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Optimal Macroprudential Policy*

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Abstract

This paper introduces financial market frictions into a standard New Keynesian model through search and matching in the credit market. Under such financial market frictions, a second-order approximation of social welfare includes a term involving credit, in addition to terms for inflation and consumption. As a consequence, the optimal monetary and macroprudential policies must contribute to both financial and price stability. This result holds for various approximated welfares that can change corresponding to macroprudential policy variables. The key features of optimal policies are as follows. The optimal monetary policy requires keeping the credit market countercyclical against the real economy. Commitment in monetary and macroprudential policy, rather than approximated welfare, justifies history dependence and pre-emptiveness. Appropriate combinations of macroprudential and monetary policy achieve perfect financial and price stability.

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1 Introduction

The serious economic disruptions caused by financial crises reveal the critical roles played by financial markets in the U.S. and the Euro area. Acknowledging that the current policy framework cannot fully mitigate nor avoid financial crises, policymakers have begun to shed light on two policy measures. The first is monetary policy, which aims to achieve, in addition to traditional policy goals, stability of the financial system. The second is a new policy tool, macroprudential policy geared toward financial stability.

Regarding this new role of monetary policy in financial stability, the Bank for International Settlements (BIS) emphasizes that central banks need to tighten monetary policy against accumulation of financial imbalances, such as overheating of mortgage, stock, and bond markets, even when the real economy seems to be stable in the near-term. This so-called BIS view is presented in BIS (2010) and Caruana (2010). Also, Taylor (2008) argues that in the U.S., the Federal Reserve Board (FRB) appears to adjust the target inflation rate in response to credit spread to stimulate the economy and maintain financial stability. These papers suggest that monetary policy can contribute to financial stability.

The role of macroprudential policy, which is independent from monetary and fiscal policy, in sustaining financial stability is also highlighted in the literature. Borio (2011) empirically shows the difference between financial and business cycles, and justifies the necessity of the coexistence of monetary policy and macroprudential policy.[1] In practice, some international organizations have begun to introduce macroprudential policy, for example, the Basel III framework, as set forth by the Basel Committee on Banking Supervision (BCBS, 2010) and BCBS (2014). Under this framework, to stabilize loan volume, banks are required to meet a particular base level of capital ratio against risk assets, where this base is changed according to economic and financial conditions. Several countries have also introduced several types of macroprudential policies, including total credit control and capital control, as described in Lim et al. (2011) and Nier et al. (2011).

In the literature, several authors have carried out quantitative studies of optimal monetary policy under well-known models with financial frictions. For example, Christiano et al. (2010) and Kannan, Rabanal, and Scott (2012) focus on the role of monetary policy in achieving financial stability in addition to real economic stability. These papers show

[1] Drehmann, Borio, and Tsatsaronis (2012) also show such empirical results.

that monetary policy reactions to credit expansion can help achieve overall macroeconomic stabilization. On the other hand, several papers explore macroprudential policies that are independent from monetary policy. Quint and Rabanal (2011) assume the dynamic stochastic general equilibrium model with real, nominal, and financial frictions, and study the optimal combination of monetary and macroprudential policies. These authors show numerically that social welfare improves when the policymaker's objective includes the credit term, implying that macroprudential policy is relevant. Suh (2012) shows that to improve welfare, macroprudential policy should respond to credit, while monetary policy should respond to the output gap and the inflation rate. These quantitative analyses, however, do not provide theoretical explanations of why monetary and macroprudential policy that takes credit into account may improve welfare.

Our goal is to derive the optimality criteria for monetary and macroprudential policy under financial market frictions. To introduce financial market frictions into a standard New Keynesian model, we assume search and matching in the credit market,[2] extending Wasmer and Weil (2000), Dell'Ariccia and Garibaldi (2000), Den Haan, Ramey, and Watson (2003), and Petrosky-Nadeau and Wasmer (2013).[3] Den Haan, Ramey, and Watson (2003) show that this search and matching procedure captures realistic properties of the credit market, such as the importance of long-term relationships between lenders and borrowers, and the time-consuming nature of building and re-building lending relationships.[4] These authors further show that search and matching frictions are costly for the economy, since the sluggish recovery of lending relationships leads to amplification of business cycle shocks. This provides an interpretation of Bernanke (1983), who shows that financial disruptions, through credit misallocation, induced the unusual length and depth of the Great

[2] Search and matching frictions are widely assumed in analyses of labor markets, as in Mortensen and Pissarides (1994) and Rogerson, Shimer, and Wright (2005).

[3] Different types of financial frictions are assumed in other studies. Bernanke, Gertler, and Gilchrist (1999) are the first to stress that credit market imperfections have a significant influence on business cycle dynamics. In the Bernanke, Gertler, and Gilchrist (1999) model, this financial market wedge is determined by time-varying leverage, where endogenous mechanisms in credit markets work to amplify and propagate shocks to the economy. In Bianchi (2010), financial market frictions are introduced by externalities in which private agents undervalue the dynamics of net worth, since agents fail to internalize the spillover effect between them.

[4] Blanchflower and Oswald (1998) and Peterson and Rajan (2002) empirically show that search and matching frictions play an important role in the credit market.

Depression. Due to symmetric dynamics, such credit market frictions explain large and long booms that lead to bubbles and financial crises. Thus, search and matching in the credit market appears to be an appropriate mechanism for explaining prolonged booms, financial crises, and resulting disruptions, against which macroprudential policies may play a role.

After developing a New Keynesian model with credit market search frictions, we approximate the representative household's welfare in the second order. This approach of approximating welfare is widely used in the New Keynesian literature. Woodford (2003) provides a prominent foundation for this approach, and shows optimal criteria for monetary policy. Aoki (2001) derives approximated welfare criteria for monetary policy in a model with both flexible-price and sticky-price sectors. Benigno (2004) extends the discussion of welfare criteria to an international macro framework. Erceg, Henderson and Levin (2000) derive a quadratic approximation of the household's utility function in a model with staggered wage contracts, while Thomas (2008) and Ravenna and Walsh (2011) pursue this approach in models with search and matching frictions in the labor market.

A novel finding of our paper is that the approximated welfare function includes, in addition to terms involving inflation and consumption, a term related to credit volume. This result theoretically justifies the necessity of financial stability for the optimal policy, and clearly indicates that policymakers should pay attention to credit volume.

Using the approximated welfare function, we explore the properties of an optimal monetary policy. We find that by taking the financial variable into account, monetary policy can contribute to financial stability and thus perform the macroprudential role. When equipped with this additional role, the optimal monetary policy must keep the credit market countercyclical against the real economy, such as setting the policy rate to induce disinflation against positive credit growth. Further, history dependence is important for achieving financial stability in addition to real economic stability.

We then examine macroprudential policy, which involves intervention in the credit market. By introducing macroprudential policy variables, the form of the approximated welfare of the representative household itself can change according to the macroprudential policy pursued. Yet, the optimal macroprudential policy needs to respond to both financial and real economic variables for various approximated welfares. Commitment in macroprudential policies, rather than approximated welfare, can bring about backward-looking and/or forward-looking terms.

We present examples of macroprudential policy, focusing on two different policy variables. In the first example, the macroprudential authority adjusts the bargaining power of banks in the credit market, which is interpreted as controlling the degree of competition among banks through financial regulations. In this case, the optimal macroprudential policy, accompanied by monetary policy, perfectly stabilizes the inflation rate against a cost-push shock. This is because the macroprudential policy contributes to price stability through the cost channel. In the second example, the macroprudential authority chooses the probability of termination of borrower–lender relationships, or the *credit separation rate*, which can be interpreted as total credit control. In this case, the approximated welfare function additionally includes a term involving credit separation, and the optimal macroprudential policy weighs the trade-off between the credit separation rate, credit market tightness, inflation rate, and consumption. In any case, macroprudential policy, primarily linked with financial stability, is closely associated with price stability, and consequently with monetary policy.

The rest of the paper is organized as follows. In Section 2, we set up the model. In Section 3, we derive the second-order approximation of the social welfare function and comment on its properties. In Section 4, we show the linearized system of the model. In Section 5, we derive the optimal monetary policy and discuss its properties. In Section 6, we derive the optimal macroprudential policy under various settings, and reveal the interaction between the optimal monetary and macroprudential policies. Finally, in Section 7, we conclude the paper.

2 Model

The model economy is populated by four types of private agent: a single representative household (consumer), and large numbers of wholesale firms, banks, and retail firms. We explain the problems faced by these agents in turn, then describe the credit market, which is characterized by search and matching frictions, as well as the goods market.

2.1 Household

An infinitely lived representative household derives utility only from consumption, and discounts the future with discount factor $\beta \in (0, 1)$. In period t, the household enjoys total real consumption C_t and receives Π_t as a real lump-sum profit from firms and banks. In addition, the household deposits D_t into a bank account, to be repaid at the end of period t with nominal interest rate $R_t^D - 1$, where R_t^D is set by the central bank.

Letting P_t denote the price of C_t, the household's problem is

$$\max_{C,D} \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i u(C_{t+i}), \tag{1}$$

subject to the budget constraint

$$C_t = \Pi_t + \frac{R_{t-1}^D D_{t-1} - D_t}{P_t}. \tag{2}$$

The household's period utility function is

$$u(C_t) \equiv \frac{C_t^{1-\sigma}}{1-\sigma},$$

where $\sigma > 0$ is the coefficient of relative risk aversion.

This optimization problem leads to

$$\lambda_t = C_t^{-\sigma},$$

$$1 = \beta \mathrm{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} R_t^D \right], \tag{3}$$

where λ_t is the Lagrange multiplier on the budget constraint, (2).

Total consumption C_t is an aggregate of differentiated retail goods, labeled by $j \in [0, 1]$. Consumption of each good $c_t(j)$ is related to C_t by

$$C_t \equiv \left[\int_0^1 c_t(j)^{\frac{\varepsilon_t - 1}{\varepsilon_t}} dj \right]^{\frac{\varepsilon_t}{\varepsilon_t - 1}},$$

where $\varepsilon_t \in (1, \infty)$ is the elasticity of substitution among retail goods, which follows a known stochastic process. In what follows, random fluctuations of ε_t are the only source of aggregate uncertainty.

