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Relative Prices and Inflation Stabilisation

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Abstract

When price adjustment is sluggish, inflation is costly in terms of welfare because it distorts various kinds of relative prices. Stabilising aggregate price inflation does not necessarily minimise these costs, but stabilising a well-designed core inflation minimises the cost of relative price fluctuations and thus the cost of inflation.

Relative prices JEL Classification: E3.

Key Words: Relative prices, inflation.

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1 Concepts of Core Inflation

In macroeconomic theories, the aggregate price level, often denoted as P, is defined as the monetary value of the minimum cost of attaining a reference utility level. Measures of the price level are called a "cost-of-living" index. Measuring the price level has been an important topic in macroeconomics, because any fluctuation in the price level —inflation—is regarded as affecting the well-being of households.

A common measure of the price level is the consumer price index (CPI). The CPI is a Laspeyres index defined as:

$$CPI_{t} = \frac{\sum_{z=1}^{n} p_{t}(z)c_{0}(z)}{\sum_{z=1}^{n} p_{0}(z)c_{0}(z)} = \frac{\sum_{z=1}^{n} \frac{p_{t}(z)}{p_{0}(z)}w_{0}(z)}{\sum_{z=1}^{n} w_{0}(z)},$$
(1)

where $p_0(z)$ and $c_0(z)$ are the nominal price and quantity purchased of good z at the reference period 0, and $p_t(z)$ is the nominal price of good z at period t. Finally, n is the number of goods in the consumption bundle. Weight $w_0(z) \equiv p_0(z)c_0(z)$ represents the nominal expenditure on good z at the reference period 0. Since the consumption bundle $\{c_0(z)\}_{z=1}^n$ is fixed, CPI is designed to capture changes in the level of nominal spending that is required to achieve the reference consumption bundle $\{c_0(z)\}_{z=1}^n$. Even though CPI is not perfectly consistent with the theoretical definition of the price level, it is widely used because it is easy to compute in practice.

Figure 1 plots CPI in Japan from 1970 to 2014. There is a secular increasing trend in the price level until the 1990s, as in many advanced economies, but the price level has decreased since the 1990s.

Figure 1 here

Figure 2 plots the CPI inflation rate, which is the year-on-year growth rate of CPI. Roughly speaking, following high inflation in the 1970s, inflation has been very stable since the 1980s. Mild deflation has been observed since the 1990s.

Figure 2 here

As equation (1) shows, CPI is a weighted average of prices of many kinds of goods. Figure 3 plots movements in the price levels of different sectors. These prices tend to move in conjunction but there are significant divergences both in the long run and in the short run. Relative prices — the value of one unit of goods relative to another — change along with these fluctuations.

Figure 3 here

Among policy makers and practitioners, it is common practice to remove the effects of fluctuations in the prices of food and energy from overall inflation measures. In Japan, the CPI inflation measure that excludes fresh food is called "core" inflation, and that which excludes both food and energy is called "core-core" inflation.¹ Figure 4 plots CPI inflation, core inflation and core-core inflation.

Figure 4 here

Core-core inflation is more stable than the other inflation measures. This is also consistent with Figure 3, which shows that the relative price of energy is very volatile. By excluding energy price inflation, core-core inflation seems to capture a stable part of inflation.

In a valuable review of core inflation, Wynne (2008) argues that core inflation should capture monetary inflation that is of concern to the central bank, and that this should be conceptually different from the cost of living index. Wynne states that:

¹From a practical point of view, core-core inflation in Japan roughly corresponds to core inflation in the United States. Both measures exclude food and energy price inflation.

...it is argued that central banks ought to target a price index whose rate of increase corresponds to the inflation that generates the cost that central banks are seeking to avoid by focusing on an inflation-control objective.

Even though some cost of inflation can be captured by the level of cost-of-living inflation and its fluctuations, it is not entirely obvious that stabilising the cost-of-living inflation minimises all the costs of inflation. As Wynne (2008) argues, core inflation is not concerned with a microeconomic theory of the cost of living, but a macroeconomic theory of the cost of inflation.

Traditionally, measures of core inflation are designed to capture "monetary inflation" that the central bank can control. CPI excluding food, or CPI excluding food and energy, have been used very commonly by central banks in many countries to guide monetary policy. As Figure 3 shows, food and energy prices are very volatile and their fluctuations tend to be temporary. Monetary policy cannot effectively offset those fluctuations. This is because there are lags in the transmission of monetary policy. For example, according to Christiano et al. (1999), it takes about one year for the overall price level to start responding to a monetary policy shock. Given that a large part of the fluctuations in food and energy prices is transitory, these short-run fluctuations are beyond the control of central banks.

Also, since there are lags in the transmission of monetary policy, central banks in practice choose their policy based on economic forecasts. A typical example is inflation-targeting central banks such as the Bank of England and the Swedish Riksbank. Such inflation-targeting central banks are in practice basing their policy on inflation *forecast* targeting. Under such policy regimes, central banks choose their policy instruments to target inflation forecasts. A typical forecast horizon is two years. It is therefore important to identify inflation measures that are accurate forecasts of future CPI inflation. Again, CPI excluding food and energy is a good measure because it is a good forecaster of future CPI. Shiratsuka (2006) reports that the core CPI inflation measure is a good forecaster of

future headline inflation in Japan. For these two reasons, CPI excluding food and energy has been the most common measure of core inflation.

However, the above arguments are not really concerned with the welfare costs of inflation fluctuations. Core inflation and core-core inflation are not the cost-of-living indices because they put zero weights on particular kinds of goods. When the relative price of food increases, the well-being of some households may decrease but the core measures do not necessarily capture those possible changes in well-being. Why is it appropriate to ignore changes in the relative price of certain goods? More generally, what are the welfare costs of inflation fluctuations? In order to minimise these costs, which price indices should be stabilised?

