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Payment Instruments and Collateral in the Interbank Payment System

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Abstract

This paper presents a three-period model to analyze the endogenous need for bank reserves in the presence of Treasury securities. The model highlights the fact that the interbank market is an over-the-counter market. It characterizes the large value payment system operated by the central bank as an implicit contract, and shows that the contract requires less liquidity than decentralized settlement of bank transfers. In this contract, bank reserves are the balances of liquid collateral pledged by banks. The optimal contract is equivalent to the floor system. A private clearing house must commit to a time-inconsistent policy to provide the contract.

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1 Introduction

Base money consists of cash and bank reserves. Banks do not hold bank reserves merely to satisfy a reserve requirement, but also to settle the transfer of deposit liabilities due to bank transfers. In fact, the average figure for the daily transfer of bank reserves is as large as a sizable fraction of annual GDP in the country.[1] Also, several countries have abandoned a reserve requirement.[2] Banks in these countries still settle bank transfers through a transfer of bank reserves.

But why do banks need bank reserves for interbank payment? Theoretically, banks should be able to pay Treasury securities, i.e., the other liquid liabilities issued by the consolidated government. Unlike payers in retail payment, banks can easily handle these wholesale assets. Also, banks obtain bank reserves in exchange for Treasury securities through open market operations. Why do banks swap liquid assets for liquid assets?

To address this question, this paper constructs a model of a decentralized interbank payment system in which banks settle bank transfers without a central bank. The model involves three assumptions: a bank must pay a penalty if it fails to send bank transfers requested by its depositors by the end of a payment cycle; banks need payment instruments to settle bank transfers due to limited commitment; and banks negotiate the terms of each transaction between them bilaterally. The first assumption reflects the fact that banks make payments between them on behalf of their depositors. This feature of interbank payment contrasts with retail payment. On the second assumption, the model incorporates liquid bonds that can be used for interbank payment. These bonds can be interpreted as Treasury securities. The last assumption reflects the fact that the interbank market is an over-the-counter (OTC) market.

The model shows that decentralized settlement of bank transfers is inefficient due to a hold-up problem in an OTC market. A bank receiving a bank transfer can require the originating bank to pay a higher value of assets than the face value of the bank transfer, because it can threaten the originating bank with the penalty for a failure to send a bank transfer. The presence of this premium increases the amount of liquidity necessary for interbank payment.

[1] For example, the figure for Japan is 23.7% in 2012.

[2] These countries include Australia, Canada, Denmark, Mexico, New Zealand, Norway, Sweden, and the U.K.

In light of this result, the interbank payment system operated by the central bank, so-called a large value payment system, can be seen as an implicit interbank settlement contract collateralized by liquid securities. This interpretation is based on the fact that banks can settle an outgoing bank transfer unilaterally by sending the same nominal balance of bank reserves in the large value payment system. Thus, the terms of settlement of bank transfers are not negotiated ex-post as if they are pre-determined in a contract. The model shows that arranging a contract has a liquidity-saving effect because it prevents a hold-up problem due to ex-post bargaining; thus it eliminates a premium in interbank payment.

To implement this contract under limited commitment, banks need to pledge collateral to a third party acting as a custodian. The custodian transfers the balance of collateral between banks according to bank-transfer requests reported by each bank. This feature of the contract is consistent with the fact that banks obtain bank reserves from the central bank in exchange for their assets through open market operations. Thus, bank reserves can be interpreted as the balances of collateral pledged by banks, while the central bank acts as the custodian of collateral. In the model, the collateral must be liquid due to an assumption that the central bank cannot handle illiquid assets. Hence, banks swap liquid collateral for bank reserves.

The optimal contract shows the two key features of the floor system: it obviates the need for OTC settlement of bank transfers by a sufficiently large prepayment of collateral; and the central bank passes on the interest on pledged collateral to banks. The first and the second feature are equivalent to an ample supply of bank reserves and interest payment on bank reserves, respectively. This result implies that the floor system is optimal not only due to interest payment on bank reserves, i.e., the Friedman's rule. The optimality of the floor system also requires the presence of a large value payment system that allows unilateral settlement of bank transfers.

At the end, this paper discusses whether a private clearing house can implement the optimal interbank settlement contract on behalf of the central bank. This question is motivated by recent developments in private large value payment systems, such as the Clearing House Interbank Payment System (CHIPS) in the U.S. and the CLS for foreign exchange settlement.[3] The model shows that after the settlement of bank transfers, the custodian of collateral must return the remaining balance of collateral to each bank. The

[3]For the function of the CLS, see Kahn and Roberds (2001).

transfer of collateral is unilateral; the custodian must release collateral in return for nothing. Thus, while the use of collateral makes the interbank settlement contract robust to limited commitment by banks, it requires the custodian of collateral not to walk away with collateral ex-post. In this regard, bank reserves are ideal collateral for a private clearing house, as they cannot be taken out of the central bank's large value payment system. Thus, there may still remain a role of the central bank even if a private clearing house operates a large value payment system. This implication of the model is consistent with the current observation that the CHIPS and the CLS use bank reserves as collateral.

1.1 Related literature

The distinction between bank reserves and Treasury securities is related to the legal restriction theory of money. Wallace (1983) discusses why money is necessary as the medium of exchange, despite the presence of interest-baring Treasury securities. He points out that the non-negotiability and the large denomination of Treasury securities provide a role for money. This paper brings this question to interbank payment, and derives a distinction between bank reserves and Treasury securities by the fact that the interbank market is an OTC market.

In this regard, Afonso and Lagos (2014) present a model of the U.S. federal funds market. They show that the cost of violating a reserve requirement, e.g., a non-negative balance requirement at the end of the day, affects the dynamics of the market through the threat point of bilateral bargaining in the market. This paper shows that a hold-up problem in OTC settlement of bank transfers gives rise to a role of the central bank in the interbank payment system.

Freeman (1996) analyzes the welfare-enhancing effect of the central bank's discount window that replaces illiquid IOUs with money. On this topic, Green (1997) discusses whether a private clearing house can take over the role of the central bank, Fujiki (2003, 2006) analyzes the effect of the central bank's liquidity provision policies on cross-border settlement, Mills (2004) proposes an alternative mechanism to the discount window based on collateralized lending, Gu, Guzman and Haslag (2011) analyze the optimal intraday interest rate, and Chapman and Martin (2013) investigate the role of tiering to limit the central bank's exposure to credit risk. This paper adds to this literature by analyzing why the central bank needs to replace liquid securities with bank reserves.

There is also a literature on private interbank payment systems. Kahn (2013) analyzes how the competition between a public and a private large value payment system limits the central bank's ability to manipulate monetary policy. Kahn (2009) brings this issue to cross-border settlement. Also, Kahn and Roberds (2009) analyze the vertical integration of a public and a private interbank payment system through tiering.

Modeling a payment system as an implicit contract is related to the mechanism design approach by Koeppl, Monnet and Temzelides (2008) and Fujiki, Green and Yamazaki (2008). Koeppl, Monnet and Temzelides show that a payment system can implement the optimal resource allocation if agents can rebalance settlement balances at sufficiently high frequency after bilateral exchanges. Fujiki, Green and Yamazaki analyze the optimal design of a payment system under asymmetric information among system participants regarding the probability of settlement failures.

This paper also adds to the literature that evaluates reserve-supply policies with an endogenous need for money. Berentsen and Monnet (2008) analyze the channel system, and Berentsen, Marchesiani and Waller (2014) compare the channel and the floor system. This paper shows that the optimality of the floor system does not hinge only on interest payment on bank reserves that implements the Friedman's rule, but also requires the presence of a large value payment system that allows unilateral settlement of bank transfers.

From a broader perspective, there is a literature on money and collateral. Shi (1996) shows useless assets except for the owner can serve as collateral to facilitate intertemporal exchange in a money-search model. Berentsen, Camera and Waller (2007) show that a bank can reduce aggregate need for collateral by collecting idle balances and lending them to the demanders of payment instruments. Ferraris and Watanabe (2008) analyze the co-existence of money and credit by introducing loans of money collateralized with illiquid capital. In contrast to these papers, this paper analyzes the distinction between bank reserves and liquid collateral in the large value payment system.