The household chooses each $c_t(j)$ to minimize cost $\int_0^1 p_t(j) c_t(j) dj$, given the level of C_t and the price of each good, $p_t(j)$. This minimization yields

$$c_t(j) = \left[\frac{p_t(j)}{P_t} \right]^{-\varepsilon_t} C_t \ ,$$

where

$$P_t \equiv \left[\int_0^1 p_t(j)^{1-\varepsilon_t} dj\right]^{\frac{1}{1-\varepsilon_t}}. \tag{4}$$

2.2 Wholesale Firms

In any period, a wholesale firm can be either a productive firm or a credit seeker firm. A productive firm produces Z units of wholesale goods. To be productive, a firm must obtain a real units of credit from a bank. The credit market is characterized by search frictions, and in each period, a credit seeker firm must buy retail goods $\kappa_t(j)$ to satisfy

$$\left[\int_0^1 \kappa_t(j)^{\frac{\varepsilon_t-1}{\varepsilon_t}} dj\right]^{\frac{\varepsilon_t}{\varepsilon_t-1}} \geq \kappa,$$

where $\kappa > 0$ is the flow cost of searching for credit. The cost minimization for $\kappa_t(j)$ parallels that for $c_t(j)$ in the household's problem. For simplicity, we assume that firms finance the cost of searching for credit by issuing stocks to the household.[5]

In period t, with probability p_t^F, a credit seeker firm is matched with a bank and engages in a credit contract. The firm then receives a real units of credit and becomes productive, sells the produced goods to retail firms, and repays $R_t^L a$ to the bank, where the loan interest rate $R_t^L - 1$ is determined in equilibrium. Finally, at the end of period t, a credit contract is exogenously terminated with probability $\rho \in (0,1)$, in which case the firm and the bank separate and search for new matches in period $t+1$. With probability $1-\rho$, a credit contract is sustained and the firm again receives credit in period $t+1$. We call ρ the credit separation rate.

There is free entry into the wholesale goods industry. Thus, in equilibrium, the value of a credit seeker firm is zero, and hence the cost of searching for credit must equal the expected revenue, or

$$\kappa = p_t^F W_t. \tag{5}$$

Here, W_t is the value of a productive wholesale firm, written as

$$W_t = \frac{Z}{\mu_t} - \left(R_t^L - 1\right) a + \beta \mathrm{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t}(1-\rho) W_{t+1}\right], \tag{6}$$

[5] In an older version of our paper, Munakata, Nakamura, and Teranishi (2013) pursue an alternative setup in which wholesales firms costlessly search for credit and banks pay the cost of posting vacancies. The form of the approximated welfare function under this setup is identical to that obtained below.

where

$$\mu_t \equiv \frac{P_t}{P_t^w}$$

is the price markup by retail firms, and P_t^w is the price of a wholesale good. The first two terms on the right-hand side (RHS) of (6) show the net current profit from production, while the third term is the discounted present value of future profit.

Given these assumptions, the demand for retail good j and total demand are

$$y_t^d(j) \equiv c_t(j) + \kappa_t(j)u_t,$$

and

$$Y_t^d \equiv C_t + \kappa u_t,$$

respectively, where u_t is the number of credit seeker firms. Note that y_t^d is related to Y_t^d by the following equation:

$$y_t^d(j) = \left[\frac{P_t(j)}{P_t}\right]^{-\varepsilon_t} Y_t^d. \tag{7}$$

2.3 Banks

Banks collect money from the household as deposits, and lend it to wholesale firms. To search for credit seeker firms, banks must post credit offers, which we call "credit vacancies". Posting credit vacancies is costless, but total funds available for lending is fixed at aL^*,[6] such that the upper limit of the number of credit contracts is L^*.[7] Therefore, the number of credit vacancies v_t is expressed as

$$v_t = L^* - (1-\rho)L_{t-1}, \tag{8}$$

where L_t is the number of productive wholesale firms. In period t, a credit vacancy is filled with probability q_t^B. Thus, L_t evolves according to

$$L_t = (1-\rho)L_{t-1} + q_t^B v_t.$$

In such settings, the value of a credit match for banks is

$$J_t^1 = a(R_t^L - 1) + \beta \mathrm{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t} \left\{ (1-\rho)J_{t+1}^1 + \rho \left[q_{t+1}^B J_{t+1}^1 + (1-q_{t+1}^B) J_{t+1}^0 \right] \right\} \right).$$

[6] For simplicity, we assume that aL^* is less than the amount of deposit.

[7] In reality, the limit of credit is determined by regulations, such as the leverage ratio regulation.

The first term on the RHS shows current profit from lending, while the second term represents discounted present value of future profit. On the other hand, the value of a credit vacancy for banks is

$$J_t^0 = \beta \mathrm{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[q_{t+1}^B J_{t+1}^1 + (1 - q_{t+1}^B) J_{t+1}^0 \right] \right\}.$$

Since a credit vacancy yields no current profit, it has only discounted future values. These two equations imply that the bank's surplus from a credit match is

$$J_t \equiv J_t^1 - J_t^0 = a(R_t^L - 1) + \beta \mathrm{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} (1-\rho)(1 - q_{t+1}^B) J_{t+1} \right]. \tag{9}$$

2.4 Retail Firms

Retail firms produce differentiated retail goods from wholesale goods, which are then sold to the household in a monopolistically competitive market. One unit of wholesale goods is converted into one unit of retail good j. To introduce price stickiness, we assume that a firm can adjust its price each period with probability $1-\omega$, as in Calvo (1983) and Yun (1996). Since the demand for good j is given by equation (7), the profit maximization problem of a retail firm that has a chance to adjust its price P_t^* becomes

$$\max_{P_t^*} \mathrm{E}_t \sum_{i=0}^{\infty} (\omega\beta)^i \left[\left(\frac{\lambda_{t+i}}{\lambda_t} \right) \left(\frac{(1+\tau) P_t^* - P_{t+i}^w}{P_{t+i}} \right) \left(\frac{P_t^*}{P_{t+i}} \right)^{-\varepsilon_{t+i}} Y_{t+i}^d \right].$$

We here assume that the subsidy for retails firms τ is set to ensure that price flexibility is achieved at the efficient steady-state equilibrium defined below. Note that P_t is related to P_{t-1} and P_t^* as

$$P_t^{1-\varepsilon_t} = (1-\omega) (P_t^*)^{1-\varepsilon_t} + \omega P_{t-1}^{1-\varepsilon_t}.$$

2.5 Credit Market

The number of new credit matches in a period is given by a Cobb-Douglas matching function

$$m(u_t, v_t) = \chi u_t^{1-\alpha} v_t^{\alpha},$$

where $\chi, \alpha \in (0,1)$ are constant parameters. Defining credit market tightness as[8]

$$\theta_t = \frac{u_t}{v_t}, \tag{10}$$

[8] Note that in our environment, u_t and v_t correspond, respectively, to the demand and the supply of credit. Thus, market tightness is defined as u_t/v_t, rather than its inverse.

we obtain

$$p_t^F = \chi\theta_t^{-\alpha}, \tag{11}$$

$$q_t^B = \chi\theta_t^{1-\alpha}, \tag{12}$$

$$L_t = (1-\rho)L_{t-1} + \chi\theta_t^{1-\alpha}v_t. \tag{13}$$

A wholesale firm and a bank forming a credit match share the match surplus according to generalized Nash bargaining. Thus, R_t^L solves

$$\max_{R_t^L} W_t^{1-b}J_t^b,$$

where $b \in (0,1)$ is the bargaining power of banks. By taking the first-order condition with respect to R_t^L,

$$bW_t = (1-b)J_t. \tag{14}$$

Using equations (5), (11), (12), and (14) to eliminate p_t^F, q_t^B, W_t, and J_t from (6) and (9), we obtain

$$\frac{\kappa}{\chi}\theta_t^\alpha = \frac{Z}{\mu_t} - (R_t^L - 1)a + \beta \mathrm{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}(1-\rho)\frac{\kappa}{\chi}\theta_{t+1}^\alpha\right]$$

and

$$\frac{b}{1-b}\frac{\kappa}{\chi}\theta_t^\alpha = (R_t^L - 1)a + \beta \mathrm{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}(1-\rho)\left(1-\chi\theta_{t+1}^{1-\alpha}\right)\frac{b}{1-b}\frac{\kappa}{\chi}\theta_{t+1}^\alpha\right]. \tag{15}$$

By further eliminating R_t^L from these equations, we obtain the following condition, which relates the markup μ_t to credit market tightness θ_t:

$$\frac{Z}{\mu_t} = \frac{1}{1-b}\frac{\kappa}{\chi}\theta_t^\alpha - \beta \mathrm{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}(1-\rho)\frac{1}{1-b}\left(\frac{\kappa}{\chi}\theta_{t+1}^\alpha - b\kappa\theta_{t+1}\right)\right]. \tag{16}$$

Equation (16) shows that the credit market affects the real economy, that is, the price setting behavior, through the cost channel.

2.6 Goods Market Clearing Condition

Since one unit of wholesale goods is needed as an input to produce one unit of each retail good j, the market clearing condition for wholesale goods is

$$ZL_t = \int_0^1 y_t^d(j)dj.$$

Together with the demand equation for retail goods (7), the following goods market clearing condition is obtained:

$$\frac{ZL_t}{Q_t} = C_t + \kappa u_t. \tag{17}$$

Here,

$$Q_t \equiv \int_0^1 \left[\frac{P_t(j)}{P_t}\right]^{-\varepsilon_t} dj \tag{18}$$

represents dispersion of prices of retail goods due to price stickiness for retail firms.

3 Welfare Criteria

This paper examines the optimal policy under the linear-quadratic approximation framework of Woodford (2003) and Benigno and Woodford (2008). In this section, we first define the efficient steady-state equilibrium, then expand the household's utility function around the equilibrium to derive the second-order approximation of the welfare function. We then comment on the implications of the approximated welfare function. Note that the linear (first order) approximation of the structural equations is discussed in the next section.