In what follows, we review the results of the literature based on the New Keynesian framework.² The New Keynesian models emphasise the role of price stickiness in inflation dynamics and are widely used for the analysis of inflation fluctuations and monetary policy. Since these models are based on the optimising behaviour of firms and consumers, it is possible to analyse what prices should be stabilised in order to maximise economic welfare. The general principle obtained from the literature is that it is the inflation of sticky price sectors that should be stabilised in order to maximise economic welfare. Therefore sticky price inflation is identified as a measure of core inflation that is consistent with economic theory.

In order to examine the robustness of this result, we consider two approaches to modelling price stickiness: time-dependent pricing, and state-dependent pricing. Under timedependent pricing, the opportunity for firms to change their prices depends on the time since they previously changed their prices. For example, price contracts can be revised every 12 months, but not every month. The Calvo (1983) model is one of the most popular models of time-dependent pricing. Under state-dependent pricing, firms are able to change their prices only if they pay a fixed menu cost of changing prices. Whether they

²For a textbook treatment of this framework, see Woodford (2003)

change their prices or not depends on the deviations of their current prices from their optimal prices. Price adjustment by firms involves two margins. The intensive margin refers to the size of price adjustment of firms who change their prices. The extensive margin refers to the number of firms who change their prices. In time-dependent pricing models, only the intensive margin is considered, while both of the margins are considered in state-dependent pricing rules. This difference has potentially significant implications for monetary policy. Under state-dependent pricing, monetary policy can use not only the intensive margin but also the extensive margin — it can change the degree of price stickiness. Intuitively speaking, if the bank chooses higher inflation, firms in more sectors may change their prices. Increased price flexibility may be welfare-improving. We analyse how the concept of core inflation may depend on different modelling assumptions regarding price stickiness.

Under state-dependent pricing, whether prices in a certain industry is flexible or not may depend on monetary policy. It is shown, at least in the simple model considered in this article, that it is not optimal for the central bank to use the intensive margin. This implies that it is not desirable to use an increase in the aggregate inflation rate to improve price flexibility. As a result, under both state-dependent pricing and time-dependent pricing, inflation of the sticky price sectors should be stabilised.

2 Canonical model:

In this section we present a very simple static general equilibrium model with price stickiness. One may wonder whether a static model is appropriate for an analysis of the cost of inflation. As is explained in detail below, the cost of inflation emphasised in the New Keynesian framework is the distortions in relative prices that are caused by price stickiness. A static framework provides some very simple analytical results on how price stickiness distorts various kinds of relative prices.

2.1 Households

The model is a static two-sector general equilibrium model with monopolistic competition. The utility of the representative household is given by

$$U = \log C - N,\tag{2}$$

where C represents a consumption composite defined by

$$C = \frac{c_1^{\alpha_1} c_2^{\alpha_2}}{\alpha_1^{\alpha_1} \alpha_2^{\alpha_2}}, \ \alpha_1 + \alpha_2 = 1,$$
(3)

and N represents labour supply. The consumption composite for sector i is defined by

$$c_i = \left[\int_0^1 c_i(z)^{\frac{\theta_i - 1}{\theta_i}} dz\right]^{\frac{\theta_i}{\theta_i - 1}}, i = 1, 2$$

$$\tag{4}$$

where $c_i(z)$ denotes consumption of differentiated good z in sector i. Parameter θ_i represents the price elasticity of demand for good z, and we assume $\theta_i > 1$. The budget constraint of the representative household is given by

$$\int_0^1 p_1(z)c_1(z)dz + \int_0^1 p_2(z)c_2(z)dz = wN + D - T$$
(5)

where $p_i(z)$, w, D, and T respectively denote the nominal price of good z in sector i, nominal wage, nominal dividend from firms, and nominal lump-sum taxes. The household chooses consumption demand for each good $(c_i(z))$ and labour supply (N) in order to maximise utility. As is well known, utility maximisation leads to the following demand functions and price indices. The aggregate price level, P, is defined by

$$P = P_1^{\alpha_1} P_2^{\alpha_2} \tag{6}$$

where sectoral price level, P_i , is defined by

$$P_{i} = \left[\int_{0}^{1} p_{i}(z)^{1-\theta_{i}} dz \right]^{\frac{1}{1-\theta_{i}}}.$$
(7)

The aggregate price index is derived from the cost minimisation problem of the household subject to attaining a unit level of aggregate consumption index C. Note also that Ccan be interpreted as a sub-utility function that is defined over c_1 and c_2 . In this sense, P represents a well-defined cost of price index — namely, the monetary value of the minimum amount of expenditure that is needed to achieve unit utility. Similarly, P_i , represents the monetary value of the minimum amount of expenditure that is needed to achieve unit sub-utility c_i . Then demand for each composite good c_i in sector i is given by

$$c_i = \alpha_i \left(\frac{P_i}{P}\right)^{-1} C,\tag{8}$$

and demand for each differentiated good is given by

$$c_i(z) = \left(\frac{p_i(z)}{P_i}\right)^{-\theta_i} c_i.$$
(9)

Finally, labour supply decision is given by

$$\frac{1}{C}\frac{W}{P} = 1.$$
(10)

The last equation (10) implies

$$W = PC. \tag{11}$$

Therefore, in this simple model, there is a one-to-one relationship between nominal wage and nominal spending.

2.2 Firms

Firms are under monopolistic competition. Consider a firm that produces good z in sector i. Its production technology is given by

$$y_i(z) = A_i n_i(z), \tag{12}$$

where A_i represents productivity of firms in sector *i*. The firm's profit function is given by

$$D_i(z) = (1 + \tau_i)p_i(z)y_i(z) - wN_i(z),$$
(13)

where τ_i denotes government subsidies for firms in sector *i*, and $N_i(z)$ represents the firm's labour demand. Following the literature (Rotemberg and Woodford (1997)) we introduce this government subsidy in order to eliminate any economic distortion that stems from monopolistic competition. Then the sole role of monetary policy is to remove economic distortions caused by the inability of firms to change their prices in response to changes in economic conditions such as productivity.