The remainder of the paper is organized as follows. The stylized features of the interbank payment system are reviewed in section 2. A model of decentralized interbank settlement is presented in section 3. An interbank settlement contract is introduced into the model in section 4. Section 5 concludes.

2 Stylized features of the interbank payment system

This section briefly summarizes the background of the baseline model in this paper. There are usually two tiers in the interbank payment system in each country. First, small-valued bank transfers from depositors enter an automated clearing house (ACH). At this stage, an ACH processes a large number of small-valued bank-transfers to calculate the net balance of bank transfers for each bank. Then, banks with outgoing net bank transfers send the corresponding nominal balances of bank reserves to an ACH's account at the central bank. Bank reserves are the balances of current accounts at the central bank. The ACH in turn passes on the received current-account balances to banks with incoming net bank transfers, so that it maintains a zero net position of bank reserves.[4] This transfer of bank reserves clears gross bank transfers bundled at the ACH. Banks settle shortfalls in bank reserves at the end of each day in an interbank money market.[5]

The system that processes the transfer of bank reserves is called a large value payment system.[6] Examples of the large value payment system operated by the central bank are Fedwire in the U.S., TARGET2 in the Eurozone, CHAPS in the U.K. and BoJ-NET in Japan.[7]

Given this structure of the interbank payment system, this paper presents a baseline model featuring an interbank payment system without bank reserves or a large value payment system operated by the central bank. In this alternative system, banks can use liquid bonds, which can be interpreted as Treasury securities, for payment instruments to settle bank transfers. Given no involvement of the central bank, banks negotiate the terms of settlement in an OTC interbank market. This assumption is motivated by the fact

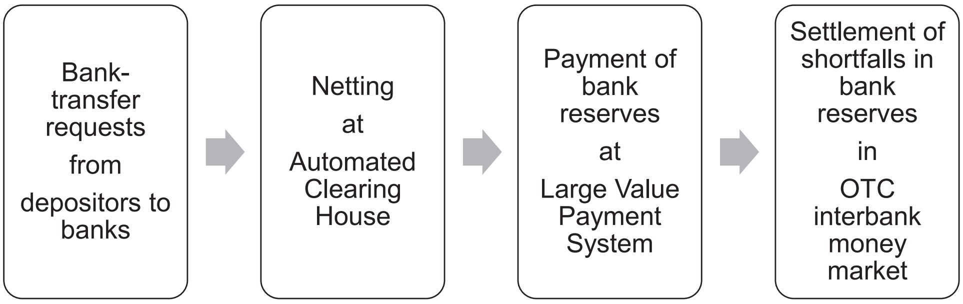
[4]Thus, a clearing house acts as a central counterparty.

[5]The central bank normally allows banks to run negative balances of bank reserves during daytime through daylight overdrafts. Banks can also fulfill expected shortfalls in bank reserves in the interbank money market in each morning.

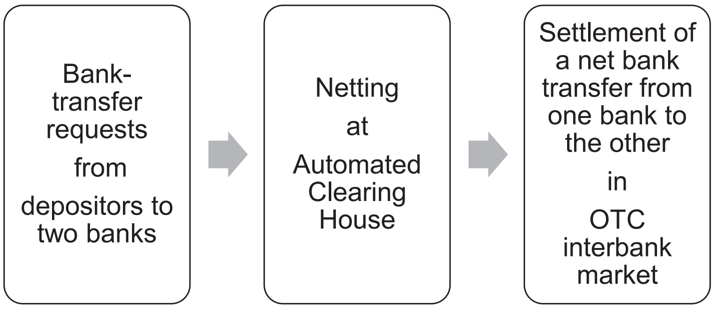
[6]This naming is due to the fact that a balance transfer in the system tends to be large-valued. In fact, if a depositor sends a large-valued bank transfer, then it is directly settled at the large value payment system without going through an ACH.

[7]In addition, some private interbank payment systems are also called large value payment systems. For example, the Clearing House Interbank Payment System (CHIPS) in the U.S. clears large-valued bank transfers related to foreign exchange transactions. The net balances of bank transfers in CHIPS are settled by the transfer of bank reserves at the Fedwire at the end of each day.

Figure 1: Comparison between the interbank payment system in reality and the baseline model



(a) Interbank payment system in reality



(b) Baseline model

that the interbank money market is an OTC market. Thus, the baseline model features a hypothetical arrangement in which an interbank market completely replaces the large value payment system operated by the central bank (see Figure 1). For simplicity, the baseline model takes as given netting at an ACH. The large value payment system will be introduced later into the baseline model to see how it affects interbank payment.

3 Baseline model of a decentralized interbank payment system

Time is discrete and indexed by $t = 0, 1, 2$. There are two banks indexed by $i = A, B$. Each bank receives a unit amount of goods from its depositors in period 0. For simplicity, the deposit interest rate is set to zero.[8]

Banks can transform deposited goods into loans and bonds. Loans generate an amount R_L of goods in period 2 per invested good. Similarly, the gross rate of return on bonds in period 2 is R_B. Assume that

$$R_L > R_B > 1, \tag{1}$$

so that the rate of return on loans dominates that on bonds. Also, loans and bonds have higher rates of return than storage, as implied by the second inequality. Depositors cannot withdraw goods from banks in period 1, as banks cannot produce any good by terminating loans or bonds in period 1. Thus, the maturity of deposits comes in period 2.

In period 1, each bank i for $i = A, B$ has orders from depositors to send a fraction λ_i of its total deposits to the other bank. The joint probability distribution of λ_A and λ_B is

$$(\lambda_A,\ \lambda_B) = \begin{cases} (\eta,\ 0) & \text{with probability } 0.5, \\ (0,\ \eta) & \text{with probability } 0.5, \end{cases} \tag{2}$$

where $\eta \in (0, 1)$.[9] Note that the two banks are symmetric before the realization of $(\lambda_A,\ \lambda_B)$ in period 1.

If the bank originating bank transfers, i.e., the bank with $\lambda_i = \eta$, fails to settle the bank transfers, then it must incur a cost $\gamma\eta$ ($\gamma > 0$). This cost can be interpreted as representing a long-term cost due to loss of reputation, or a cost payable in period 2 due to a litigation filed by depositors for failed payments. In contrast, the cost of failed settlement of bank transfers for the receiving bank, i.e., the bank with $\lambda_i = 0$, is normalized to zero. Thus, the originating bank must pay a higher penalty for failed settlement of bank

[8] A zero deposit interest rate can be derived as an endogenous equilibrium outcome. See Appendix A for the formal assumption about depositors.

[9] For simplicity, assume that overlapping gross flows of bank transfers between banks are automatically canceled out at an ACH, so that banks only need to settle a net flow of bank transfers at the end of period 1.

transfers than the receiving bank. The underlying assumption is that a deposit contract includes the right to send a bank transfer on demand, for which the originating bank is liable, but the receiving bank is not.

Given this environment, assume that banks cannot commit to any future behavior between them.[10] This assumption implies that banks cannot write a pledgeable contract in period 0 to set the terms of settlement of bank transfers in period 1. Even if banks swap some amounts of loans and bonds as collateral between them in period 0, they take an equal amount of collateral from each other, given the ex-ante symmetry between them in period 0. As a result, a bank does not lose anything by reneging on a contract in period 1, because it can cancel out the collateral taken by, and from, the other bank.

Thus, banks need to pay loans or bonds to settle bank transfers between them after the realization of λ_A and λ_B in period 1. Assume that the interbank market is an OTC market; so banks determine the terms of settlement through bilateral bargaining. The outcome of bargaining is determined by Nash bargaining in which each bank has equal bargaining power. If banks do not reach an agreement, then no bank transfer is made. In this case, the originating bank receives a penalty, as assumed above.