3.1 Efficient Steady-State Equilibrium

The efficient steady-state equilibrium is defined as a steady-state equilibrium of the deterministic (i.e., $\varepsilon_t = \bar{\varepsilon}$ for all t)[9] model, whose allocation coincides with that of a benevolent social planner who maximizes the discounted lifetime utility of the representative household. Such an equilibrium can be achieved only when the model exhibits neither credit matching inefficiency nor price markup. Specifically, in the efficient steady-state equilibrium, (1) the Hosios (1990) condition holds, that is, the bargaining power of banks (b) equals the elasticity of the matching function with respect to credit vacancies (α), and (2) the subsidy for retail firms τ is chosen to ensure $\bar{\mu} = \frac{\bar{\varepsilon}}{(\bar{\varepsilon}-1)(1+\tau)} = 1$.

By definition, the allocation in the efficient steady-state equilibrium can be obtained by solving the following optimization problem of the social planner:

$$\max_{C,L,v,\theta} \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \{ \frac{C_{t+i}^{1-\sigma}}{1-\sigma} + \phi_{t+i} \left[Z L_{t+i} - \kappa \theta_{t+i} v_{t+i} - C_{t+i} \right]$$
$$+ \psi_{t+i} \left[(1-\rho) L_{t+i-1} + \chi \theta_{t+i}^{1-\alpha} v_{t+i} - L_{t+i} \right] + s_{t+i} \left[v_{t+i} - L^* + (1-\rho) L_{t+i-1} \right] \},$$

where ϕ_t, ψ_t, and s_t are the Lagrange multipliers for the constraints. The solution to this problem yields the condition that characterizes the efficient steady-state equilibrium:

$$Z - \frac{1}{1-\alpha}\frac{\kappa}{\chi}\bar{\theta}^{\alpha} = -\beta(1-\rho)\frac{1}{1-\alpha}\frac{\kappa}{\chi}\bar{\theta}^{\alpha}\left(1-\alpha\chi\bar{\theta}^{1-\alpha}\right). \tag{19}$$

[9] Throughout, a bar above each variable implies its value in the efficient steady-state equilibrium.

For later convenience, let

$$\delta_1 \equiv Z - \frac{1}{1-\alpha}\frac{\kappa}{\chi}\bar{\theta}^{\alpha}$$

and

$$\delta_2 \equiv (1-\rho)\frac{1}{1-\alpha}\frac{\kappa}{\chi}\bar{\theta}^{\alpha}\left(1-\alpha\chi\bar{\theta}^{1-\alpha}\right)$$

to simplify (19) as

$$\delta_1 = -\beta\delta_2. \tag{20}$$

Since $q^B\left(\bar{\theta}\right) = \chi\bar{\theta}^{1-\alpha} \leq 1$, it follows that $\delta_2 \geq 0$ and thus $\delta_1 \leq 0$.

3.2 Policy Objective Function

We now derive a second-order approximation to the welfare function following Woodford (2003). The second-order expansion of the household's utility function around the efficient steady-state equilibrium yields

$$\sum_{i=0}^{\infty}\beta^i u(C_{t+i}) = -\frac{1}{2}\sum_{i=0}^{\infty}\beta^i N_{t+i} + t.i.p., \tag{21}$$

where a period policy objective function N is given by

$$N_{t+i} = \lambda_\pi \pi_{t+i}^2 + \lambda_\theta \widehat{\theta}_{t+i}^2 + \lambda_c \widehat{c}_{t+i}^2,$$

where $\pi_t \equiv \widehat{p}_t - \widehat{p}_{t-1}$ is the inflation rate, $\lambda_\pi \equiv u_c Z\bar{L}\bar{\varepsilon}/\delta$, $\lambda_\theta \equiv u_c\kappa\bar{u}\alpha$, $\lambda_c \equiv \sigma u_c\bar{C}$, $u_c \equiv u'(\bar{C})$, $\delta \equiv \frac{(1-\omega)(1-\omega\beta)}{\omega}$, and $t.i.p.$ denotes "terms independent of policy."[10] Note that the log-deviation of a variable (e.g., C_t) from its efficient steady-state value ($\bar{C}$) is expressed by placing a hat $\widehat{\ }$ over its lower case ($\widehat{c}_t$). In particular, we call $\widehat{c}_t$ the consumption gap.

Equation (21) shows that the optimal policy faces a trade-off between the inflation rate, consumption, and credit market tightness. The presence of credit market tightness in the approximated welfare function has a novel implication for the optimal policy. Under credit market frictions, even when the real economy is perfectly stable, with zero inflation and consumption equaling the efficient steady state level, the optimal policy should respond

[10] We provide detailed derivations in Appendix A. As we discuss in Appendix B, even when productivity shock Z_t is introduced into the model, we can derive a mathematically similar formula for the approximated welfare function by taking the difference from the efficient stochastic state. Here, the efficient stochastic state is defined as a time-dependent state whose allocation coincides with that of a benevolent social planner who maximizes the discounted lifetime utility of the representative household given the stochastic variation of productivity Z_t.

to an inefficient state of the credit market. Note that the period policy objective function includes components of inflation, consumption, and credit at time t.

Moreover, the approximated welfare function can be transformed as

$$\begin{aligned}&\sum_{i=0}^{\infty}\beta^i u(C_{t+i}) \\ &= -\frac{1}{2}\sum_{i=0}^{\infty}\beta^i\left[\lambda_\pi\pi_{t+i}^2+\frac{\lambda_\theta}{(1-\alpha)^2\rho^2}\left(\widehat{l}_{t+i}-\rho_u\widehat{l}_{t+i-1}\right)^2+\lambda_c\widehat{c}_{t+i}^2\right]+t.i.p.,\end{aligned} \tag{22}$$

where

$$\rho_u \equiv (1-\rho)\left(1-\rho\frac{\bar{L}}{\bar{v}}\right).$$

Equation (22) clearly shows that the optimal policy should respond to the volume of credit. Note that $\rho_u \in (0,1)$, since (12) and (13) yield $\rho\bar{L}/\bar{v} = q^B\left(\bar{\theta}\right) \in [0,1]$. In particular, as the separation rate ρ approaches 1, ρ_u approaches zero. This implies that the optimal policy should focus on the current volume of credit, because the history of the credit market is irrelevant when all matches are replaced each period. In contrast, as ρ approaches zero, ρ_u approaches 1. In this limit, all existing loans continue to the next period, so the optimal policy should focus on the volume of new loans, or equivalently, on the growth of credit.

The result that the criteria for optimal policy directly includes the volume of credit is a nontrivial finding for the macroprudential policy. It justifies that the macroprudential policy works for eliminating inefficient dynamics of lending, such as an over-lending. This is quite consistent with the aim of the macroprudential policy to stabilize the volume of loan.[11]

Moreover, the result here provides valuable implications for setting simple policy rules, including simple monetary policy rules and simple macroprudential policy rules. In the case of the model without the credit market, Woodford (2003) analytically shows that simple monetary policy rules should respond to inflation rate and consumption terms, since approximated welfare includes these terms and their stabilization can improve welfare. This author suggests that the optimal monetary policy can be replicated by a simple policy rule and justifies the conventional simple monetary policy rule, the Taylor rule.

Recently, a number of studies have claimed that a simple policy rule should include variables related to credit. For example, Taylor (2008) empirically points out that a spread-adjusted Taylor rule that additionally includes the credit spread term in the standard Taylor

[11] For example, see BCBS (2010, 2014).

rule can well explain the easing of monetary policy by the FRB in response to the subprime mortgage crisis. Also, as mentioned in the introduction, Christiano et al. (2010), Quint and Rabanal (2011), Suh (2012), and Kannan, Rabanal, and Scott (2012), from numerical simulations, claim that policy should respond to credit in addition to inflation and the output gap to improve welfare. Our results extend Woodford (2003) and theoretically support the mentioned studies. Equations (21) and (22) imply that, under credit market frictions, the simple policy rules should include terms related to credit.

3.3 Analysis for Welfare Criteria

We now analyze the dependence of the welfare function on several parameters, especially focusing on the weight of the term corresponding to financial frictions.

Straightforward calculation in Appendix C yields

$$\frac{\partial}{\partial \kappa}\left(\frac{\lambda_\theta}{\lambda_c}\right) > 0, \tag{23}$$

where the parameters (e.g., α) except κ are fixed, but the efficient steady-state value of the variables (e.g., $\bar{\theta}$) are allowed to vary with κ. The relation (23) implies that, as the cost of searching for credit κ increases, the relative weight for credit to that for consumption in the approximated welfare function (λ_θ/λ_c of equation (21)) increases. This is because the degree of financial friction increases with κ, and the optimal policy responding to an inefficient state of the credit market induces greater welfare improvement.

A similar relation holds for the credit separation rate ρ, namely:

$$\frac{\partial}{\partial \rho}\left(\frac{\lambda_\theta}{\lambda_c}\right) > 0. \tag{24}$$

Again, this is because the increase in ρ raises the cost of holding credit.

These results imply that the relative weight for the credit term increases when the cost of obtaining credit increases. In other words, as the degree of market imperfection increases, the optimal policy should react more strongly to the credit market condition.

4 Linearization

In this section, we log-linearize the structural equations around the efficient steady-state equilibrium. For a general stochastic non-efficient state, the Calvo-type stickiness introduced in the retail sector leads to the standard Phillips curve with a cost-push shock $\widehat{\varepsilon}_t$,

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} - \delta \left(\frac{1}{\bar{\varepsilon} - 1} \widehat{\varepsilon}_t + \widehat{\mu}_t \right). \tag{25}$$

The retail price markup term $\widehat{\mu}_t$ in this equation can be obtained from the log-linearized version of equation (16),

$$Z\widehat{\mu}_t = -\frac{\alpha}{1-\alpha} \frac{\kappa}{\chi} \bar{\theta}^{\alpha} \left(\widehat{\theta}_t - \beta \rho_u \mathrm{E}_t \widehat{\theta}_{t+1} \right) - \beta \delta_2 \sigma \left(\mathrm{E}_t \widehat{c}_{t+1} - \widehat{c}_t \right), \tag{26}$$

where the Hosios condition $b = \alpha$ is used.