2.3 Monetary Policy and Fiscal Policy

We assume for simplicity that monetary policy controls nominal spending:

$$M = PC. \tag{14}$$

Using (11) and (14), we obtain

$$W = M. \tag{15}$$

Therefore, in our model, controlling nominal spending is equivalent to controlling nominal wages. We assume a balanced budget

$$T = \tau_1 \int_0^1 p_1(z)y_1(z)dz + \tau_2 \int_0^1 p_2(z)y_2(z)dz.$$
 (16)

The right hand side represents total subsidies to firms in sectors 1 and 2.

2.4 Benchmark: the Flexible Price Equilibrium

As a benchmark case, let us define the flexible price equilibrium in which all firms change their prices. Firm z in sector i chooses $p_i(z)$ to maximise its profit (13) subject to production function (12) and demand function (9). This leads to

$$p_i(z) = \mu_i (1 + \tau_i) \frac{W}{A_i},$$
(17)

where $\mu_i \equiv \frac{\theta_i}{\theta_i - 1}$ represents the markup of firms in sector *i*, and W/A_i represents the nominal marginal cost of firms in sector *i*. In what follows we assume that

$$\mu_i (1 + \tau_i) = 1. \tag{18}$$

Market clearing for each good is given by

$$y_i(z) = c_i(z), i = 1, 2, \quad z \in [0, 1].$$
 (19)

Finally, labour market also clears.

Let us define aggregate productivity by

$$A \equiv A_1^{\alpha_1} A_2^{\alpha_2}. \tag{20}$$

Then, it is easily shown that the flexible price equilibrium is given by

$$y_i(z) = Y_i = \alpha_i A_i \tag{21}$$

$$C = A \tag{22}$$

$$\frac{P_1}{P} = \frac{A}{A_1} \tag{23}$$

$$\frac{P_2}{P} = \frac{A}{A_2} \tag{24}$$

$$\frac{W}{P} = A \tag{25}$$

$$N = 1 \tag{26}$$

Finally, the aggregate price level is

$$P = \frac{M}{C} = \frac{M}{A} \tag{27}$$

Since the production subsidy satisfies (18), this equilibrium is Pareto efficient.

2.5 Initial State of the Economy

In the next section, we introduce price stickiness to the model economy. For this purpose, assume that, before any shocks occur, the economy is at the efficient equilibrium described in Section 2.4. Assume that the initial technology levels are \bar{A}_1 and \bar{A}_2 and the corresponding aggregate productivity is $\bar{A} \equiv \bar{A}_1^{\alpha_1} \bar{A}_2^{\alpha_2}$. Then the initial state of the economy is

$$\bar{y}_i(z) = \bar{Y}_i = \alpha_i \bar{A}_i \tag{28}$$

$$\bar{p}_i(z) = \bar{p}_i(z') = P_i \text{ for any } z, z'$$
(29)

$$\bar{C} = \bar{A} \tag{30}$$

$$\frac{\bar{P}_1}{\bar{P}} = \frac{\bar{A}}{\bar{A}_1} \tag{31}$$

$$\frac{\bar{P}_2}{\bar{P}} = \frac{\bar{A}}{\bar{A}_2} \tag{32}$$

$$\frac{\bar{W}}{\bar{P}} = \bar{A} \tag{33}$$

$$\bar{N} = 1 \tag{34}$$

Finally, for simplicity, we normalise \bar{P} as

$$\bar{P} = 1 \tag{35}$$

so that

$$\bar{W} = \bar{M} = \bar{A} \tag{36}$$

Note that the relative price of any pair of goods in the same sector is equal to one. This is because the firms face symmetric demand curves (9) and their production technology (12) is identical. Now we analyse how the economy responds to changes in A_1 , A_2 and M.

3 Optimal Monetary Policy under Time-Dependent Sticky Prices

In this section we discuss optimal policy under sticky prices. We focus our analysis on the question of what kind of price level should be stabilised in the optimal equilibrium. Is it P, P_1 , or P_2 that should be stabilised? Or is it something else that should be stabilised?

3.1 One Sector Time-Dependent Sticky Price Model

Firstly, we replicate a standard result in the literature. Assume that there is only one sector. This corresponds to the case where $\alpha_1 = 1$ and $\alpha_2 = 0$. Then, $C = c_1$, $P = P_1$. Regarding the price stickiness, we start with time-dependent pricing. Under time-dependent pricing, a fraction of firms changes their prices in each period while the others do not, and the number of such firms is exogenously given³. Assume that a fraction $1-\kappa$ of firms changes their prices in response to shocks, while the remaining fraction κ of firms keeps their prices at their original levels \bar{p} . Let p^* denote the price of the firms that change their prices. We call those firms "flexible-price firms". Since they are identical, all of them choose the same price. The optimal price of the flexible-price firms is given by

$$p^* = \frac{W}{A} = \frac{M}{A},\tag{37}$$

while the price of the sticky-price firms is

$$\bar{p} = \frac{\bar{W}}{\bar{A}}.$$
(38)

In this case, the price level becomes

$$P = \left[(1 - \kappa)(p^*)^{1-\theta} + \kappa \bar{p}^{1-\theta} \right]^{\frac{1}{1-\theta}}.$$
(39)

Output for the firms that change their prices is

$$y(z;p^*) = \left(\frac{p^*}{P}\right)^{-\theta} C,$$
(40)

and for firms that keep their prices fixed is

$$y(z;\bar{p}) = \left(\frac{\bar{p}}{P}\right)^{-\theta} C.$$
(41)

³On the contrary, under the state-dependent pricing that we analyse later, whether or not firms choose to change their prices is endogenously determined.