Bonds are transferable at no cost between banks. In contrast, if a bank sells its loans to the other bank in period 1, then the bank buying the loans must monitor the loans by itself to generate returns. In this case, the net return per loan in period 2 becomes δ ($\in (0, R_L]$). The difference between R_L and δ is due to a loan monitoring cost.[11] Also, assume that a bank cannot commit to monitoring loans if its loans are submitted to the other bank as collateral for a repo. Thus, a repo and a spot sale are indifferent in the model.

In period 2, each bank receives returns on its loans and bonds, repays deposits given a zero deposit interest rate, and consumes the residual as its profit. Each bank is risk-neutral, and chooses its portfolio of loans and bonds

[10] This assumption can be compatible with each bank's ability to commit to deposit contracts. Suppose that depositors can seize loans and bonds in period 2, if a bank defaults on deposit contracts. Denote by v the rate of return on loans and bonds for depositors in case of seizure. Assume $R_L > R_B > 1 + v$. Under this assumption, banks can commit to repaying a deposit interest rate up to v. If $v = 0$, then banks can commit to deposit contracts considered in the model.

[11] The loan interest rate, R_L, can be interpreted as the rate of return on loans net of the loan monitoring cost for the originator bank. Thus, this assumption does not imply that an originator bank does not have to monitor loans.

Table 1: Summary of events in the baseline model

Period 0	Period 1	Period 2
There are two banks; each bank receives a unit amount of goods from depositors, given a zero deposit interest rate.	One of the banks has an outflow of bank transfers, η, to the other bank. The probability to be the originating bank is 0.5 for each bank.	Banks receive returns on loans and bonds, repay deposits, and consume the residual.
Banks invest deposited goods into loans and bonds.	A bank must incur a penalty, $\gamma\eta$, if it fails to send bank transfers requested by its depositors within period 1.	The return of goods per loan equals R_L if loans are not transferred in period 1, and δ ($\leq R_L$) if loans are transferred in the period.
	Banks bargain over how much amounts of loans and bonds the originating bank must pay to the receiving bank to settle bank transfers.	The return of goods per bond always equals R_B ($< R_L$).

in period 0 to maximize the expected profit in period 2. An equilibrium is a Perfect Bayesian Nash equilibrium for the two banks. See Table 1 for the summary of events in the model.

Hereafter, assume that

Assumption 1. $R_B > (1 + \gamma)\,\eta$.

Under this assumption, the value of bank transfers, η, is small enough that a bank can always choose to settle bank transfers by investing into a sufficient amount of bonds. Also, assume that the penalty per failed bank transfer, γ, is sufficiently high:

Assumption 2. $\gamma > 4\left(\dfrac{R_L}{R_B} - 1\right)$.

This assumption ensures that each bank does not ignore bank-transfer requests from its depositors in any case considered below.

3.1 Efficiency of a decentralized interbank payment system in case of liquid bank loans

For the benchmark, let us start from the case in which loans are transferable at no cost between banks:

Assumption 3. $\delta = R_L$.

Solve the model backward from the settlement of bank transfers in period 1. Throughout the paper, call the bank with $\lambda_i = \eta$ the "originating bank", and the bank with $\lambda_i = 0$ the "receiving bank". Under Assumption 3, the bargaining problem between the originating and the receiving bank in period 1 takes the following form:

$$\max_{l\in[0,k],\, b\in[0,a]} [-(R_L l + R_B b - \eta) - (-\gamma\eta)]^{0.5}(R_L l + R_B b - \eta)^{0.5}, \tag{3}$$

where: k and a are the amounts of loans and bonds, respectively, held by the originating bank at the beginning of period 1; l and b denote the amounts of loans and bonds, respectively, that the originating bank pays to the receiving bank; and η is the value of bank transfers in the period. In (3), the left square bracket contains the trade surplus for the originating bank, and the right parenthesis contains the trade surplus for the receiving bank. The first term in the left square bracket, $-(R_L l + R_B b - \eta)$, is a change in profit in period 2 for the originating bank. The second term in the bracket, $-\gamma\eta$, is the penalty for a failed settlement of bank transfers. This penalty determines the threat point for the originating bank.

The solution for the bargaining problem is

$$R_L l + R_B b = \eta + \frac{\gamma\eta}{2}, \tag{4}$$

which is feasible under Assumption 1.[12] This equation implies that the originating bank must pay an extra value of assets, $\gamma\eta/2$, above the value of bank

[12]Given Assumption 1 and the flow of funds constraint for each bank in period 0, $k + a = 1$, there exists a pair of l and b satisfying (4), $l \leq k$, and $b \leq a$ for every possible pair of k and a.

transfers, η. This result is due to bilateral bargaining in an OTC interbank market. The originating bank must complete the bank transfers within period 1 to avoid incurring a penalty, $\gamma\eta$, for failed settlement of bank transfers. The receiving bank takes advantage of this time constraint, charging an extra amount of assets for the settlement of bank transfers.

Now move back to period 0. The profit maximization problem for each bank in the period is:

$$\begin{aligned}\max_{\{k\geq 0,\ a\geq 0\}} \ & R_L k + R_B a - 1 + \frac{1}{2}\frac{\gamma\eta}{2} + \frac{1}{2}\left(-\frac{\gamma\eta}{2}\right), \\ \text{s.t. } & k + a = 1,\end{aligned} \tag{5}$$

where the constraint is a flow of funds constraint that the sum of investments into loans, k, and bonds, a, by each bank in period 0 must equal the total deposits at each bank in the period. The first two terms in the objective function are the returns on loans and bonds in period 2. The third term is the face value of deposit liabilities issued in period 0. The last two terms are the expected net gain and loss due to incoming and outgoing bank transfers, i.e., $\pm(R_L l + R_B b - \eta)$, as implied by (4).

Given $R_L > R_B > 1$ as assumed in (1), the solution for this problem is

$$(k,\ a) = (1,\ 0). \tag{6}$$

Thus, each bank invests only into the assets with the highest rate of return:

Proposition 1. Suppose Assumption 1 holds. Under Assumption 3, each bank chooses the efficient resource allocation, (6), in period 0.

3.2 Inefficiency of a decentralized interbank payment system in case of illiquid bank loans

The efficiency result described above is overturned if bank loans are illiquid. Now suppose that the cost of liquidating loans, $R_L - \delta$, is sufficiently large:

Assumption 4. $\delta < \dfrac{R_L}{1+\gamma}$.

For a general value of δ, the bargaining problem for the settlement of bank transfers in period 1 takes the following form:

$$\max_{l\in[0,k],\ b\in[0,a]} [-(R_L l + R_B b - \eta) - (-\gamma\eta)]^{0.5}(\delta l + R_B b - \eta)^{0.5}. \tag{7}$$

The left square bracket contains the trade surplus for the originating bank, and the right parenthesis contains the trade surplus for the receiving bank. Note that the gross rate of return on transferred loans, l, in the right parenthesis is changed from R_L to δ.

Denote by $\theta(a)$ and $\phi(a)$ the net changes in profit for the originating bank and the receiving bank, respectively, after the bargaining, (7). Both $\theta(a)$ and $\phi(a)$ are functions of a, given $k = 1 - a$ as implied by the flow of fund constraint for each bank in period 0. The following result holds for all $\delta \in (0, R_L)$ under Assumption 1:

$$
\begin{aligned}
&(\theta(a),\ \phi(a)) \\
&= \begin{cases} (-\gamma\eta,\ 0), & \text{if } R_B a - \eta < -\frac{\delta\gamma\eta}{R_L-\delta}, \\ (-[R_L l(a) + R_B b(a) - \eta],\ \delta l(a) + R_B b(a) - \eta), & \text{otherwise,} \end{cases}
\end{aligned} \tag{8}
$$

where $l(a)$ and $b(a)$ denote the optimal values of l and b, given a:

$$
(l(a),\ b(a)) = \begin{cases} \left(\frac{\delta\gamma\eta - (R_L+\delta)(R_B a - \eta)}{2R_L\delta},\ a\right), & \text{if } R_B a - \eta \in \left[-\frac{\delta\gamma\eta}{R_L-\delta},\ \frac{\delta\gamma\eta}{R_L+\delta}\right], \\ (0,\ a), & \text{if } R_B a - \eta \in \left[\frac{\delta\gamma\eta}{R_L+\delta},\ \frac{\gamma\eta}{2}\right], \\ \left(0,\ \frac{1}{R_B}\left(\eta + \frac{\gamma\eta}{2}\right)\right), & \text{if } R_B a - \eta > \frac{\gamma\eta}{2}. \end{cases} \tag{9}
$$

See Appendix B for the proof.