From equation (3), the IS relation is given as

$$\widehat{c}_t = \mathrm{E}_t \widehat{c}_{t+1} - \frac{1}{\sigma} \left(\widehat{r}_t^D - \mathrm{E}_t \pi_{t+1} \right). \tag{27}$$

On the other hand, by linearizing equations (8) and (13), we can express the credit market tightness term $\widehat{\theta}_t$ by using the loan volume term $\widehat{l}_t$ as

$$\widehat{\theta}_t = \frac{1}{(1-\alpha)\rho} \left(\widehat{l}_t - \rho_u \widehat{l}_{t-1} \right). \tag{28}$$

By combining equation (28) with the linearized equation of the market clearing condition (equation (17)), the consumption gap $\widehat{c}_t$ is given by

$$\widehat{c}_t = \frac{\bar{L}\delta_2}{\bar{C}} \left(-\beta \widehat{l}_t + \widehat{l}_{t-1} \right). \tag{29}$$

It is also noteworthy that, by linearizing equation (15), it is shown that the loan interest rate term $\widehat{r}_t^L$ is related to $\widehat{\theta}_t$ and $\widehat{r}_t^D$ as

$$\begin{aligned} a\bar{R}^L \widehat{r}_t^L = \frac{\alpha}{1-\alpha} \frac{\kappa}{\chi} \bar{\theta}^{\alpha} \Big[& \alpha \widehat{\theta}_t - \beta(1-\rho) \left(\alpha - \chi \bar{\theta}^{1-\alpha} \right) \mathrm{E}_t \widehat{\theta}_{t+1} \\ & + \beta(1-\rho) \left(1 - \chi \bar{\theta}^{1-\alpha} \right) \left(\widehat{r}_t^D - \mathrm{E}_t \pi_{t+1} \right) \Big]. \end{aligned} \tag{30}$$

In particular, equation (30) implies that when the deposit interest rate and credit market tightness increase, so does the loan interest rate. Moreover, by using equations (27), (28), and (29), this equation can be transformed as

$$\widehat{r}_t^L = h_1 \mathrm{E}_t \widehat{l}_{t+1} + h_2 \widehat{l}_t + h_3 \widehat{l}_{t-1}, \tag{31}$$

where

$$h_1 \equiv \frac{1}{h_4}\left[-\beta\frac{(1-\rho)}{\rho}\frac{(\alpha-\chi\overline{\theta}^{1-\alpha})}{1-\alpha}-\beta^2\sigma\rho_u\frac{\bar{L}}{\overline{C}}\delta_2\right],$$
$$h_2 \equiv \frac{1}{h_4}\left[\frac{\alpha}{(1-\alpha)\rho}+\beta\frac{(1-\rho)}{\rho}\frac{(\alpha-\chi\overline{\theta}^{1-\alpha})}{1-\alpha}\rho_u+\beta\sigma\rho_u\frac{\bar{L}}{\overline{C}}\delta_2(1+\beta)\right],$$
$$h_3 \equiv \frac{1}{h_4}\left[-\frac{\alpha}{1-\alpha}\frac{\rho_u}{\rho}-\beta\sigma\rho_u\frac{\bar{L}}{\overline{C}}\delta_2\right],$$
$$h_4 \equiv \frac{a\,(1-\alpha)\,\chi\bar{R}^L}{\alpha\kappa\overline{\theta}^{\alpha}}.$$

Thus, the deposit rate, the loan interest rate, and credit volume are closely related.[12]

5 Optimal Monetary Policy

5.1 Derivation

We now investigate the optimal monetary policy when the central bank is the unique authority responsible for financial as well as real economic stability. In other words, monetary policy is expected to play a macroprudential role as well.

Following Woodford (2003), we assume that the central bank controls the nominal interest rate on deposits, R_t^D, and accordingly the real interest rate on deposits, to maximize social welfare. By changing the real interest rate, the central bank can affect consumption, and thus the entire economy, through the IS relation given by equation (27). This is the typical transmission channel of monetary policy in the literature.

For the approximated welfare function in equation (21), the optimal commitment policy under the timeless perspective for the central bank is then obtained by solving

$$\min_{\pi,\widehat{\theta},\widehat{c},\widehat{l},\widehat{r}^D} \mathrm{E}_t \sum_{i=0}^{\infty}\frac{1}{2}\beta^i\left(\lambda_\pi\pi_{t+i}^2+\lambda_\theta\widehat{\theta}_{t+i}^2+\lambda_c\widehat{c}_{t+i}^2\right)$$

subject to the Phillips curve equation (25), the markup equation (26), the IS equation (27), the credit market tightness equation (28), and the consumption equation (29).

[12] By using equation (31), it is possible to include the loan rate term in the approximated welfare function. The welfare function that includes the loan interest rate is consistent with those in Teranishi (2008) and Cúrdia and Woodford (2009). Teranishi (2008) shows that under the staggered cost channel model, an approximated welfare function includes growth of the loan interest rate. Cúrdia and Woodford (2009) show that an approximated welfare function includes the credit spread term under a model where households face financial market frictions.

The first-order conditions with respect to π_t, $\widehat{\theta}_t$, $\widehat{c}_t$, $\widehat{l}_t$, and $\widehat{r}_t^D$ are

$$\lambda_\pi \pi_t + \varphi_{1t} - \varphi_{1t-1} - \beta^{-1}\sigma^{-1}\varphi_{2t-1} = 0, \tag{32}$$

$$\lambda_\theta \widehat{\theta}_t - \frac{\delta}{Z}\frac{\alpha}{1-\alpha}\frac{\kappa}{\chi}\bar{\theta}^\alpha(\varphi_{1t} - \rho_u\varphi_{1t-1}) + \varphi_{3t} = 0, \tag{33}$$

$$\lambda_c \widehat{c}_t - \frac{\delta\delta_2\sigma}{Z}(\varphi_{1t-1} - \beta\varphi_{1t}) + \varphi_{2t} - \beta^{-1}\varphi_{2t-1} + \varphi_{4t} = 0, \tag{34}$$

$$-\frac{1}{(1-\alpha)\rho}\varphi_{3t} + \frac{\beta\rho_u}{(1-\alpha)\rho}\mathrm{E}_t\varphi_{3t+1} + \frac{\bar{L}\delta_2\beta}{\bar{C}}\varphi_{4t} - \frac{\bar{L}\delta_2\beta}{\bar{C}}\mathrm{E}_t\varphi_{4t+1} = 0, \tag{35}$$

$$\varphi_{2t} = 0, \tag{36}$$

where φ_{1t}, φ_{2t}, φ_{3t}, and φ_{4t} are the Lagrange multipliers for equations (25) (combined with (26)), (27), (28), and (29), respectively.

As shown in Appendix D, these first-order conditions yield the following condition, which clearly characterizes the optimal monetary policy:

$$\pi_t + \frac{Z\bar{L}}{\alpha\delta\kappa\bar{u}}\frac{\lambda_\theta}{\lambda_\pi}\left(\widehat{l}_t - \widehat{l}_{t-1}\right) = 0. \tag{37}$$

The central bank adjusts the deposit interest rate R_t^D (and thus $\widehat{r}_t^D$) to satisfy the optimal targeting rule (37). This optimal targeting rule and the linearized structural equations (25)–(29) define the paths of π_t, $\widehat{\theta}_t$, $\widehat{c}_t$, $\widehat{l}_t$, and $\widehat{r}_t^D$ under the optimal monetary policy.

5.2 Features of Optimal Monetary Policy

The optimal targeting rule (37) has several important features. First, the optimal targeting rule includes both financial variables ($\widehat{l}_t$ and $\widehat{l}_{t-1}$) and a real economic variable (π_t), implying that the optimal targeting rule must maintain a balance between financial and real economic conditions. By taking financial variables into account, monetary policy may contribute to financial stability and perform the macroprudential role. This result contrasts with the standard result shown in Woodford (2003): Under the model with frictions in the goods market, that is, price stickiness, the loan volume gap $\widehat{l}_t$ is replaced by the consumption gap $\hat{c}_t$, so the optimal monetary policy focuses on the relation between inflation and consumption.

Second, since $\lambda_\theta = u_c\kappa\bar{u}\alpha > 0$ and $\lambda_\pi = u_c Z\bar{L}\bar{\varepsilon}/\delta > 0$, the coefficient on $\widehat{l}_t - \widehat{l}_{t-1}$ in equation (37) is positive, and can be simplified as

$$\frac{Z\bar{L}}{\alpha\delta\kappa\bar{u}}\frac{\lambda_\theta}{\lambda_\pi} = \frac{1}{\bar{\varepsilon}} > 0.$$

Thus, when monetary policy also serves a macroprudential role, optimality requires keeping negative comovement between price and credit. In the optimal targeting rule, the policy rate is set to induce disinflation against positive credit growth, so as to avoid overheating or overcooling of the economy. This finding is consistent with the recent argument claiming that preventing pro-cyclicality of financial markets can reduce the occurrences of, and dampen the disruptions from, financial crises.[13]

Third, the presence of $\widehat{l}_{t-1}$ in equation (37) implies that the optimal monetary policy is history dependent. Woodford (2003) shows that history dependence is one of the fundamental features of monetary policy for management of expectations. By managing expectations, the central bank can control the real interest rate and guarantee the effectiveness of the monetary policy. This property extends to our model's environment, where monetary policy aims to achieve financial stability in addition to real economic stability. Here, the approximated welfare itself does not provide an answer to the question of superiority of ex ante or ex post policy in conducting optimal policy. This is seen from equation (21), where the period policy objective function N includes only components of inflation, consumption, and credit at time t. However, such ex ante and ex post features of the policy arise through the introduction of the style of the policy, such as a commitment policy.

6 Optimal Macroprudential Policy

6.1 Features of Optimal Macroprudential Policy

We now introduce the macroprudential policy, which involves interventions in the credit market by the macroprudential authority. Before assuming detailed policy tools, we describe basic features of the optimal macroprudential policy from approximated welfare.

First, depending on the macroprudential policy variable, the form of the approximated welfare of the representative household may differ from equation (21). This is because macroprudential policies themselves become a part of the economy and change the economic structure. This simple but new finding can be demonstrated via the approach of a model-consistent welfare approximation, and is very important in conducting optimal policies under new macroprudential policies.