Aggregate output is determined by

$$C = \frac{M}{P},\tag{42}$$

and equilibrium labour is

$$N = \left[(1 - \kappa) \left(\frac{p^*}{P}\right)^{-\theta} + \kappa \left(\frac{\bar{p}}{P}\right)^{-\theta} \right] \frac{C}{A}.$$
 (43)

Now we show that it is possible to replicate the flexible price equilibrium by choosing M appropriately, and that this policy results in price stability⁴. By choosing

$$M = \frac{A}{\bar{A}}\bar{W},\tag{44}$$

the central bank can set

$$p^* = \bar{p} \tag{45}$$

so that the flexible-price firms keep their prices constant even if they can change their prices. As a result,

$$P = \bar{P}.\tag{46}$$

Namely, the aggregate price level remains constant, and the relative price of any firm is

$$\frac{p(z)}{P} = \frac{\bar{p}(z)}{P} = 1.$$
(47)

Therefore, demand for each good is identical:

$$c(z) = C. (48)$$

Then,

$$C = \frac{M}{P} = \frac{A}{\bar{A}}\bar{W}\frac{1}{\bar{P}} = A,$$
(49)

⁴For a more general analysis of the optimality, see, for example, Rotemberg and Woodford (1997) and King and Wolman (1999).

and, from the production function, equilibrium labour is

$$N = \frac{1}{A} \int_0^1 c(z) dz = 1.$$
 (50)

Therefore, the equilibrium is identical to the efficient flexible-price equilibrium in Section 2.4. The intuition behind this result is as follows. We assume that the government removes distortions caused by monopolistic competition, and this is why the flexible-price equilibrium is efficient. The only distortion that monetary policy seeks to minimise is the distortion caused by price stickiness. In the one-sector model, the bank can replicate the flexible price allocation by creating an environment in which firms that can change their prices do not have to change their prices. This is efficient because, in the efficient allocation, the relative price of any good should be equal to one.

The above analysis shows that it is optimal to stabilise the aggregate price index. But since there is only one sector, there is no meaningful distinction between inflation of the aggregate price level and core inflation. Next we consider a two-sector economy.

3.2 One Flexible-Price Sector and One Sticky-Price Sector

Let us go back to the two-sector setting: $\alpha_2 > 0$. Assume that all firms in sector 2 can change their prices. We call sector 2 the "flexible-price sector". Firms in sector 1, which we call the "sticky-price sector", are subject to price stickiness. A fraction $1 - \kappa_1$ of firms can change their prices in response to shocks while the remaining fraction κ_1 keeps their prices fixed.

Consider the firms in the flexible-price sector. Their pricing behaviour is

$$p_2^*(z) = \frac{W}{A_2} = \frac{M}{A_2} = P_2,$$
(51)

so that the relative price of any pair of goods in sector 2 is equal to 1. In equilibrium,

$$c_2(z) = c_2(z') = \alpha_2 c_2 = \alpha_2 \left(\frac{P_2}{P}\right)^{-1} C.$$
 (52)

Next, consider the firms in the sticky-price sector. The firms that can change their prices set their prices as

$$p_1^*(z) = \frac{W}{A_1} = \frac{M}{A_1},\tag{53}$$

while the other firms keep their prices fixed

$$\bar{p}_1(z) = \frac{\bar{W}}{\bar{A}_1}.\tag{54}$$

Then the price index of sector 1 is

$$P_1 = \left[(1 - \kappa_1) (p_1^*)^{1 - \theta_1} + \kappa_1 \bar{p}_1^{1 - \theta_1} \right]^{\frac{1}{1 - \theta_1}}$$
(55)

and the demand for each firm that changes its price is

$$c_1(z; p_1^*) = \left(\frac{p_1^*}{P_1}\right)^{-\theta_1} c_1 = \alpha_1 \left(\frac{p_1^*}{P_1}\right)^{-\theta_1} \left(\frac{P_1}{P}\right)^{-1} C,$$
(56)

and for firms that keep their prices fixed is

$$c_{1}(z;\bar{p}_{1}) = \left(\frac{\bar{p}_{1}}{P_{1}}\right)^{-\theta_{1}} c_{1} = \alpha_{1} \left(\frac{\bar{p}_{1}}{P_{1}}\right)^{-\theta_{1}} \left(\frac{P_{1}}{P}\right)^{-1} C.$$
 (57)

Here the aggregate price index is

$$P = P_1^{a_1} P_2^{a_2} \tag{58}$$

and aggregate demand is

$$C = \frac{M}{P}.$$
(59)

Now we show that it is also possible to replicate the flexible-price equilibrium. Suppose

that the central bank chooses for the money supply

$$M = \frac{A_1}{\bar{A}_1} \bar{W}.$$
(60)

Then, equation (53) implies that the optimal price for firms in sector 1 is given by

$$p_1^*(z) = \frac{\bar{W}}{\bar{A}_1} = \bar{p}_1(z).$$
 (61)

Therefore, firms in the sticky price sector will not change their prices even if they can do so. The aggregate price index of sector 1 is

$$P_1 = \frac{\bar{W}}{\bar{A}_1}.\tag{62}$$

The firms in the flexible price sector (sector 2) set their prices as

$$p_2^*(z) = \frac{M}{A_2} = \frac{A_1}{A_2} \frac{\bar{W}}{\bar{A}_1} = P_2.$$
(63)

Therefore, the relative price of any pair of goods in sector 1 and sector 2 is

$$\frac{P_1}{P_2} = \frac{A_2}{A_1}.$$
(64)

From (58), (59), (60) and (64), aggregate output is

$$C = A_1^{\alpha_1} A_2^{\alpha_2} = A \tag{65}$$

and equilibrium labour is given by

$$N = \frac{c_1}{A_1} + \frac{c_2}{A_2} = \left[\frac{\alpha_1 \left(\frac{P_1}{P}\right)^{-1}}{A_1} + \frac{\alpha_2 \left(\frac{P_2}{P}\right)^{-1}}{A_2}\right]C = 1.$$
 (66)

Therefore, by choosing monetary policy as in (60), the central bank can replicate the efficient flexible-price equilibrium.