These equations imply that banks fail to agree on the settlement of bank transfers (i.e., $\theta(a) = -\gamma\eta$), if a is too small under Assumption 4. This result holds because the cost of liquidating loans, $R_L - \delta$, is too large. If a is sufficiently large for banks to settle bank transfers, then the value of loan transfer, l, is weakly decreasing in a. In this case, the originating bank must pay a higher value of assets than the value of bank transfers, i.e., $R_L l(a) + R_B b(a) > \eta$, whether $l(a)$ is positive or zero. This result is due to bilateral bargaining in the OTC interbank market: the receiving bank takes advantage of the constraint that the originating bank must complete the bank transfers within period 1 to avoid a penalty. This hold-up problem is as same as the reason behind the second term on the right-hand side of (4).

Given (8) and (9), the profit maximization problem for each bank in period 0 can be written as

$$\max_{\{k\geq 0,\ a\geq 0\}} R_L k + R_B a - 1 + \frac{1}{2}\theta(a) + \frac{1}{2}\phi(a'), \qquad \text{s.t. } k + a = 1, \tag{10}$$

where a' denotes the amount of bonds held by the other bank at the end of the period, which is taken as given. Under Assumptions 2 and 4, banks invest into the just enough amount of bonds in period 0 to avoid liquidation of loans in period 1:

Proposition 2. Suppose Assumptions 1, 2 and 4 hold. Each bank chooses

$$(k,\ a) = \left(1 - a,\ \frac{1}{R_B}\left(\eta + \frac{\delta\gamma\eta}{R_L + \delta}\right)\right), \tag{11}$$

in period 0. Given this value of a, the originating bank pays no loan to the receiving bank for the settlement of bank transfers in period 1:

$$(l,\ b) = \left(0,\ \frac{1}{R_B}\left(\eta + \frac{\delta\gamma\eta}{R_L + \delta}\right)\right). \tag{12}$$

Proof. See Appendix C. □

As implied by (8), each bank can reduce the amount of bonds necessary to settle bank transfers by limiting its bond holdings, a, ex-ante, while maintaining $l(a) = 0$. The liquidity-saving effect of limiting the ex-ante bond holdings is not perfect, however, as the originating bank still has to pay an extra value of bonds above the value of bank transfers, η, as implied by (12). This effect of bilateral bargaining in an OTC interbank market increases the amount of bonds that each bank must invest into in period 0.

4 Role of the central bank in interbank payment

Now introduce the central bank into the baseline model. Two cases will be considered. In the first case, the central bank issues bank reserves just as liquid assets. In the second case, the central bank is introduced as the custodian of collateral in an interbank settlement contract. It will be shown that the central bank can improve the efficiency of the payment system only in the second case.

4.1 No efficiency gain from the introduction of bank reserves just as liquid assets

Suppose that the central bank allows each bank to exchange its bonds for bank reserves in period 0. The central bank repays bank reserves by the whole return on the bonds in period 2. Thus, the central bank just holds bonds on behalf of banks. Bank reserves are transferable between banks at no cost in period 1, just like bonds. The central bank cannot accept a transfer of loans from banks, because it does not have enough ability to monitor loans.

In this case, bank reserves and bonds are identical as liquid assets. Thus, the bargaining problem over the settlement of bank transfers in period 1 remains essentially the same as (3) and (7) under Assumptions 3 and 4, respectively.[13] Hence, Propositions 1 and 2 remain to hold.

4.2 Introduction of the central bank as the custodian of collateral in an interbank settlement contract

Next, suppose that the central bank offers an interbank settlement contract in period 0. A contract,

$$f : (\hat{\lambda}_A,\ \hat{\lambda}_B) \in \{\emptyset,\ \lambda_A\} \times \{\emptyset,\ \lambda_B\} \mapsto (b_A(\hat{\lambda}_A,\ \hat{\lambda}_B),\ b_B(\hat{\lambda}_A,\ \hat{\lambda}_B)) \in \mathbb{R}^2_+, \tag{13}$$

maps the outflows of bank transfers reported by bank A, $\hat{\lambda}_A$, and bank B, $\hat{\lambda}_B$, to a contingent flow of bonds, $b_i(\hat{\lambda}_A,\ \hat{\lambda}_B)$, from bank i to the central bank for $i = A, B$. A negative value of $b_i(\hat{\lambda}_A,\ \hat{\lambda}_B)$ indicates a flow of bonds from the central bank to bank i. If $\hat{\lambda}_i = \emptyset$, then it implies that bank i opts out of the contract in period 1. Otherwise, $\hat{\lambda}_i = \lambda_i$, that is, bank i reports bank-transfer requests from its depositors truthfully. The central bank does not have any endowment in any period. Thus:

$$\sum_{i=A,B} b_i(\hat{\lambda}_A,\ \hat{\lambda}_B) = 0 \quad \text{for all } (\hat{\lambda}_A,\ \hat{\lambda}_B). \tag{14}$$

The equality implies that the net flow of bonds for the central bank must be always zero.

[13] Only the following changes in the notations are necessary: $R_B a$ is redefined as the sum of the par value of bonds and bank reserves held by each bank in period 1; and b is redefined as the sum of the par value of bonds and bank reserves transferred in period 1.

The central bank offers only a symmetric contract between banks, given their ex-ante symmetry in period 0. Hence:

$$\hat{b} \equiv b_A(\eta,\ 0) = b_B(0,\ \eta), \tag{15}$$

$$b_A(0,\ \eta) = b_B(\eta,\ 0) = -\hat{b}, \tag{16}$$

where $\hat{b}$ denotes the value of bonds to be transfered from the originating bank to the receiving bank through the central bank. The central bank aims to maximize each bank's expected profit in period 2.

To implement a contract, the central bank requires each bank to pledge an amount $\hat{b}$ of bonds in period 0. Then the central bank transfers bond balances between banks according to the contract in period 1. Assume that the central bank can commit to returning the resulting balance of bonds to each bank only in period 2, even if a bank opts out of the contract in period 1. It cannot accept loans held by banks as collateral, as it does not have enough ability to monitor loans.

To maintain consistency with the baseline model, assume that banks cannot commit to any future behavior:

Assumption 5. If either bank rejects the offer of a contract in period 0, or opts out of a contract in period 1, then banks settle bank transfers through bilateral bargaining in period 1.

Thus, the central bank cannot enforce a contract if either bank has a higher ex-post profit in bilateral bargaining in period 1 than under the contract. See Table 2 for the summary of the model with an interbank settlement contract offered by the central bank.

4.3 Optimal interbank settlement contract

Under Assumption 5, a contract must ensure that the receiving bank does not incur a loss from receiving a bank transfer, because the receiving bank would not incur a loss even if no bank transfer were settled. Thus,

$$R_B\hat{b} - \eta \geq 0, \text{ for } i = A, B, \tag{17}$$

where the left-hand side is the net gain in profit for the receiving bank in case that it stays in the contract in period 1.