Second, even when the form of the approximated welfare changes via the macropru-

[13] See, e.g., BIS (2009).

dential policy, it still includes both financial variables, such as credit, and real economic variables, such as inflation and consumption. This finding clearly contrasts with recent argument that insists that the macroprudential authority should focus on the financial variables only.[14]

Third, as shown in the following subsection, an appropriate combination of macroprudential and monetary policy can secure perfect financial and price stability. Given the cost-push shock, the macroprudential policy for the bargaining power of banks achieves perfect financial and price stability while the macroprudential policy for the credit separation rate fails in both perfect financial and price stability.

Fourth, style of the policy such as a commitment policy, rather than the approximated welfare, determines the superiority of ex ante or ex post policy in conducting macroprudential policy. Under commitment, the optimal macroprudential policies can include backward-looking and forward-looking terms.

In the next subsection, we discuss the optimal macroprudential policy assuming two different policy variables. As for the monetary policy, we consider two cases – in the former, the deposit interest rate is fixed at the efficient steady-state level, and in the latter, the macroprudential authority chooses the macroprudential policy variable taking the monetary policy variable as given, and similarly for the monetary authority.[15]

6.2 Examples

6.2.1 Intervention in Nash Bargaining

We first consider a macroprudential policy that intervenes in the setting of loan interest rates. More precisely, we assume that the macroprudential authority controls the bargaining power of banks b_t in the Nash bargaining problem,

$$\max_{R_t^L} W_t^{1-b_t} J_t^{b_t}.$$

In reality, the macroprudential authority controls the degree of bank competition in the credit market via financial regulations.

[14] For example, Drehmann, Borio, and Tsatsaronis (2012) show that the credit–GDP ratio is a good predictive indicator of financial crisis and emphasize that policymakers should use this ratio as the criterion for implementing macroprudential policy.

[15] Our formulation of the latter case here resembles that of Benigno and Woodford (2003), who analyze optimal monetary and fiscal policy.

In this case, the efficient steady-state condition (20) is unchanged. On the other hand, the markup equation (26) becomes

$$
\begin{aligned}
Z\widehat{\mu}_t = & -\frac{\alpha}{1-\alpha}\frac{\kappa}{\chi}\bar{\theta}^{\alpha}(\widehat{\theta}_t - \beta\rho_u \mathrm{E}_t\widehat{\theta}_{t+1}) \\
& - \beta\delta_2\sigma\left(\mathrm{E}_t\widehat{c}_{t+1} - \widehat{c}_t\right) - \frac{\bar{b}}{(1-\bar{b})^2}\frac{\kappa}{\chi}\bar{\theta}^{\alpha}\left(\widehat{b}_t - \beta\rho_u \mathrm{E}_t\widehat{b}_{t+1}\right), \qquad (38)
\end{aligned}
$$

where $\bar{b} = \alpha$ due to the Hosios condition. The macroprudential policy controls the economy through the cost channel; it affects the retail price markup, and thus the standard Phillips curve given by equation (25). The form of the approximated welfare given by equation (21) is not changed in this case. Also, there are no changes to the rest of the model.

The macroprudential authority optimally chooses $\widehat{b}_t$, taking as given the monetary policy variable (i.e., the deposit interest rate). Then, under the commitment policy in the timeless perspective, the first-order condition for $\widehat{b}_t$ implies

$$\varphi_{1t} - \rho_u\varphi_{1t-1} = 0. \qquad (39)$$

First, let us consider the case in which only the macroprudential policy is at work and the deposit interest rate is fixed at the efficient steady-state level. In this case, the first-order conditions (32)–(35) and (39) yield

$$
\begin{aligned}
\frac{\lambda_\theta \bar{C}}{\bar{L}\rho\beta\delta_2(1-\alpha)}\mathrm{E}_t\left(1+\beta\rho_u F\right)\left(1-\rho_u L\right)\widehat{\theta}_t + \lambda_c \mathrm{E}_t\left(1-\rho_u L\right)(F-1)\widehat{c}_t \\
+\sigma\lambda_\pi \mathrm{E}_t\left(1-\rho_u L\right)\left(1-\beta F\right)\left(1-F\right)\pi_t = 0, \qquad (40)
\end{aligned}
$$

where F is a forward operator and L is a lag operator that yield, for example, $\widehat{c}_{t-1} = L\widehat{c}_t$ and $\widehat{c}_t = F\widehat{c}_{t-1}$. While the optimal targeting rule is not expressed in a simple form, we observe that it includes backward-looking and forward-looking terms. Together with equation (40), the Phillips curve equation (25), the IS equation (27), the credit market tightness equation (28), the consumption equation (29), and the new markup equation (38) give the paths of π_t, $\widehat{\theta}_t$, $\widehat{c}_t$, $\widehat{l}_t$, and $\widehat{b}_t$ under the optimal macroprudential policy.[16]

When we further assume that the economy is in the efficient steady-state equilibrium before a sudden cost-push shock arrives, equation (39) immediately yields

$$\varphi_{1t} = 0. \qquad (41)$$

[16] For reasonable parameters, this economy is well defined. This result holds for other optimal policies in the paper.

By using equation (41), it can be shown that the optimal macroprudential policy is given by

$$\widehat{b}_t = \frac{\chi Z(1-\bar{b})^2}{\bar{b}\bar{\theta}^\alpha \kappa\,(\bar{\varepsilon}-1)}\widehat{\varepsilon}_t + \beta\rho_u \mathrm{E}_t \widehat{b}_{t+1},$$

and that the inflation rate as well as the gaps in consumption $\widehat{c}_t$ and loan volume $\widehat{l}_t$ are perfectly stabilized.[17] This is because financial stability leads to price stability via the cost channel. Therefore, in our model, the macroprudential policy, which is normally assumed to be primarily linked with financial stability, holds a close relationship with price stability and ultimately with monetary policy.

Second, let us consider the case in which both the macroprudential and monetary policies are optimal. This time, the deposit interest rate $\widehat{r}_t^D$ is an endogenous variable, and thus the associated first-order condition (36) is added to optimality conditions. Accordingly, the first-order conditions (32)–(36) and (39), along with equations (25), (27)–(29), and (38), give the paths of π_t, $\widehat{\theta}_t$, $\widehat{c}_t$, $\widehat{l}_t$, $\widehat{b}_t$, and $\widehat{r}_t^D$. As shown in Appendix D, using equations (32)–(36), equation (39) is transformed as

$$\widehat{l}_t = \rho_u \widehat{l}_{t-1}. \tag{42}$$

In particular, if the economy is in the efficient steady-state equilibrium before the arrival of a shock, equation (42) implies

$$\widehat{l}_t = 0 \tag{43}$$

for all t. From structural equations (27)–(29), we reconfirm that the inflation rate as well as the gaps in other variables from their efficient steady-state values are fixed at zero under the combination of the optimal macroprudential policy and the optimal monetary policy.

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[17] We can easily confirm this statement by substituting equation (38) into the Phillips curve equation (25):

$$\begin{aligned}
&(\pi_t - \beta \mathrm{E}_t \pi_{t+1}) - \frac{\delta}{Z}\frac{\alpha}{1-\alpha}\frac{\kappa}{\chi}\bar{\theta}^\alpha\left(\widehat{\theta}_t - \beta\rho_u \mathrm{E}_t\widehat{\theta}_{t+1}\right) - \frac{\delta}{Z}\beta\delta_2\sigma\left(\mathrm{E}_t\widehat{c}_{t+1} - \widehat{c}_t\right) \\
&= -\frac{\delta}{\bar{\varepsilon}-1}\widehat{\varepsilon}_t + \frac{\delta}{Z}\frac{\bar{b}}{(1-\bar{b})^2}\frac{\kappa}{\chi}\bar{\theta}^\alpha\left(\widehat{b}_t - \beta\rho_u \mathrm{E}_t\widehat{b}_{t+1}\right).
\end{aligned}$$

Since the cost-push shock $\widehat{\varepsilon}_t$ does not appear in any of the other structural equations (27), (28), or (29), the shock to the economy can be exactly cancelled by choosing the macroprudential tool $\widehat{b}_t$ such that the RHS of the above equation is zero.

6.2.2 Intervention in Credit Separation

Another possible intervention in the credit market is through control of the credit separation rate ρ. This macroprudential policy can be interpreted as a control of total credit. In this case, by introducing the macroprudential policy, the form of the approximated welfare of the representative household changes and is given by[18]

$$\begin{aligned}\sum_{i=0}^{\infty}\beta^i u(C_{t+i}) = &-\frac{1}{2}\sum_{i=0}^{\infty}\beta^i\left(\lambda_\pi \pi_{t+i}^2 + \lambda_\theta \widehat{\theta}_{t+i}^2 + \lambda_c \widehat{c}_{t+i}^2\right) \\ &-\sum_{i=0}^{\infty}\beta^i \frac{u_c \bar{L}\bar{\rho}\delta_2}{1-\bar{\rho}}\left[\frac{1}{2}\widehat{\rho}_{t+i-1}^2 + \widehat{\rho}_{t+i-1}\widehat{l}_{t+i-1} + \widehat{\rho}_{t+i-1}\right] + t.i.p..\end{aligned} \tag{44}$$

This example shows that the macroprudential policy itself changes the criteria for the optimal policy. Note that the new criteria for optimal policy includes both real economic variables (inflation and consumption terms) and financial variables (credit market tightness, credit separation rate, and loan volume terms).[19]

Now, when the change in ρ affects no other parameters, there is no benefit from match destruction, so welfare is clearly maximized by always setting $\rho = 0$. In the real economy, however, constraining natural separation of bank–firm credit relationships is likely to result in deterioration of productivity.[20] To capture such a feature in the simplest fashion, we introduce the following relationship between the credit separation rate ρ_t, controlled by the macroprudential authority, and the productivity of wholesale firms Z_{t+1} in the subsequent period:[21]

$$Z_{t+1} = f(\rho_t). \tag{45}$$

[18] The derivation of the approximated welfare in this case is similar to the derivation of equation (48), which is given in Appendix E. Specifically, by setting $\widehat{z}_t = 0$ in equation (75), equation (44) is immediately obtained.

[19] When we assume κ (the flow cost of searching for credit) and χ (the constant parameter of a Cobb-Douglas matching function) as macroprudential policy variables, the forms of the approximated welfare change, but still includes financial variables and real economic variables.

