresponds to the case of no storage costs of money. The red line corresponds to the case of linear storage costs with  $\theta$  set to  $\theta = 0.01$ . The green line corresponds to the case of convex storage costs with  $\mu$  set to  $\mu = 100$ .

Figure 3: M1 and the Opportunity Cost of Holding Money



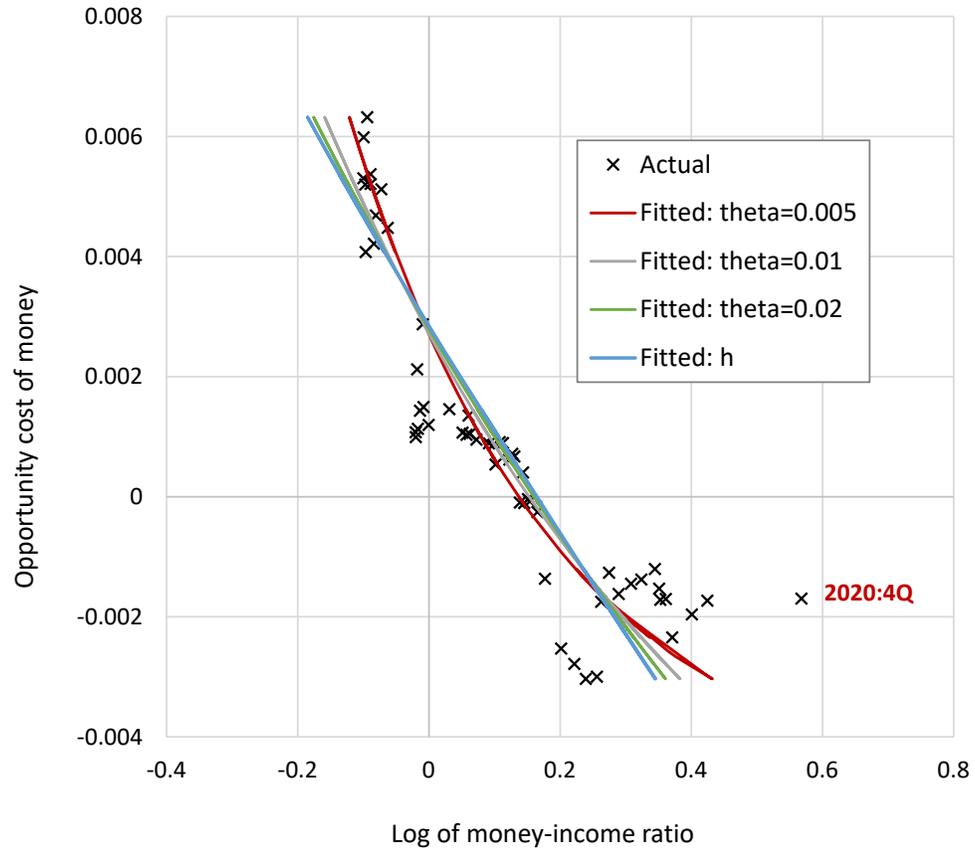
**Note:** The money-income ratio is defined as M1 divided by nominal GDP. The opportunity cost of money is defined as the 1-year JGB yield minus the interest rate on saving deposits.

Figure 4: Demand for Money in 1979-2020



**Note:** The horizontal axis shows the log of the money-income ratio, while the vertical axis shows the log of the opportunity cost of money plus  $\theta$ , which is set to  $\theta = 0.005$ .

Figure 5: Estimated Money Demand Functions



**Note:** The observation period is 2006:Q2-2020:Q2. The estimated results are as follows:

$$\begin{aligned}
 \ln m &= -0.3162 \times \ln(x + 0.005) - 1.5382 && \text{for } \theta = 0.005; \\
 \ln m &= -0.6362 \times \ln(x + 0.01) - 2.7770 && \text{for } \theta = 0.01; \\
 \ln m &= -1.2217 \times \ln(x + 0.02) - 4.6196 && \text{for } \theta = 0.02; \\
 \ln m &= -0.5863 \times h(x) - 2.9417 && \text{for } h(\cdot) \text{ function.}
 \end{aligned}$$

Figure 6: Estimate of the Marginal Utility of Money

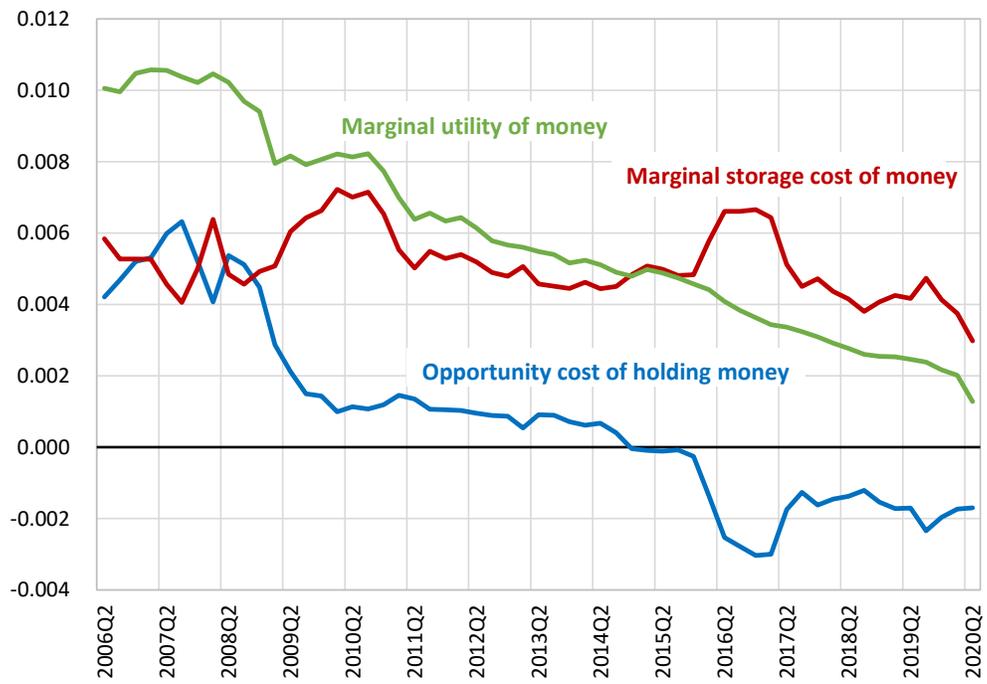


Figure 7: Welfare Loss Due to Massive Money Injection