Table 2: Summary of the model with an interbank settlement contract

Period 0	Period 1	Period 2
There are two banks; each bank receives a unit amount of goods from depositors, given a zero deposit interest rate.	One of the banks has an outflow of bank transfers, η, to the other bank. The probability to be the originating bank is 0.5 for each bank.	If banks enter into a contract in period 0, the central bank returns the remaining balance of bonds to each bank.
Banks invest deposited goods into loans and bonds.	A bank must incur a penalty, $\gamma\eta$, if it fails to send bank transfers from its depositors within period 1.	Banks receive returns on loans and bonds, repay deposits, and consume the residual.
The central bank offers an interbank settlement contract for banks, which requires each bank to pledge bonds to the central bank in period 0.	If neither bank rejected the offer of a contract in period 0 or opts out of a contract in period 1, then the central bank transfers bond balances between banks according to bank-transfer requests reported by each bank, as specified by the contract.	The return of goods per loan equals R_L if loans are not transferred in period 1, and δ ($< R_L$) if loans are transferred in the period.
	Otherwise, banks bargain over how much amounts of loans and bonds the originating bank must pay to the receiving bank to settle bank transfers.	The return of goods per bond always equals R_B ($< R_L$).

For now, suppose that the originating bank does not have incentive to opt out of the contract in period 1. This conjecture will be verified later. Given this conjecture, the optimal contract problem for the central bank is specified as follows:

$$
\begin{aligned}
\max_{\{k\geq 0,\ a\geq 0,\ \hat{b}\}} \quad & R_L k + R_B a - 1 - \frac{1}{2}(R_B\hat{b} - \eta) + \frac{1}{2}(R_B\hat{b} - \eta), \\
\text{s.t. } & k + a = 1, \\
& R_B\hat{b} - \eta \geq 0, \\
& a \geq \hat{b},
\end{aligned} \tag{18}
$$

where: k and a are the amounts of loans and bonds, respectively, that each bank invests into in period 0; and $\hat{b}$ is the amount of bonds to be transferred from the originating bank to the receiving bank in period 1. The first constraint is the flow of funds constraint for each bank in period 0. The second constraint is the incentive-compatibility constraint for the receiving bank to remain in the contract, (17). The third constraint indicates that each bank must invest into the amount of bonds to pledge to the central bank, $\hat{b}$, in period 0 under the contract. It is straightforward to show that the solution to this problem is characterized by

$$
(k,\ a,\ \hat{b}) = \left(1 - a,\ \hat{b},\ \frac{\eta}{R_B}\right), \tag{19}
$$

which is feasible under Assumption 1.

Now confirm that the originating bank does not have incentive to deviate from this contract in period 1. Suppose that banks enter into the contract characterized by (19) in period 0, but one of the banks opts out of the contract to initiate bilateral bargaining in period 1. In this case, banks can transfer only loans between them, because the central bank keeps their entire bond holdings until period 2 given $a = \hat{b}$. Thus, the bargaining problem in this case is

$$
\max_{\tilde{l}\in[0,k]} [-(R_L\tilde{l} - \eta) - (-\gamma\eta)]^{0.5}(\delta\tilde{l} - \eta)^{0.5}, \tag{20}
$$

where: the left square bracket and the right parenthesis are the trade surpluses for the originating bank and the receiving bank, respectively; and $\tilde{l}$ is the amount of loans transfered from the originating bank to the receiving bank.

Under Assumption 4, the total trade surplus, $(1+\gamma)\eta/R_L - \eta/\delta$, is negative due to a high loan-liquidation cost. Banks cannot avoid this cost by transferring bonds because of no bond holdings in period 1. Thus, banks do not settle bank transfers outside the contract. Also, each bank has a weakly higher payoff from the contract than no settlement of bank transfers, because the contract characterized by (19) leaves each bank break-even, while an originating and a receiving bank's payoff are $-\gamma\eta$ and 0, respectively, in no settlement of bank transfers. Hence, no bank has incentive to opt out of the contract in period 1.

Given Assumptions 2 and 4, banks participate into the optimal contract, (19), in period 0 because they can save the amount of bonds necessary for the settlement of bank transfers. Compare (11) and (19) to confirm this result. Hence:

Proposition 3. Suppose that Assumptions 1, 2, 4, and 5 hold. Banks participate into the interbank settlement contract characterized by (19).

Note that the central bank does not need to commit to confiscating the bonds of a bank opting out of the contract. To implement the contract, it only needs to retain bonds until the end of period 1. Thus, $b_i(\emptyset,\ \cdot) = b_i(\cdot,\ \emptyset) = 0$ for $i = A, B$ in the optimal contract.

4.4 Large value payment system as an implicit interbank settlement contract

In the large value payment system operated by the central bank, a bank can settle an outgoing bank transfer unilaterally by sending the corresponding nominal balance of bank reserves to the receiving bank. This observation implies that the large value payment system is an implicit interbank settlement contract, as it allows banks to settle bank transfers without bargaining. Also, banks in reality obtain bank reserves in exchange for liquid assets, such as Treasury securities, through open market operations. In light of the model, this transaction can be interpreted as the submission of liquid collateral to the central bank. Thus, bank reserves are the balances of liquid collateral pledged by banks in an implicit interbank settlement contract. This result explains why banks swap Treasury securities for bank reserves despite that both are liquid liabilities of the consolidated government.

In addition, the model is consistent with the fact that banks are not bound to stay in the large value payment system by any explicit contract.

In the model, the central bank can implement the contract only by retaining pledged collateral until the end of the settlement of bank transfers. This result accords with the fact that the large value payment system typically does not include a rule to confiscate bank reserves in case of a settlement failure.

4.5 Implementation of the optimal interbank settlement contract by the floor system

The optimal interbank settlement contract in the model is equivalent to the floor system, which is a type of reserve-supply policy. In the floor system, the central bank supplies a sufficient amount of bank reserves for interbank payment in advance, so that banks do not need to borrow bank reserves in the interbank money market. To give banks incentive to hold the supplied amount of bank reserves, the central bank pays interest on bank reserves. Consequently, this interest rate determines the short-term nominal interest rate in the financial market. This system has been adopted by New Zealand since July 2006.

In the optimal interbank settlement contract in the model, banks pledge to the central bank the enough amount of bonds to settle possible bank transfers in the future. Thus, banks do not settle any bank transfer through bilateral bargaining. Also, the central bank passes on to banks the whole return on bonds pledged as collateral. This policy is equivalent to interest payment on bank reserves. Moreover, the interest paid by the central bank equals that on bonds, i.e., the short-term interest rate in the financial market. Thus, the optimal interbank settlement contract shows the features of the floor system.

This result implies a new aspect of the efficiency of the floor system. In the standard result in the literature, the floor system is optimal because it implements the Friedman's rule: by paying the market interest rate on money, the central bank can eliminate the opportunity cost to hold money intertemporally. Once the presence of liquid securities, such as Treasury securities, is taken into account, however, this feature of the floor system is no longer sufficient for its optimality. As shown in section 4.1, merely supplying interest-baring bank reserves does not change any result in the model. Instead, the key feature of the optimal contract is to eliminate the need for OTC settlement of bank transfers. Thus, the efficiency of the floor system requires the presence of a large value payment system that allows unilateral settlement of bank transfers.

4.6 Can a private clearing house provide a large value payment system on behalf of the central bank?

A remaining question is whether a private clearing house can be the custodian of collateral in an interbank settlement contract. For this question, the key characteristic of the custodian of collateral is its ability to commit to returning the remaining balance of bonds to each bank after the settlement of bank transfers. Note that this is a commitment to an time-inconsistent policy, as the custodian must return bonds to banks in exchange for nothing ex-post. Thus, while the use of collateral makes the interbank settlement contract robust to limited commitment by banks, the contract still relies on the commitment ability of the custodian of collateral.

Two interpretations are possible on why the central bank can act as the custodian of collateral in reality. First, as a non-profit organization with a heavy oversight by the government, the central bank can commit to returning collateral to banks. This interpretation is consistent with the fact that the central bank releases their assets in exchange for bank reserves through open market operations, despite that bank reserves are worthless for the central bank. Second, given the general acceptability of cash as fiat money, the convertibility between bank reserves and cash can provide incentive for banks to swap their collateral for bank reserves even without the central bank's commitment to returning collateral. In terms of profit maximization, the central bank is indifferent between cash and bank reserves as both of them are central-bank liabilities. Thus, there is no time-inconsistency problem in the conversion between bank reserves and cash.