In this equilibrium, the prices of the sticky price sector, P_1 , are fully stabilised, but the aggregate price level, P, is not. However, by stabilising P_1 , the relative price between the two sectors, P_1/P_2 , is efficient. This is the result reached by Aoki (2001), in which core inflation is identified as the inflation in the sticky price sector.

3.3 Generalisation

Behind the concept of core inflation is the idea that it should capture the macroeconomic cost of inflation fluctuations. In the New Keynesian models, the macroeconomic cost of inflation fluctuations is the relative-price distortion caused by price stickiness. Then, it is not entirely obvious that there is a one-to-one relationship between fluctuations in the cost of living index and relative-price distortions. In the previous section, we have shown that, when there is only one sector, the central bank should stabilise the aggregate price index —the cost of living index. When there is one sticky price sector and one flexible-price sector, then the central bank should stabilise the sticky price sector, not the overall cost of living index. Thus, in the latter case, core inflation and cost-of-living inflation are different.

The finding that it is optimal to stabilise sticky price inflation can be extended to more general settings. Benigno (2004) considers an economy with two sticky price sectors. In this case, it is no longer possible to replicate the flexible price equilibrium. The intuition behind this result is very simple. When the economy is subject to sector-specific shocks, the relative price of the two goods must reflect these shocks. In our model, relative productivity is represented by A_1/A_2 (see equation (64)). Relative price dispersion in sector 1 is eliminated when the price level in sector 1, P_1 , is fully stabilised. The same is also true for sector 2. But when P_1 and P_2 are both completely stabilised, the relative price, P_1/P_2 cannot respond to changes in relative productivity, A_2/A_2 . This is why monetary policy cannot replicate the flexible price equilibrium. However, Benigno (2004) shows that the optimal measure of inflation to be targeted is a weighted sum of the inflation measures in the two sectors, and the optimal weight depends on the degree of price stickiness in each sector. As a quantitative exercise, Eusepi et al. (2011) construct a price index whose weights minimise the welfare cost of nominal price stickiness. They compute these weights in a 15-sector New Keynesian model for the US economy and show that their weights depend mainly on the degree of price stickiness. They also show that their optimal inflation measure, which they call a "cost of nominal distortions index," corresponds closely to core inflation figures in the United States. Core inflation in the US excludes food and energy from its headline inflation figures.

4 Optimal Monetary Policy under State-Dependent Sticky Prices

We now turn our analysis to state-dependent pricing, which is regarded as capturing firms pricing decisions more accurately. In this modelling approach, firms are subject to a menu cost of changing prices. Recent contributions include Dotsey et al. (1999), Gertler and Leahy (2008) and Golosov and Lucas Jr (2007). In this section we attempt to examine how the results we obtained in the previous sections may change under state-dependent pricing. Analysis of optimal monetary policy under state-dependent pricing is still scarce. A notable exception are Nakov and Thomas (2014).

We assume that the firm pays a cost when it changes its price. When it changes its price, it chooses its optimal price given by

$$p_i^*(z) = p_i^* = \frac{W}{A_i}.$$
(67)

The maximised profit (before paying menu cost) is given by

$$\Pi_i(z; p_i^*) = (1 + \tau_i) p_i^* y(z; p_i^*) - \frac{W}{A_i} y(z; p_i^*)$$
$$= \frac{W}{A_i} \left(\frac{p_i^*}{P_i}\right)^{-\theta_i} c_i(\mu_i - 1).$$

When the firm keeps its price, \bar{p}_i , the profit is given by

$$\Pi_i(z;\bar{p}_i) = \frac{W}{A_i} \left(\frac{\bar{p}_i}{P_i}\right)^{-\theta_i} c_i \left[\mu_i \frac{\bar{p}_i}{p_i^*} - 1\right].$$
(68)

We assume that in order to change its price the firm needs to pay a fraction χ_i of its optimal profit⁵. Therefore, the firm does not choose to change its price when

$$\Pi_i(z; \bar{p}_i) \ge (1 - \chi_i) \Pi_i(z; p_i^*).$$
(69)

Equation (69) is equivalent to

$$x_i^{-\theta_i}\left(x_i - \mu_i^{-1}\right) \ge (1 - \chi_i)(1 - \mu_i^{-1}),\tag{70}$$

where

$$x_i \equiv \frac{\bar{p}_i}{p_i^*} \tag{71}$$

represents the deviation of the firm's old price, \bar{p}_i , from its optimal level, p_i^* . As is well known, the firm changes its price when x_i falls outside the "inaction" region X_i :

$$X_{i} = \left\{ x_{i} \mid x_{i}^{-\theta_{i}} \left(x_{i} - \mu_{i}^{-1} \right) \ge (1 - \chi_{i})(1 - \mu_{i}^{-1}) \right\} = \left[x_{L,i}, x_{H,i} \right]$$
(72)

$$\chi_i \frac{\Pi_i(z; p_i^*)}{P} = \chi_i \frac{1}{P} \frac{W}{A_i} \left(\frac{W/A_i}{P_i}\right)^{-\theta_i} c_i(\mu - 1)$$

⁵This assumption makes the firm's decision to change its price independent of the firm's production size, thus making the model tractable. In real term, the menu cost is given by

where

$$0 < x_{L,i} < 1 < x_{H,i}. (73)$$

Equation (70) implies that the firm is more likely to change its price when menu cost parameter, χ_i , is small, and when its price elasticity, θ_i , is large. A large elasticity means that the firm's profit is sensitive to its price and therefore the firm has a strong incentive to choose its price near its optimal level.

An important property of the model is that the degree of price stickiness depends on monetary policy. Equation (15) implies that monetary policy can change nominal wage Wby changing M. An inflationary policy (an increase in M) increases p_i^* and thus decreases x_i . If an increase in M is large enough to move x_i outside the inaction region X_i , then the firm will change its price and set it equal to p_i^* .