[20] In labor search and matching models with endogenous separations (e.g., Mortensen and Pissarides (1994)), such deterioration of productivity arises naturally when firing costs lower the reservation match productivity. Empirically, Caballero, Hoshi, and Kashyap (2008) report how “zombie lendings" by Japanese banks led to lower firm productivity.

[21] Introducing such a relationship is also necessary for a technical reason: The approximated welfare given by equation (44) contains a linear term ($\widehat{\rho}_t$), and thus does not fit into the linear-quadratic approximation framework.

Here, the function $f : (0,1) \to R_{++}$ is strictly increasing, strictly concave, and continuously differentiable, and satisfies $\lim_{\rho \to 0} f'(\rho) = \infty$ and $\lim_{\rho \to 1} f'(\rho) = 0$. By expanding this relation up to the second order, we have

$$\bar{Z}\left(\widehat{z}_{t+1} + \frac{1}{2}\widehat{z}_{t+1}^2\right) = \bar{\rho}\left[\bar{f}_1\widehat{\rho}_t + \frac{1}{2}\left(\bar{f}_1 + \bar{f}_2\bar{\rho}\right)\widehat{\rho}_t^2\right], \tag{46}$$

where $\bar{f}_1 \equiv f'(\bar{\rho}) > 0$, $\bar{f}_2 \equiv f''(\bar{\rho}) < 0$, and $\bar{\rho} \in (0,1)$.

In this scenario, the social-planner problem for obtaining the condition for efficient steady-state equilibrium treats ρ_t as a choice variable. As shown in Appendix E, we then obtain, in addition to equation (20), the following condition for the efficient steady-state equilibrium:

$$(1-\bar{\rho})\bar{f}_1 = \delta_2. \tag{47}$$

Furthermore, the welfare function becomes

$$\begin{aligned}\sum_{i=0}^{\infty}\beta^i u(C_{t+i}) = &-\frac{1}{2}\sum_{i=0}^{\infty}\beta^i\left(\lambda_\pi\pi_{t+i}^2 + \lambda_\theta\widehat{\theta}_{t+i}^2 + \lambda_c\widehat{c}_{t+i}^2 + \lambda_\rho\widehat{\rho}_{t+i-1}^2\right) \\ &+\sum_{i=0}^{\infty}\beta^i\psi_\rho\widehat{\rho}_{t+i-1}\left(\widehat{l}_{t+i} - \widehat{l}_{t+i-1}\right) + t.i.p.,\end{aligned} \tag{48}$$

where $\lambda_\rho \equiv u_c\bar{L}\bar{\rho}^2(-\bar{f}_2) > 0$ and $\psi_\rho \equiv u_c\bar{L}\bar{\rho}\bar{f}_1 > 0$.

Equation (48) has a clear implication. The fact that the last term on the RHS is linear in $\widehat{\rho}$ suggests that, when the volume of credit is expected to increase, society is better off by setting ρ above the efficient equilibrium value, and thus by reducing the volume of credit. The second-order term $\lambda_\rho\widehat{\rho}_{t+i-1}^2$ is the cost incurred by excessive control of $\widehat{\rho}$, which results from the concavity of f.

The time-variation in the credit separation rate modifies the markup equation (26) as

$$\begin{aligned}\bar{Z}\widehat{\mu}_t = &-\frac{\alpha}{1-\alpha}\frac{\kappa}{\chi}\bar{\theta}^\alpha\left(\widehat{\theta}_t - \beta\rho_u \mathrm{E}_t\widehat{\theta}_{t+1}\right) \\ &-\beta\delta_2\sigma\left(\mathrm{E}_t\widehat{c}_{t+1} - \widehat{c}_t\right) + \delta_2\frac{\bar{\rho}}{1-\bar{\rho}}\left(\widehat{\rho}_{t-1} - \beta\widehat{\rho}_t\right),\end{aligned} \tag{49}$$

and the relationship (28) between credit market tightness and the volume of credit as

$$\widehat{\theta}_t = \frac{1}{(1-\alpha)\bar{\rho}}\left[\widehat{l}_t - \rho_u\widehat{l}_{t-1} + \bar{\rho}\left(1-\chi\bar{\theta}^{1-\alpha}\right)\widehat{\rho}_{t-1}\right]. \tag{50}$$

On the other hand, the efficient steady-state condition (47) ensures that the relation (29) between consumption and credit is unchanged in the first order.

The optimal policy is obtained by maximizing the approximated welfare in (48) subject to the Phillips curve equation (25), the modified markup equation (49), the IS equation (27), the credit market tightness equation (50), and the consumption equation (29). Then, under the commitment policy in the timeless perspective, the first-order condition (35) is replaced by

$$-\psi_\rho \left(\widehat{\rho}_{t-1} - \beta \widehat{\rho}_t\right) - \frac{1}{(1-\alpha)\,\bar{\rho}} \varphi_{3t} + \frac{\beta \rho_u}{(1-\alpha)\,\bar{\rho}} \mathrm{E}_t \varphi_{3t+1} + \frac{\bar{L}\delta_2 \beta}{\bar{C}} \varphi_{4t} - \frac{\bar{L}\delta_2 \beta}{\bar{C}} \mathrm{E}_t \varphi_{4t+1} = 0, \quad (51)$$

whereas the first-order condition for $\widehat{\rho}_t$ is given by

$$\lambda_\rho \widehat{\rho}_t - \psi_\rho \left(\mathrm{E}_t \widehat{l}_{t+1} - \widehat{l}_t\right) + \frac{\delta}{\bar{Z}} \delta_2 \frac{\bar{\rho}}{1-\bar{\rho}} \left(\mathrm{E}_t \varphi_{1t+1} - \varphi_{1t}\right) - \frac{\rho_u}{(1-\alpha)\,(1-\bar{\rho})} \mathrm{E}_t \varphi_{3t+1} = 0. \quad (52)$$

When the deposit interest rate is fixed at the efficient steady-state level, the first-order conditions (32)–(34), (51), and (52), along with equations (25), (27), (29), (49), and (50), give the optimal paths of π_t, $\widehat{\theta}_t$, $\widehat{c}_t$, $\widehat{l}_t$, and $\widehat{\rho}_t$.

When both the macroprudential and monetary policies are optimal, in addition to these conditions, equation (36) defines the optimal paths of π_t, $\widehat{\theta}_t$, $\widehat{c}_t$, $\widehat{l}_t$, $\widehat{\rho}_t$, and $\widehat{r}_t^D$. In this case, by using equations (32) and (36), we can transform equation (52) as

$$\mathrm{E}_t \varphi_{3t+1} = \frac{(1-\alpha)\bar{\rho}}{\rho_u} u_c \bar{L} \left[\bar{\rho}\,(1-\bar{\rho})\,|\bar{f}_2| \widehat{\rho}_t - \delta_2 \mathrm{E}_t \left(\bar{\varepsilon}\pi_{t+1} + \widehat{l}_{t+1} - \widehat{l}_t\right)\right]. \quad (53)$$

By substituting equations (34) and (53) into equation (51) to eliminate φ_{3t} and φ_{4t}, and by using (29) to eliminate $\widehat{c}_t$, the condition characterizing the optimal macroprudential policy is obtained as

$$\widehat{\rho}_t = k_1 \mathrm{E}_t \widehat{\rho}_{t+1} + k_2 \mathrm{E}_t \left(\bar{\varepsilon}\pi_{t+1} + \widehat{l}_{t+1} - \widehat{l}_t\right) - k_3 \mathrm{E}_t \left(\bar{\varepsilon}\pi_{t+2} + \widehat{l}_{t+2} - \widehat{l}_{t+1}\right), \quad (54)$$

where

$$\begin{aligned}
k_1 &\equiv \frac{1}{k_4}\left[\frac{\overline{\rho}}{1-\overline{\rho}} + \frac{\overline{\rho}(1-\overline{\rho})|f_2|}{\delta_2}\right],\\
k_2 &\equiv \frac{1}{k_4}\left[\frac{\overline{L}\delta_2\beta\sigma}{\overline{C}} + \frac{1}{\rho_u}\right],\\
k_3 &\equiv \frac{1}{k_4}\left[\frac{\overline{L}\delta_2\beta^2\sigma}{\overline{C}} + \beta\right],\\
k_4 &\equiv \frac{\overline{\rho}\beta}{1-\overline{\rho}} + \frac{\overline{\rho}(1-\overline{\rho})\beta|f_2|}{\rho_u\delta_2}.
\end{aligned}$$

Equation (54) shows that the optimal macroprudential policy is given by a simple instrumental rule whereby the policy variable $\widehat{\rho}_t$ is explained by two endogenous variables, the inflation rate and the volume of credit. Note that the simple instrumental rule is forward-looking, even though the period policy objective function includes, as observed from equation (48), lags of the credit separation rate and volume of credit.

Unlike the case in the previous section 6.2.1, in response to a cost-push shock, the inflation rate as well as the gaps in other variables deviate from zeros under the optimal macroprudential and monetary policies.[22] In other words, perfect financial stability and price stability are not secured.

7 Concluding Remarks

We extend a standard New Keynesian model by introducing search and matching frictions into the credit market. In this model, the second-order approximation of social welfare includes terms related to credit, such as credit market tightness, volume of credit, and credit separation rate, in addition to inflation rate and consumption. This is a new finding in the field of optimal policy. Using the approximated welfare function, we reveal several important features for the optimal monetary and macroprudential policies.

For future research, the following points may be of interest. Establishing simple and optimal macroprudential and monetary policy rules that include credit terms is one extension of this paper. Also, in the present model, the monetary and macroprudential authorities coordinate their policies to maximize a single welfare function. An alternative assumption, however, would be that two policymakers have distinct welfare criteria and set their policies in a non-cooperative way. Moreover, we can imagine a situation where the macroprudential policy can stabilize economic disturbances, even though the monetary policy cannot; such disturbances could arise from credit spreads on the policy interest rate. Finally, we need to estimate the model to reach a quantitatively robust assessment of the optimal and simple macroprudential policy.