The second interpretation is not applicable to a private clearing house, as it does not issue money. Thus, a private clearing house needs to have commitment ability to take over the role of the central bank in the large value payment system. If it cannot commit to the same behavior as the central bank, then it needs some commitment device. In this regard, bank reserves are ideal collateral for a private clearing house, as they cannot be taken out of the central bank's large value payment system. In this case, the role of the central bank as the custodian of collateral will remain even if a private clearing house operates a large value payment system. This implication of the model is consistent with the current observation that existing private large value payment systems, such as the CHIPS and the CLS, use bank reserves as collateral.

5 Conclusions

This paper has shown that a hold-up problem in an OTC interbank market leads to an endogenous need for a large value payment system operated the central bank. In this result, the large value payment system is interpreted as an implicit interbank settlement contract, and bank reserves are the balances of liquid collateral pledged by banks in the contract. While the optimal contract is equivalent to the floor system, this result does not solely hinge on interest payment on bank reserves. The key to the optimality of the floor system is unilateral settlement of bank transfers in the large value payment system: without this feature of the large value payment system, an ample supply of bank reserves does not obviate the need for the settlement of bank transfers in an OTC interbank market.

On the general nature of money, this paper demonstrates that money is a transitory vehicle that replaces financial assets when asset holders must make payments. This view on money coincides with the analysis of Freeman (1996) that focuses on the elastic supply of money, but contrasts with the standard view in macroeconomics that money is a stock variable. It is left for future research to explore the implications of the transitory nature of money for monetary policy and aggregate economic activity.

It also remains an open question whether a private clearing house can take over the role of the central bank in interbank payment in the future. One advantage of a private large value system is the fast adoption of new technology to save the operational cost. On the other hand, a private clearing house may still need the central bank as the custodian of collateral, if it cannot commit to retuning collateral to its members in any circumstance. In this case, it may be inefficient to add a private large value payment system as another layer between the central bank and commercial banks. Analyzing this trade-off is left for future research.

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A Baseline model with a formal assumption about depositors

A.1 Preference and technology

Time is discrete and indexed by $t = 0, 1, 2$. There are two banks indexed by $i = A, B$. Each bank has a fixed customer base consisting of a unit continuum of risk-neutral depositors. Each depositor is endowed with a unit of goods in period 0. A depositor can save its good in two ways. One is storage technology, in which a depositor can store its good without depreciation or appreciation between consecutive periods. The other is a bank deposit. If a depositor deposits its good in period 0, then the depositor's bank can transform the good into a loan or a bond in that period. A Loan generates an amount R_L of goods in period 2 per invested good. Similarly, the gross rate of return on a bond in period 2 is R_B Assume that

$$R_L > R_B > 1, \tag{21}$$

the last term is the gross rate of return on storage.

Each depositor becomes a buyer or a seller due to an idiosyncratic shock in period 1. A buyer can consume goods produced by sellers at the other bank in period 1, but cannot consume goods in period 2. A seller can produce goods at a unit utility cost per good in period 1, and consume goods in period 2. Each depositor maximizes the following expected utility:

$$U = p_1 c_{b,1} + (1 - p_1)(-h_{s,1} + c_{s,2}), \tag{22}$$

where: p_1 is the probability to be a buyer in period 1 for each depositor in period 0; $c_{b,1}$ is the consumption in period 1 in case of becoming a buyer; and $h_{s,1}$ and $c_{s,2}$ are the production in period 1 and the consumption in period 2, respectively, in case of becoming a seller.

A.2 Deposit contract

Depositors are anonymous to each other; thus, buyers cannot buy goods on credit in period 1. To pay the price of goods in period 1, buyers can order their bank to send the equivalent balance of deposits from their accounts, if any, to the sellers' bank accounts in period 1. The goods market in period 1 is competitive: every depositor takes the price of goods as given.

Banks, however, cannot commit to future behavior. If a bank fails to complete the bank transfers requested from its depositors, then it must incur a cost γ per depositor ($\gamma > 0$). This cost can be interpreted as representing a long-term cost due to loss of reputation, or a cost payable in period 2 due to a litigation filed by depositors for failed payments. In contrast, the cost of failed settlement of bank transfers for the receiving bank, i.e., the bank with $\lambda_i = 0$, is normalized to zero. Thus, the originating bank must pay a higher penalty for failed settlement of bank transfers than the receiving bank. The underlying assumption is that a deposit contract includes the right to send a bank transfer on demand, for which the originating bank is liable, but the receiving bank is not.

Each bank sets the deposit interest rate for its depositors monopolistically in period 0. If a bank reneges on the redemption of a deposit in period 2, then depositors can seize the loans and bonds of the bank and convert them into goods in period 2. The gross rate of return on seized loans and bonds declines to one due to liquidation cost. Thus, the pledgeable deposit interest rate is zero. To satisfy the participation constraint for depositors, a bank cannot set a deposit rate lower than zero because depositors would be better off by storing goods by themselves in such a case. As a result, banks set the deposit interest rate to zero in period 0.

Neither depositor or bank can generate goods by terminating loans or bonds in period 1. Also, depositors cannot seize bonds and loans in period 1 due to a high asset management cost for them. Hence, the maturity of deposits comes in period 2.

A.3 Settlement of bank transfers

The buyer fraction of depositors at each bank is stochastic. At each bank, a fraction λ_i of depositors become buyers. The joint probability distribution of λ_A and λ_B is

$$(\lambda_A,\ \lambda_B) = \begin{cases} (\eta,\ 0) & \text{with probability } 0.5, \\ (0,\ \eta) & \text{with probability } 0.5, \end{cases} \tag{23}$$

where $\eta \in (0, 1)$. Given (2), the unconditional probability for each depositor to be a buyer, i.e., p_1, is

$$p_1 = 0.5\eta. \tag{24}$$

The assumption that banks cannot commit to any future behavior implies that banks cannot write a pledgeable contract in period 0 to set the terms of settlement of bank transfers in period 1. Even if banks swap some amounts of loans and bonds as collateral between them in period 0, they take an equal amount of collateral from each other, given the ex-ante symmetry between them in period 0. As a result, a bank does not lose anything by reneging on a contract in period 1, because it can cancel out the collateral taken by, and from, the other bank.

Thus, banks need to pay loans or bonds to settle bank transfers between them after the realization of λ_A and λ_B in period 1. Assume that the interbank market is an OTC market; so banks determine the terms of settlement through bilateral bargaining. The outcome of bargaining is determined by Nash bargaining in which each bank has equal bargaining power. If banks do not reach an agreement, then no bank transfer is made. In this case, the originating bank receives a penalty, as assumed above.

Bonds are transferable at no cost between banks. In contrast, if a bank sells its loans to the other bank in period 1, then the bank buying the loans must monitor the loans by itself to generate returns. In this case, the net return per loan in period 2 becomes δ ($\in (0, R_L]$). The difference between R_L and δ is due to a loan monitoring cost.[14] Also, assume that a bank cannot commit to monitoring loans if its loans are submitted to the other bank as collateral for a repo. Thus, a repo and a spot sale are indifferent in the model.

A.4 Each bank's objective and the definition of equilibrium

In period 2, each bank receives returns on its loans and bonds, repays deposits given a zero deposit interest rate, and consumes the residual as its profit. Each bank is risk-neutral, and chooses its portfolio of loans and bonds in period 0 to maximize the expected profit in period 2. An equilibrium is a Perfect Bayesian Nash equilibrium for the two banks.

[14] The loan interest rate, R_L, can be interpreted as the rate of return on loans net of the loan monitoring cost for the originator bank. Thus, this assumption does not imply that an originator bank does not have to monitor loans.