4.1 Optimal Policy in a One-Sector Menu-Cost Model.

When there is only one sector, then, as in the case of the time-dependent pricing model, it is possible to replicate the efficient flexible price equilibrium. To show this, suppose monetary policy is given by

$$M = \frac{A}{\bar{A}}\bar{W}.$$
(74)

Then,

$$p^*(z) = p^* = \frac{1}{A} \frac{A}{\bar{A}} \bar{W} = \frac{\bar{W}}{\bar{A}}$$
(75)

and therefore

$$x = 1. \tag{76}$$

The firms will not change their prices. In this case,

$$C = \frac{M}{P} = A, \quad \frac{W}{P} = A \tag{77}$$

and

$$N = 1. \tag{78}$$

The resulting utility is

$$U = \log A - 1. \tag{79}$$

Therefore the equilibrium allocation is efficient.

The central bank can also choose a very high level of M so as to induce all firms to change their prices. Then the sectoral relative price becomes efficient, and aggregate consumption level is A. But since in this case the firms pay menu cost (in terms of consumption composite), the goods market clearing condition is

$$Y = C + \chi(\mu - 1)Y.$$
 (80)

Here the second term of equation (80) represents the menu cost in terms of consumption goods when all the firms change their prices. Therefore equilibrium output is

$$Y = \frac{A}{1 - \chi(\mu - 1)} > A.$$
 (81)

The resulting utility is

$$U = \log A - \frac{1}{1 - \chi(\mu - 1)}.$$
(82)

It is now obvious that it is optimal to induce firms not to change their prices (equation (79)) rather than induce all firms to change their prices (equation (82)). Nakov and Thomas (2014) formally show that the optimal policy under state dependent pricing is price stability. The desirability of price stability is shown to be robust against different assumptions on price stickiness.

4.2 Optimal Policy in a Two-Sector Menu Cost Model

Next, consider a two-sector economy. As in Section 3.2, suppose that the firms in sector 1 are subject to a menu cost while the firms in sector 2 are not. In this case, too, it is possible to replicate the flexible price equilibrium. To show this, suppose that monetary policy chooses M such that

$$M = \frac{A_1}{\bar{A}_1} \bar{M}.$$
(83)

Then, since

$$\bar{p}_1 = \frac{M}{\bar{A}_1},\tag{84}$$

$$p_1^* = \frac{M}{A_1},$$
(85)

and

$$x_1 \equiv \frac{\bar{p}_1}{p_1^*} = 1,$$
(86)

the firms in sector 1 do not change their prices. The firms in sector 2 adjust their prices optimally:

$$p_2 = p_2^* = \frac{M}{A_2} = \frac{1}{A_2} \frac{A_1}{\bar{A}_1} \bar{M}$$
(87)

and

$$\frac{P_1}{P_2} = \frac{p_1}{p_2} = \frac{A_2}{A_1}.$$
(88)

Therefore, the optimal relative price is efficient. And it is easy to show that

$$C = A, \quad \frac{W}{P} = A \tag{89}$$

and

$$N = 1 \tag{90}$$

in this equilibrium. Therefore, by choosing monetary policy as in (83), the central bank can achieve the efficient allocation.

In this optimal equilibrium, the firms in sector 1 do not change their prices by paying a menu cost. Therefore, core inflation is again defined as inflation of the sticky price sector and the result obtained in the case of time-dependent pricing (Section 3.2) is extended to the case of state-dependent pricing. It is worth noting that, in order to make the firms in sector 1 keep their prices constant, the central bank need not set M equal to the exact value given by equation (83) — firms do not change their prices as long as their prices are in the inaction region:

$$x_1 \in X_1. \tag{91}$$

However, in order to achieve the efficient equilibrium, the bank should set M as in (83). Comparison of this result and the result of the time-dependent pricing discussed in Section 3.2 reveals an interesting point. What is important is to make the old price optimal even under the presence of menu cost. It is not enough that the prices of the sticky price sector remain fixed.

What about the case in which the two sectors are subject to menu cost? As reviewed in Section 3.3, under time-dependent price stickiness, it is not possible to replicate the flexible price equilibrium when the prices of the two sectors are fully stabilised because the relative price, P_1/P_2 , cannot be adjusted in response to changes in A_2/A_1 . The question arises as to whether this result depends on the assumption of time-dependent pricing. Time-dependent pricing assumes that price stickiness is exogenously given and is not dependent on monetary policy. Monetary policy only affects the size of price adjustment of firms who have the opportunity to change their prices. The size of price adjustment is called the "intensive" margin of price adjustment. However, in state-dependent pricing, whether or not firms change their prices depends on monetary policy. In other words, monetary policy affects how many firms change their prices — monetary policy can use both the intensive margin and extensive margin of price adjustment. Under state-dependent price stickiness, the bank can choose a highly inflationary or deflationary policy so that all firms will change their prices. In this case, there is no relative price distortion either within sectors or between the two sectors. The cost of generating high inflation is that firms need to pay menu costs.

Now we consider whether it is possible for the central bank to use the extensive margin to improve economic welfare. Recall that the deviation of the firm's old price from its optimal price is given by $x_i = p_i/\bar{p}_i = \frac{M/A_i}{M/A_i}$. This deviation increases with an increase in A_i/\bar{A}_i . The firm in sector *i* chooses to change its price when x_i is outside the inaction region. Note that the size of $\left|\frac{A_2/A_1}{A_2/A_1}\right|$ determines the distance between x_1 and x_2 . Firstly, suppose that $\left|\frac{A_2/A_1}{A_2/A_1}\right|$ is large enough. This is shown in Figure 5, as Case 1.

Figure 5 here

In this case, the central bank cannot make the prices of both sectors fixed. However, it is possible to replicate the efficient sectoral relative price by setting $M = \frac{A_1}{A_1}\overline{M}$ so as to achieve $x_1 = 1^6$. Then the firms in sector 1 will keep their prices fixed while the firms in sector 2 pay menu cost to adjust their prices. As a result, the relative price, P_1/P_2 , becomes efficient. This equilibrium is shown to be efficient, except that the firms in sector 2 pay menu cost⁷.