[22]The different outcomes between the two cases ($\widehat{b}_t$ and $\widehat{\rho}_t$) can be understood by examining the effect of the introduction of each policy variable on the structural equations. As equations (38) and (49) suggest, both macroprudential policy variables enter the markup equation (26) in a similar way. However, while the introduction of $\widehat{b}_t$ alters no other structural equation, introduction of $\widehat{\rho}_t$ modifies the relation between the credit market tightness and loan volume, as shown in equations (28) and (50).

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Appendix

A Detailed Derivation of the Welfare Function

The second-order expansion of the household's utility function around the efficient steady state yields

$$u(C_t) \simeq u(\bar{C}) + u_c\bar{C}\left(\widehat{c}_t + \frac{1}{2}\widehat{c}_t^2\right) - \frac{1}{2}\sigma u_c\bar{C}\widehat{c}_t^2. \tag{55}$$

The goal of calculation in this section is to rewrite the RHS of (55) using $\widehat{l}_t$, $\widehat{l}_{t-1}$, and the inflation rate $\pi_t \equiv \widehat{p}_t - \widehat{p}_{t-1}$. By using the market clearing condition (17), we obtain

$$\widehat{c}_t + \frac{1}{2}\widehat{c}_t^2 = -\frac{Z\bar{L}}{\bar{C}}\widehat{q}_t + \frac{Z\bar{L}}{\bar{C}}\left(\widehat{l}_t + \frac{1}{2}\widehat{l}_t^2\right) - \frac{\kappa\bar{u}}{\bar{C}}\left(\widehat{u}_t + \frac{1}{2}\widehat{u}_t^2\right). \tag{56}$$

Note that the efficient steady-state value of the price dispersion term Q_t is $\bar{Q} = 1$, and the log-deviation of this term $\widehat{q}_t$ is already in the second order, as shown below.

Expansion of equation (8) yields

$$\widehat{v}_t + \frac{1}{2}\widehat{v}_t^2 = -\eta\left(\widehat{l}_{t-1} + \frac{1}{2}\widehat{l}_{t-1}^2\right),$$

where $\eta \equiv (1-\rho)\bar{L}/\bar{v}$, while that of equation (13) yields

$$\begin{aligned}
&\frac{1}{\rho}\left(\widehat{l}_t + \frac{1}{2}\widehat{l}_t^2\right) - \frac{1-\rho}{\rho}\left(\widehat{l}_{t-1} + \frac{1}{2}\widehat{l}_{t-1}^2\right) \\
&\qquad = (1-\alpha)\left(\widehat{\theta}_t + \frac{1-\alpha}{2}\widehat{\theta}_t^2\right) + \left(\widehat{v}_t + \frac{1}{2}\widehat{v}_t^2\right) + (1-\alpha)\widehat{\theta}_t\widehat{v}_t.
\end{aligned}$$

By using these equations and equation (10), we obtain up to the second order

$$\begin{aligned}
\widehat{u}_t + \frac{1}{2}\widehat{u}_t^2 = &\frac{1}{\rho(1-\alpha)}\left(\widehat{l}_t + \frac{1}{2}\widehat{l}_t^2\right) - \frac{\bar{L}}{\kappa\bar{u}}\delta_2\left(\widehat{l}_{t-1} + \frac{1}{2}\widehat{l}_{t-1}^2\right) \\
&+ \frac{1}{2}\frac{\alpha}{\rho^2(1-\alpha)^2}\left(\widehat{l}_t - \rho_u\widehat{l}_{t-1}\right)^2.
\end{aligned} \tag{57}$$

By substituting equations (56) and (57) into equation (55), we obtain

$$\begin{aligned}
u(C_t) = &\, u(\bar{C}) - u_cZ\bar{L}\widehat{q}_t \\
&+ u_c\bar{L}\left[\left(\delta_1\widehat{l}_t + \delta_2\widehat{l}_{t-1}\right) + \frac{1}{2}\left(\delta_1\widehat{l}_t^2 + \delta_2\widehat{l}_{t-1}^2\right)\right] \\
&- \frac{u_c\kappa\bar{u}}{2}\frac{\alpha}{(1-\alpha)^2\rho^2}\left(\widehat{l}_t - \rho_u\widehat{l}_{t-1}\right)^2 - \frac{1}{2}\sigma u_c\bar{C}\left(\frac{\bar{L}}{\bar{C}}\right)^2\left(\delta_1\widehat{l}_t + \delta_2\widehat{l}_{t-1}\right)^2,
\end{aligned}$$

where

$$\rho_u \equiv 1 - \rho - \rho\eta.$$

Thus, the household's lifetime utility in equation (1) is rewritten as

$$\begin{aligned}\sum_{i=0}^{\infty}\beta^i u(C_{t+i}) = &\frac{u(\bar{C})}{1-\beta} - u_c Z\bar{L}\sum_{i=0}^{\infty}\beta^i \widehat{q}_{t+i} \\ &+ u_c\bar{L}\sum_{i=0}^{\infty}\beta^i\left(\delta_1\widehat{l}_{t+i}+\delta_2\widehat{l}_{t+i-1}\right) \\ &+\frac{1}{2}u_c\bar{L}\sum_{i=0}^{\infty}\beta^i\left(\delta_1\widehat{l}^2_{t+i}+\delta_2\widehat{l}^2_{t+i-1}\right) \\ &-u_c\kappa\bar{u}\frac{\alpha}{2(1-\alpha)^2\rho^2}\sum_{i=0}^{\infty}\beta^i\left(\widehat{l}_{t+i}-\rho_u\widehat{l}_{t+i-1}\right)^2 \\ &-\frac{1}{2}\sigma u_c\bar{C}\left(\frac{\bar{L}}{\bar{C}}\right)^2\sum_{i=0}^{\infty}\beta^i\left(\delta_1\widehat{l}_{t+i}+\delta_2\widehat{l}_{t+i-1}\right)^2.\end{aligned}$$

By using the efficient steady-state condition (20), we can show that the third and fourth terms on the RHS of this equation depend only on $\widehat{l}_{t-1}$. Therefore, we can write

$$\begin{aligned}\sum_{i=0}^{\infty}\beta^i u(C_{t+i}) = &-u_c Z\bar{L}\sum_{i=0}^{\infty}\beta^i \widehat{q}_{t+i} \\ &-u_c\kappa\bar{u}\frac{\alpha}{2(1-\alpha)^2\rho^2}\sum_{i=0}^{\infty}\beta^i\left(\widehat{l}_{t+i}-\rho_u\widehat{l}_{t+i-1}\right)^2 \\ &-\frac{1}{2}\sigma u_c\bar{C}\left(\frac{\bar{L}}{\bar{C}}\delta_2\right)^2\sum_{i=0}^{\infty}\beta^i\left(-\beta\widehat{l}_{t+i}+\widehat{l}_{t+i-1}\right)^2 \\ &+t.i.p.\end{aligned} \tag{58}$$

Next, we consider the sum over the price dispersion terms $\widehat{q}_{t+i}$. From equation (18),

$$\begin{aligned}\widehat{q}_t &= \int_0^1 dj \exp\left[-\varepsilon_t\left(\widehat{p}_t(j)-\widehat{p}_t\right)\right]-1 \\ &\simeq -\bar{\varepsilon}(\Delta_t^E-\widehat{p}_t)(1+\widehat{\varepsilon}_t)+\frac{1}{2}\bar{\varepsilon}^2\left[\Delta_t^V+\left(\Delta_t^E-\widehat{p}_t\right)^2\right],\end{aligned}$$

where $\Delta_t^E \equiv E_j\widehat{p}_t(j) = \int_0^1 \widehat{p}_t(j)dj$ and $\Delta_t^V \equiv Var_j\widehat{p}_t(j) = E_j\widehat{p}_t(j)^2-(E_j\widehat{p}_t(j))^2$. The definition of the aggregate price P_t given by equation (4) can be used to show that

$$\Delta_t^E-\widehat{p}_t \simeq -\frac{1}{2}(1-\bar{\varepsilon})\Delta_t^V.$$

Thus, up to the second order in $\widehat{p}_t$, we can rewrite $\widehat{q}_t$ as

$$\widehat{q}_t \simeq \frac{1}{2}\bar{\varepsilon}\Delta_t^V,$$

leading to

$$\sum_{i=0}^{\infty}\beta^i\widehat{q}_{t+i} = \frac{1}{2}\bar{\varepsilon}\sum_{i=0}^{\infty}\beta^i\Delta_{t+i}^V. \tag{59}$$

On the other hand, the equation to obtain Δ_t^V is written as

$$\Delta_t^V = E_j(\widehat{p}_t(j) - \Delta_{t-1}^E)^2 - (\Delta_t^E - \Delta_{t-1}^E)^2. \tag{60}$$

Here, we remember that only the fraction $1-\omega$ of all firms adjust their prices to P_t^*, while other firms do not change their prices $p_{t-1}(j)$. This condition leads to the following relation:

$$P_t^{1-\varepsilon_t} = (1-\omega)(P_t^*)^{1-\varepsilon_t} + \omega P_{t-1}^{1-\varepsilon_t}. \tag{61}$$

We also observe that, by using the same condition, equation (60) can be transformed into

$$\Delta_t^V = \omega E_j(\widehat{p}_{t-1}(j) - \Delta_{t-1}^E)^2 + (1-\omega)(\widehat{p}_t^* - \Delta_{t-1}^E)^2 - (\Delta_t^E - \Delta_{t-1}^E)^2, \tag{62}$$

where $\widehat{p}_t^*$ is the log-deviation of P_t^*.