B Proof for (8) and (9)

Given $\delta < R_L$, the first-order conditions for the bargaining problem, (7), with respect to l and b are:

$$-\frac{R_L}{-[R_L l + R_B b - \eta] + \gamma\eta} + \frac{\delta}{\delta l + R_B b - \eta} + \underline{\theta}_l - \overline{\theta}_l = 0, \tag{25}$$

$$-\frac{R_B}{-[R_L l + R_B b - \eta] + \gamma\eta} + \frac{R_B}{\delta l + R_B b - \eta} - \overline{\theta}_b = 0, \tag{26}$$

where $\underline{\theta}_l$, $\overline{\theta}_l$, and $\overline{\theta}_b$ are proportional to the non-negative Lagrange multipliers for $0 \leq l$, $l \leq k$, and $b \leq a$. The Lagrange multiplier for the other constraint, $0 \leq b$, is always zero, because it is positive only if $\underline{\theta}_l > \overline{\theta}_l$. Note that if $\underline{\theta}_l > \overline{\theta}_l$, then $b = l = 0$, under which $\delta l + R_B b - \eta$ is negative.

Given that the denominator in each side is the same across the two conditions and the assumption that $R_L \geq \delta$, $\underline{\theta}_l = \overline{\theta}_l = \overline{\theta}_b = 0$ cannot hold. Thus, there are four cases to consider: $\{l = 1 - a,\ b = a\}$; $\{l \in (0,\ 1 - a),\ b = a\}$; $\{l = 0,\ b = a\}$; and $\{l = 0,\ b \in (0, a)\}$.

In the first case, $\underline{\theta}_l = 0$ and $\overline{\theta}_l \geq 0$. For this case to happen, it must hold that

$$\frac{R_L}{-[R_L(1-a) + R_B a - \eta] + \gamma\eta} \leq \frac{\delta}{\delta(1-a) + R_B a - \eta}. \tag{27}$$

Given (26) and the assumption $R_L \geq \delta$, $\overline{\theta}_b > 0$.

In the second case, $\underline{\theta}_l = \overline{\theta}_l = 0$. In this case, (25) implies that

$$\exists l \in (0,\ 1-a),\ \text{s.t.}\ \frac{R_L}{-[R_L l + R_B a - \eta] + \gamma\eta} = \frac{\delta}{\delta l + R_B a - \eta}. \tag{28}$$

Given (26) and the assumption $R_L \geq \delta$, $\overline{\theta}_b > 0$.

In the third case, $\underline{\theta}_l \geq 0$, $\overline{\theta}_l = 0$, and $\underline{\theta}_b \geq 0$. Thus, (26) implies

$$\frac{R_B}{-[R_B a - \eta] + \gamma\eta} \leq \frac{R_B}{R_B a - \eta}. \tag{29}$$

Also, (25) implies

$$\frac{R_L}{-[R_B a - \eta] + \gamma\eta} \geq \frac{\delta}{R_B a - \eta}. \tag{30}$$

In the fourth case, $\underline{\theta}_l \geq 0$, $\overline{\theta}_l = 0$, and $\underline{\theta}_b = 0$. Hence:

$$\exists b \in (0,\ a),\ \text{s.t.}\ \frac{R_B}{-[R_B b - \eta] + \gamma\eta} = \frac{R_B}{R_B b - \eta}. \tag{31}$$

This condition is sufficient for (25) under $l = 0$ and $\underline{\theta}_l \geq \overline{\theta}_l = 0$, given the assumption $R_L \geq \delta$.

Summarizing the four cases, the solutions for l and b under $\delta < R_L$ take the following form:

$$(l(a),\ b(a)) = \begin{cases} (1-a,\ a)\,, & \text{if } R_B a - \eta \leq \frac{\delta\gamma\eta - 2R_L\delta(1-a)}{R_L+\delta}, \\ \left(\frac{\delta\gamma\eta - (R_L+\delta)[R_B a - \eta]}{2R_L\delta},\ a\right), & \text{if } R_B a - \eta \in \left(\frac{\delta\gamma\eta - 2R_L\delta(1-a)}{R_L+\delta},\ \frac{\delta\gamma\eta}{R_L+\delta}\right), \\ (0,\ a)\,, & \text{if } R_B a - \eta \in \left[\frac{\delta\gamma\eta}{R_L+\delta},\ \frac{\gamma\eta}{2}\right], \\ \left(0,\ \frac{1}{R_B}\left[\eta + \frac{\gamma\eta}{2}\right]\right), & \text{if } R_B a - \eta > \frac{\gamma\eta}{2}, \end{cases} \tag{32}$$

if both banks have non-negative trade surpluses in each case.

In the third and the fourth case, it is immediate that both banks have non-negative trade surpluses. In the second case, the necessary and sufficient condition for non-negative trade surpluses for both banks is

$$\delta\gamma\eta + (R_L - \delta)[R_B a - \eta] \geq 0. \tag{33}$$

In the first case, the necessary and sufficient conditions for non-negative trade surpluses are:

$$\gamma\eta \geq R_L(1-a) + R_B a - \eta, \tag{34}$$

$$\delta(1-a) + R_B a - \eta \geq 0. \tag{35}$$

If (33) and (34)-(35) are not satisfied in the second and the first case, respectively, then banks do not settle bank transfers in period 1.

Now show the following lemma:

Lemma 1. Under Assumption 1, $(l(a),\ b(a)) = (1-a, a)$ never occurs in equilibrium.

Proof. This lemma is equivalent to say that the first case of (32) does not exist for any $a \in [0,1]$, or violates (34) or (35). First, a necessary condition for the existence of the first case is that there exists $a \in [0,1]$ such that

$$R_B a - \eta \leq \frac{\delta\gamma\eta - 2R_L\delta(1-a)}{R_L+\delta}, \tag{36}$$

as implied by (32). Note that both sides of this condition are increasing functions of a and also that the left-hand side is higher than the right-hand side at $a = 1$ under Assumption 1. Thus, there exists $a \in [0, 1]$ satisfying (36) if and only if the intercept of the left-hand side is lower than that of the right-hand side:

$$-\eta < \frac{\delta\gamma\eta - 2R_L\delta}{R_L + \delta}. \tag{37}$$

If this condition is violated, then the first case does not exist for any $a \in [0, 1]$.

Suppose that (37) holds. This condition is equivalent to

$$(\eta - \delta)R_L > \delta[R_L - (1 + \gamma)\eta]. \tag{38}$$

Thus, $\eta > \delta$, and hence $R_B > \delta$, follows given Assumption 1. For the first case to exist in this case, both (34) and (35) must be satisfied. Given $R_B > \delta$, these two conditions can be written as

$$a \geq \max\left\{\frac{R_L - (1 + \gamma)\eta}{R_L - R_B}, \frac{\eta - \delta}{R_B - \delta}\right\}, \tag{39}$$

where the first and the second term in the max operator are derived from (34) and (35), respectively. Under (37) and Assumption 1, it can be shown that:

$$\begin{aligned}
&\frac{R_L - (1 + \gamma)\eta}{R_L - R_B} - \frac{\eta - \delta}{R_B - \delta} \\
&\quad \propto R_L(R_B - \delta) - (1 + \gamma)\eta(R_B - \delta) - \eta(R_L - R_B) + \delta(R_L - R_B) \\
&\quad = R_L(R_B - \eta) + (\eta - \delta)R_B - (1 + \gamma)\eta(R_B - \delta) \\
&\quad = R_L(R_B - \eta) + (\eta - \delta)R_B - (1 + \gamma)\eta(R_B - \eta + \eta - \delta) \\
&\quad = [R_L - (1 + \gamma)\eta](R_B - \eta) + (\eta - \delta)[R_B - (1 + \gamma)\eta] > 0.
\end{aligned} \tag{40}$$

The inequality holds due to $\eta > \delta$ under (37). Thus, (34) is sufficient for (35) under (37) and Assumption 1.