An interesting case is the one in which the size of changes in relative productivity, $\left|\frac{A_2/A_1}{A_2/A_1}\right|$, is intermediate. This situation is also shown in Figure 5, as Case 2. The central bank has two options. One option is to choose $M = \frac{A_1}{A_1}\overline{M}$ so as to achieve $x_1 = 1$, and let the firms in sector 2 change their prices. This case is shown in Figure 5 as Case 2a. The other option is to choose M so as to keep the prices in both sectors fixed. This case is shown in Figure 5, as Case 2b. One advantage of the first option is that the relative price, P_1/P_2 , is efficient, while its disadvantage is that the firms in sector 2 pay menu cost. On

⁶Alternatively, the central bank could also replicate the flexible price equilibrium by setting $M = \frac{A_2}{\bar{A}_2} \bar{M}$ to achieve $x_2 = 1$. Then the firms in sector 2 keep their prices fixed, but those in sector 1 will adjust their prices.

⁷The sectoral relative price also becomes efficient in an equilibrium where the firms in both sectors change their prices. But because the firms in both sectors pay menu cost, utility level is not as high as that of the case in which only one sector changes price.

the other hand, an advantage of the second option is that no firms pay menu cost, while its disadvantage is that the relative price is inefficient since $P_1/P_2 = \bar{p}_1/\bar{p}_2 \neq A_2/A_1$. The question is which is better. Should the central bank use the extensive margin to make the relative price efficient? In what follows we answer to this question.

4.2.1 Optimal Equilibrium in Which Monetary Policy Uses Extensive Margin

Without loss of generality, here we consider the case in which firms in sector 2 adjust their prices while those in sector 1 keep their prices fixed. By allowing the firms in sector 2 to change their prices, the central bank uses the extensive margin of price adjustment as well as the intensive margin. This situation arises when the menu cost in sector 1 is large enough compared with the menu cost in sector 2.

As discussed in Section 4.2, in order to ensure the relative price (p_1/p_2) is efficient, the central bank should set M by equation (83). When the firms in sector 2 change their prices they pay menu cost. Therefore the goods market clearing condition is

$$Y = C + \chi_2(\mu_2 - 1)\alpha_2 Y.$$
(92)

In order to achieve the efficient equilibrium, we need $x_i = 1$ for both sectors. As in the equilibrium discussed in Section 4.2, consumption in this equilibrium is

$$C = A. (93)$$

Since the goods market clearing condition is (92), the labour required is

$$N = \frac{y_1}{A_1} + \frac{y_2}{A_2}$$

= $\left[\frac{1}{A_1} \left(\frac{\bar{p}_1}{P}\right)^{-1} + \frac{1}{A_2} \left(\frac{p_2^*}{P}\right)^{-1}\right] Y$
= $\left[\frac{1}{A_1} \left(\frac{A}{A_1}\right)^{-1} + \frac{1}{A_2} \left(\frac{A}{A_2}\right)^{-1}\right] Y$
= $\frac{1}{1 - \chi_2(\mu_2 - 1)\alpha_2}.$

This equation shows that N > 1 because the firms in sector 2 pay menu cost (in terms of goods). Therefore the maximised utility is

$$U^* = \log A - \frac{1}{1 - \chi_2(\mu_2 - 1)\alpha_2}.$$
(94)

This equilibrium exists if and only if x_2 is outside the inaction region:

$$x_{2}^{*} = \frac{\bar{p}_{2}}{p_{2}} = \frac{\bar{W}/\bar{A}_{2}}{W/A_{2}} = \frac{\bar{W}\bar{A}_{2}}{\left(\frac{A_{1}}{\bar{A}_{1}}\bar{W}\right)/A_{2}} = \frac{\bar{A}_{1}/\bar{A}_{2}}{A_{1}/A_{2}} \notin X_{2}.$$
(95)

4.2.2 Optimal Equilibrium in Which Monetary Policy Does Not Use Extensive Margin

When the firms in the two sector keep their prices fixed, no firms pay menu cost. Then the utility is given by

$$U = \log C - N$$

= $\log C - \left(\frac{c_1}{A_1} + \frac{c_2}{A_2}\right)$
= $\log Y - \left[\frac{\alpha_1}{A_1} \left(\frac{\bar{p}_1}{\bar{P}}\right)^{-1} + \frac{\alpha_2}{A_2} \left(\frac{\bar{p}_2}{\bar{P}}\right)^{-1}\right] Y,$ (96)

From the second line to the third line, we use the fact that no firms pay menu cost so that Y = C. Since we assumed that the steady state is efficient (equations (28) -(34)), (96) can be written as

$$U = \log Y - \left(\alpha_1 \frac{\bar{A}_1}{A_1} + \alpha_2 \frac{\bar{A}_2}{A_2}\right) \frac{Y}{A}.$$
(97)

The bank can maximise (97) by choosing M optimally. The optimal M is given by

$$M^{**} = \bar{P} \frac{\bar{A}}{\alpha_1 \frac{\bar{A}_1}{A_1} + \alpha_2 \frac{\bar{A}_2}{A_2}}.$$
(98)

The maximised utility is

$$U^{**} = \log\left[\frac{\bar{A}}{\alpha_1 \frac{\bar{A}_1}{A_1} + \alpha_2 \frac{\bar{A}_2}{A_2}}\right] - 1.$$
 (99)

This equilibrium exists if and only if the firms in both sectors are in their inaction regions:⁸

$$x_i^{**} = \left(\alpha_1 \frac{\bar{A}_1}{A_1} + \alpha_2 \frac{\bar{A}_2}{A_2}\right) \frac{A_i}{\bar{A}_i} \in X_i$$
(100)

Recall that X_2 depends on menu cost parameter χ_2 . It is particularly interesting to analyse the parameter range of χ_2 in which both (100) and (95) hold. This situation arises when χ_2 is not too large and not too small.⁹ The parameter space of χ_2 will be specified more rigorously below. The central bank has two options. One is to choose M by (98) so as to keep all prices fixed. In this case, while no firms need to pay menu cost, the relative price between the two sectors is inefficient. The other is to choose M by (83) so as to make the firms in sector 2 adjust their prices but make the firms in sector 1 keep their prices fixed. In this case, while the firms in sector 2 pay menu cost, the equilibrium relative price is efficient. This policy uses the extensive margin of price adjustment to make the relative

⁸For simplicity, we assume that the menu cost parameter of sector 1 is large enough so that the firms in sector 1 are in their inaction region. So we focus our analysis on the firms in sector 2.