Finally, show that (34) is violated in the first case of (32), if (37) and Assumption 1 hold. In this case, the first case of (32) can exist only for $a \in [0, a^*]$ such that

$$R_B a^* - \eta = \frac{\delta\gamma\eta - 2R_L\delta(1 - a^*)}{R_L + \delta}. \tag{41}$$

The root for this equation can be explicitly solved as

$$a^* = \frac{R_L(\eta - \delta) - \delta[R_L - (1+\gamma)\eta]}{R_L(R_B - \delta) - \delta(R_L - R_B)}. \tag{42}$$

It can be shown that

$$\begin{aligned}
a^* - \frac{R_L - (1+\gamma)\eta}{R_L - R_B}& \\
\propto R_L(\eta - \delta)(R_L - R_B) &- \delta[R_L - (1+\gamma)\eta](R_L - R_B) \\
- R_L(R_B - \delta)[R_L - (1+\gamma)\eta] &+ \delta(R_L - R_B)[R_L - (1+\gamma)\eta] \\
= R_L(\eta - \delta)(R_L - R_B) &- R_L(R_B - \delta)[R_L - (1+\gamma)\eta] \\
&\propto \frac{\eta - \delta}{R_B - \delta} - \frac{R_L - (1+\gamma)\eta}{R_L - R_B} < 0, \qquad (43)
\end{aligned}$$

where the last inequality is implied by (40). Thus, a^* is below the lower bound for a that satisfies (34). Hence, banks cannot have non-negative trade surpluses in the first case of (32), if (37) holds. □

It remains to pin down the range of a for the second case of (37). The necessary and sufficient condition for non-negative trade surpluses in the second case of (37), (33), implies

$$a \geq \frac{1}{R_B}\left(\eta - \frac{\delta\gamma\eta}{R_L - \delta}\right). \tag{44}$$

As shown in the proof for Lemma 1, (36) never holds for $a \in [0, 1]$ if (37) is violated. Thus, in this case, (44) becomes the lower bound for a in the second case of (37).

If (37) is satisfied, it can be shown that the right-hand side of (44) is greater than a^* in (42), that is, the root for (41):

$$\begin{aligned}
&\frac{1}{R_B}\left(\eta - \frac{\delta\gamma\eta}{R_L - \delta}\right) - a^* \\
&\propto \eta[R_L - (1+\gamma)\delta][(R_L + \delta)R_B - 2\delta R_L] - \{\eta[R_L + (1+\gamma)\delta] - 2\delta R_L\}(R_L - \delta)R_B \\
&= \eta R_L(2\delta R_B - 2\delta R_L) - \eta(1+\gamma)\delta(2R_L R_B - 2\delta R_L) + 2\delta R_L(R_L - \delta)R_B \\
&= 2\delta R_L[-\eta(R_L - R_B) - \eta(1+\gamma)(R_B - \delta) + R_B(R_L - \delta)] \\
&\propto -\eta(R_L - R_B) - \eta(1+\gamma)(R_B - \delta) + R_B(R_L - R_B + R_B - \delta) \\
&= (R_L - R_B)(R_B - \eta) + (R_B - \delta)[R_B - (1+\gamma)\eta] > 0. \qquad (45)
\end{aligned}$$

The last inequality follows from $R_B > \delta$ under Assumption 1 and (37), as implied by (38). Thus, (44) is the lower bound for a in the second case of (37) regardless of whether (37) is satisfied. Banks do not make any deal in period 1 if the value of a is lower than the right-hand side of (44).

C Proof for Proposition 2

If $l(a) = 0$ at the optimum of the bargaining problem (10), then each bank chooses the lower bound for a such that $l(a) = 0$, because an increase in the bond holdings only results in a transfer of more bonds in case of an outflow of bank transfers as long as $l(a) = 0$, as implied by (8). Hereafter, denote the lower bound for a such that $l(a) = 0$ by $\hat{a}$.

Next, compare $\hat{a}$ and the value of a such that $l(a) > 0$. For the range of a such that $l(a) > 0$, the objective function in the profit maximization problem for a bank in period 0, (10), can be written as

$$\begin{aligned} \Pi(a,\ a') \equiv\ & R_L(1-a) + R_B a - 1 \\ & + \frac{1}{2}\left\{ -R_L \frac{\delta\gamma\eta - (R_L+\delta)(R_B a - \eta)}{2R_L\delta} - R_B a + \eta \right\} + \frac{1}{2}\phi(a'). \end{aligned} \tag{46}$$

The derivative of this function with respect to a is

$$\begin{aligned} \frac{\partial \Pi(a,\ a')}{\partial a} &= -R_L + R_B + \frac{1}{2}\left[\frac{R_L(R_L+\delta)R_B}{2R_L\delta} - R_B \right] && (47) \\ &= -R_L + R_B + \frac{1}{2}\frac{R_L-\delta}{2\delta}R_B && (48) \\ &\propto R_L R_B - \delta(4R_L - 3R_B). && (49) \end{aligned}$$

Because the objective function in the profit maximization problem for a bank in period 0, (10), is continuous at $\hat{a}$ and the upper bound for a such that $l(a) > 0$, choosing a value of a such that $l(a) > 0$ is dominated by choosing $a = \hat{a}$ in period 0 if

$$\delta < \frac{R_L R_B}{4R_L - 3R_B}. \tag{50}$$

Finally, find the condition under which choosing $a = \hat{a}$ in period 0 dominates no settlement of outgoing bank transfers. If no settlement of outgoing bank transfers is optimal for a bank, then each bank sets $a = 0$ in period 0 because it does not need any liquidity for interbank settlement in period 1. Thus, choosing $a = \hat{a}$ in period 0 dominates no settlement of outgoing bank transfers if and only if

$$\begin{aligned}R_L\left[1-\frac{1}{R_B}\left(\eta+\frac{\delta\gamma\eta}{R_L+\delta}\right)\right]+R_B\left[\frac{1}{R_B}\left(\eta+\frac{\delta\gamma\eta}{R_L+\delta}\right)\right]-1\\+\frac{1}{2}\left[-R_B\frac{1}{R_B}\left(\eta+\frac{\delta\gamma\eta}{R_L+\delta}\right)+\eta\right]+\frac{1}{2}\phi(a')\\>R_L-1+\frac{1}{2}(-\gamma\eta)+\frac{1}{2}\phi(a'),\end{aligned}\quad(51)$$

where the left- and the right-hand side are the expected payoff for a bank with $a = \hat{a}$ and $a = 0$, respectively. This condition is equivalent to

$$\delta<\frac{R_L[(2+\gamma)R_B-2R_L]}{2(R_L-R_B)(1+\gamma)},\quad(52)$$

because

$$\begin{aligned}-\left(\frac{R_L}{R_B}-1\right)\left(\eta+\frac{\delta\gamma\eta}{R_L+\delta}\right)+\frac{1}{2}\left(-\frac{\delta\gamma\eta}{R_L+\delta}\right)-\frac{1}{2}(-\gamma\eta)\\\propto-2\left(\frac{R_L}{R_B}-1\right)\eta[R_L+(1+\gamma)\delta]+\gamma\eta R_L\\=-2\left(\frac{R_L}{R_B}-1\right)\eta(1+\gamma)\delta+\eta R_L\left[\gamma-2\left(\frac{R_L}{R_B}-1\right)\right].\end{aligned}\quad(53)$$

If both (50) and (52) hold, then it is optimal for a bank to choose $a = \hat{a}$ in period 0. Under Assumption 2, (50) is sufficient for (52) as

$$\begin{aligned}\frac{R_L[(2+\gamma)R_B-2R_L]}{2(R_L-R_B)(1+\gamma)}-\frac{R_LR_B}{4R_L-3R_B}\\\propto[(2+\gamma)R_B-2R_L](4R_L-3R_B)-2(R_L-R_B)(1+\gamma)R_B\\=\gamma R_B(4R_L-3R_B)-2(R_L-R_B)(4R_L-3R_B)\\-2(R_L-R_B)(1+\gamma)R_B\\=(2R_L-R_B)[\gamma R_B-4(R_L-R_B)].\end{aligned}\quad(54)$$

Assumption 4, in turn, is sufficient for (50) under Assumption 2, because

$$\frac{R_L R_B}{4R_L - 3R_B} - \frac{R_L}{1+\gamma} \propto R_B(1+\gamma) - (4R_L - 3R_B) \tag{55}$$

$$= -4(R_L - R_B) + R_B\gamma. \tag{56}$$

Thus, Assumptions 2 and 4 are sufficient for $a = \hat{a}$ in period 0.