⁹If χ_2 is too large then (95) does not hold so that the firms in sector 2 will not change their prices under policy (83). They will not change their prices under policy (98) either. On the other hand, if χ_2 is too small then (100) does not hold so that the firms in sector 2 will change their prices under policy (98). (And they also change their prices under policy (83)).

price efficient.

4.2.3 Welfare Comparison of the Two Policies

Now we ask which policy delivers a higher welfare. Figure 6 shows a numerical example¹⁰.

Figure 6 here

The inaction region for sector 2, X_2 , is given by equation (72), which is presented here for the reader's reference:

$$X_2 = \left\{ x_2 \mid x_2^{-\theta_2} \left(x_2 - \mu_2^{-1} \right) \le (1 - \chi_2)(1 - \mu_2^{-1}) \right\}.$$
 (101)

In the optimal equilibrium in which the firms in sector 2 change their prices, the inaction region is given by equation (95). In the optimal equilibrium in which the firms in both sectors keep their prices fixed, the inaction region is (100). We continue to assume that the menu cost of sector 1 is large enough so that for the parameter space considered in Figure 6, the firms in sector 1 keep their prices fixed.

When we compare the welfare of the two equilibria, we focus on the menu cost parameter χ_2 and the variations in the relative technology $\frac{A_1/A_2}{A_1/A_2}$. Intuitively speaking, given a certain value of $\frac{A_1/A_2}{A_1/A_2}$, a small value of χ_2 makes the second option attractive because menu cost is small. Similarly, given a certain value of χ_2 , a large deviation of $\frac{A_1/A_2}{A_1/A_2}$ from unity makes the second option more attractive because the optimal relative price change, $\frac{p_1/p_2}{\bar{p}_1/\bar{p}_2}$, is large. In other words, the cost of deviation from the optimal relative price is large.

The line denoted as "using extensive margin" represents the pair of $(\frac{A_1/A_2}{A_1/A_2}, \chi_2)$ that satisfies

$$(x_2^*)^{-\theta_2} \left(x_2^* - \mu_2^{-1} \right) = (1 - \chi_2)(1 - \mu_2^{-1}), \tag{102}$$

¹⁰In this numerical example, we use the following parameter values: $\theta_i = 5$, $\alpha_i = 0.5$.

which is the boundary of the inaction region in the equilibrium in which the firms in sector 2 change their prices. Here x_2^* is given by (95). This equilibrium is analysed in Section 4.2.1 and such equilibrium exists below the line. Intuitively speaking, given the size of changes in relative productivity, menu cost parameter χ_2 needs to be small enough for firms in sector 2 to change their prices. Similarly, for the equilibrium in which all firms keep their prices fixed, we plot the boundary of the inaction region by the line denoted as "not using extensive margin". This is the equilibrium analysed in Section 4.2.2 and such equilibrium exists above the line because, given the size of changes in relative productivity, menu cost parameter χ_2 needs to be large enough for firms in sector 2 to keep their prices fixed. Figure 6 shows that both equilibria exist between the two lines. Therefore, in these regions, the central bank has two options — whether or not to use the extensive margin of price adjustment. Finally, the line denoted as "indifferent" refers to the boundary such that the welfare levels are identical between the two policies:

$$U^* = U^{**}.$$
 (103)

Above the line, it is better not to use the extensive margin because the cost of price adjustment is high.

Figure 6 shows that when both policy options are available, it is always better not to use the extensive margin. The policy that does not use the extensive margin, i.e., the policy under which all firms keep their prices fixed, always delivers a higher welfare. What are the implications? The policy that uses the extensive margin can be interpreted as the one that generates a high inflation rate to decrease the degree of price stickiness. The benefit of this policy option is to achieve the efficient relative price. Its cost is that firms pay menu cost. In our numerical example, when the bank can maintain overall price stability, it should always do so and should not attempt to use inflation to align relative price distortions.

5 Conclusion

This article reviews research on the price index to be targeted in order to maximise economic welfare. The literature shows that it is the price index of the sticky price sector that should be targeted. Much of the literature is based on time-dependent pricing. We also examine whether the results may change under state-dependent pricing. The difference between the two assumptions on price stickiness is that, under state-dependent pricing, the degree of price stickiness can be controlled by monetary policy. So the definition of "sticky price sector" under state-dependent pricing is not as clear as that under timedependent pricing. We show that, at least in the simple static model considered here, it is not optimal for the central bank to use the extensive margin to achieve efficient relative price changes. This implies that, if the bank can stabilise prices, it should do so as much as possible. However, since the model is highly stylised, it is important to investigate whether the results continue to hold in a more general, dynamic setting. This is left for future research.

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Figure 1: Consumer Price Index of Japan: 1970=1

Figure 2: CPI Inflation in Japan





Figure 3: Sectoral Price Levels (1970=1) 3(a) Goods





Note: The sharp drop in the price of 'Public services related to education' is caused by the policy that makes public highschool education free of charge in 2010.



Figure 4: CPI, core CPI, and core-core CPI inflation: 2000-2013

Figure 5:



Figure 6: Welfare comparison