By taking the log-deviation of both sides of equation (61), $\widehat{p}_t^*$ can be expressed by $\widehat{p}_t$ and $\widehat{p}_{t-1}$. We can then substitute this equation into equation (62), yielding

$$\Delta_t^V \simeq \omega\Delta_{t-1}^V + (1-\omega)\left(\frac{1}{1-\omega}\widehat{p}_t - \frac{\omega}{1-\omega}\widehat{p}_{t-1} - \widehat{p}_{t-1}\right)^2 - \left(\widehat{p}_t - \widehat{p}_{t-1}\right)^2,$$

up to the second order in $\widehat{p}_t$. Using $\pi_t = \widehat{p}_t - \widehat{p}_{t-1}$, we thus have

$$\Delta_t^V \simeq \omega\Delta_{t-1}^V + \frac{\omega}{1-\omega}\pi_t^2.$$

The sum of Δ_t^V becomes

$$\sum_{i=0}^{\infty}\beta^i\Delta_{t+i}^V = \omega\Delta_{t-1}^V + \omega\beta\sum_{i=0}^{\infty}\beta^i\Delta_{t+i}^V + \frac{\omega}{1-\omega}\sum_{i=0}^{\infty}\beta^i\pi_{t+i}^2.$$

We therefore obtain

$$\sum_{i=0}^{\infty}\beta^i\Delta_{t+i}^V = \frac{\omega}{(1-\omega)(1-\omega\beta)}\sum_{i=0}^{\infty}\beta^i\pi_{t+i}^2 + t.i.p..$$

Combining this equation with equations (58) and (59),

$$\begin{aligned}
\sum_{i=0}^{\infty}\beta^i u(C_{t+i}) = &-\frac{1}{2}u_c Z\bar{L}\frac{\bar{\varepsilon}}{\delta}\sum_{i=0}^{\infty}\beta^i\pi_{t+i}^2 \\
&- u_c\kappa\bar{u}\frac{\alpha}{2(1-\alpha)^2\rho^2}\sum_{i=0}^{\infty}\beta^i\left(\widehat{l}_{t+i} - \rho_u\widehat{l}_{t+i-1}\right)^2 \\
&- \frac{1}{2}\sigma u_c\bar{C}\left(\frac{\bar{L}}{\bar{C}}\delta_2\right)^2\sum_{i=0}^{\infty}\beta^i\left(-\beta\widehat{l}_{t+i} + \widehat{l}_{t+i-1}\right)^2 \\
&+ t.i.p.,
\end{aligned}$$

where $\delta = \frac{(1-\omega)(1-\omega\beta)}{\omega}$.

We also use approximations up to the second order

$$\widehat{\theta}^2_{t+i} = \frac{1}{(1-\alpha)^2\rho^2}\left(\widehat{l}_{t+i} - \rho_u \widehat{l}_{t+i-1}\right)^2 = \frac{1}{(1-\alpha)^2(\rho\eta)^2}\left(\widehat{v}_{t+i+1} - \rho_u \widehat{v}_{t+i}\right)^2,$$

and

$$\widehat{c}^2_{t+i} = \left(\frac{\bar{L}}{\bar{C}}\delta_2\right)^2 \left(-\beta\widehat{l}_{t+i} + \widehat{l}_{t+i-1}\right)^2,$$

and finally, we have the second-order expansion of the welfare function, (21).

B Policy Objective Function with Productivity Shock

The models in the main text do not consider the effect of the productivity shock Z_t. This is because, even if Z_t is taken into account as a shock, the mathematical forms of the model would remain the same by taking the difference from an efficient stochastic state equilibrium. In this appendix, we show that the productivity shock alters neither the policy objective function nor any of the linearized structural equations.

When we have a stochastic exogenous productivity Z_t, the second-order expansion of the household's utility function around the efficient steady-state equilibrium becomes

$$\begin{aligned}\sum_{i=0}^{\infty}\beta^i u(C_{t+i}) = &-\frac{1}{2}\sum_{i=0}^{\infty}\beta^i\left(\lambda_\pi \pi^2_{t+i} + \lambda_\theta \widehat{\theta}^2_{t+i} + \lambda_c \widehat{c}^2_{t+i}\right)\\ &+u_c\bar{Z}\bar{L}\sum_{i=0}^{\infty}\beta^i\left(\widehat{z}_{t+i} + \frac{1}{2}\widehat{z}^2_{t+i}\right) + u_c\bar{Z}\bar{L}\sum_{i=0}^{\infty}\beta^i\widehat{z}_{t+i}\widehat{l}_{t+i} + t.i.p..\end{aligned}$$

Here, although the term

$$u_c\bar{Z}\bar{L}\sum_{i=0}^{\infty}\beta^i\left(\widehat{z}_{t+i} + \frac{1}{2}\widehat{z}^2_{t+i}\right)$$

is clearly *t.i.p.*, the cross term between $\widehat{z}_{t+i}$ and $\widehat{l}_{t+i}$ seems relevant. To eliminate this term, we consider the log-linearized deviation from the dynamics of an efficient stochastic state. This efficient stochastic state is obtained by imposing the Hosios condition for credit market $b = \alpha$ and no price markup $\bar{\mu} = 1$, but allowing the productivity shock Z_t to move. We write the log-linearized value of a variable X_t at the efficient stochastic state as x^e_t and the deviation from the efficient stochastic state as $\widetilde{x}_t \equiv \widehat{x}_t - x^e_t$. At the efficient stochastic state, the consumption up to the first order is

$$c^e_t = \frac{\bar{L}}{\bar{C}}\left(\bar{Z}\widehat{z}_t - \beta\delta_2 l^e_t + \delta_2 l^e_{t-1}\right). \tag{63}$$

On the other hand, at the efficient stochastic equilibrium, the Euler equation (3) becomes

$$c_t^e = \mathrm{E}_t c_{t+1}^e - \frac{1}{\sigma} r_t^e,$$

where r_t^e is the real interest rate. Substituting equation (63) into this equation yields

$$\widehat{z}_t = \mathrm{E}_t \widehat{z}_{t+1} + \frac{\delta_2}{\bar{Z}}(-\beta \mathrm{E}_t l_{t+1}^e + l_t^e + \beta l_t^e - l_{t-1}^e) - \frac{\bar{C}}{\sigma \bar{Z} \bar{L}} r_t^e. \tag{64}$$

The policy objective function can be expressed as

$$\sum_{i=0}^{\infty} \beta^i u(C_{t+i}) = A_t + B_t + t.i.p., \tag{65}$$

where

$$A_t = u_c \bar{Z} \bar{L} \sum_{i=0}^{\infty} \beta^i \widehat{z}_{t+i} \widehat{l}_{t+i} - \sigma u_c \bar{Z} \bar{C} \delta_2 \left(\frac{\bar{L}}{\bar{C}} \right)^2 \sum_{i=0}^{\infty} \beta^i \left(\widehat{z}_{t+i} \widehat{l}_{t+i-1} - \beta \widehat{z}_{t+i} \widehat{l}_{t+i} \right)$$

is the collection of the terms that include $\widehat{z}_t$, and

$$B_t = -\frac{1}{2} \lambda_\pi \sum_{i=0}^{\infty} \beta^i \pi_{t+i}^2 - \frac{1}{2} u_c \kappa \bar{u} \alpha \sum_{i=0}^{\infty} \beta^i \widehat{\theta}_{t+i}^2 - \frac{1}{2} \sigma u_c \bar{C} \left(\frac{\bar{L}}{\bar{C}} \delta_2 \right)^2 \sum_{i=0}^{\infty} \beta^i \left(-\beta \widehat{l}_{t+i} + \widehat{l}_{t+i-1} \right)^2$$

represents the other terms. By using equation (64) only for the last term in A_t,

$$\begin{aligned}
A_t &= u_c \bar{Z} \bar{L} \sum_{i=0}^{\infty} \beta^i \widehat{z}_{t+i} \widehat{l}_{t+i} - \sigma u_c \bar{Z} \bar{C} \delta_2 \left(\frac{\bar{L}}{\bar{C}} \right)^2 \sum_{i=0}^{\infty} \beta^i \left(\widehat{z}_{t+i} \widehat{l}_{t+i-1} - \beta \widehat{z}_{t+i+1} \widehat{l}_{t+i} \right) \\
&\quad + \beta \sigma u_c \bar{Z} \bar{C} \delta_2 \left(\frac{\bar{L}}{\bar{C}} \right)^2 \sum_{i=0}^{\infty} \beta^i \left[\frac{\delta_2}{\bar{Z}} \left(-\beta l_{t+i+1}^e + l_{t+i}^e + \beta l_{t+i}^e - l_{t+i-1}^e \right) - \frac{\bar{C}}{\sigma \bar{Z} \bar{L}} r_{t+i}^e \right] \widehat{l}_{t+i} \\
&= u_c \bar{Z} \bar{L} \sum_{i=0}^{\infty} \beta^i \widehat{z}_{t+i} \widehat{l}_{t+i} + t.i.p. \\
&\quad + \beta \sigma u_c \bar{Z} \bar{C} \delta_2 \left(\frac{\bar{L}}{\bar{C}} \right)^2 \sum_{i=0}^{\infty} \beta^i \left[\frac{\delta_2}{\bar{Z}} \left(-\beta l_{t+i+1}^e + l_{t+i}^e + \beta l_{t+i}^e - l_{t+i-1}^e \right) - \frac{\bar{C}}{\sigma \bar{Z} \bar{L}} r_{t+i}^e \right] \widehat{l}_{t+i}.
\end{aligned}$$

On the other hand, equation (16) implies that, at the efficient stochastic equilibrium,

$$\bar{Z} \widehat{z}_t = \frac{\alpha}{1-\alpha} \frac{\kappa}{\chi} \bar{\theta}^\alpha \left(\theta_t^e - \beta \rho_u \mathrm{E}_t \theta_{t+1}^e \right) + \beta \delta_2 r_t^e.$$

By substituting this, A_t can be further simplified as

$$
\begin{aligned}
A_t &= u_c \bar{L} \frac{\alpha}{1-\alpha} \frac{\kappa}{\chi} \bar{\theta}^{\alpha} \sum_{i=0}^{\infty} \beta^i \left(\theta^e_{t+i} \widehat{l}_{t+i} - \beta \rho_u \theta^e_{t+i+1} \widehat{l}_{t+i} \right) \\
&\quad + \beta \sigma u_c \bar{C} \left(\delta_2 \frac{\bar{L}}{\bar{C}} \right)^2 \sum_{i=0}^{\infty} \beta^i \left(-\beta l^e_{t+i+1} + l^e_{t+i} + \beta l^e_{t+i} - l^e_{t+i-1} \right) \widehat{l}_{t+i} + t.i.p. \\
&= u_c \bar{L} \frac{\alpha}{1-\alpha} \frac{\kappa}{\chi} \bar{\theta}^{\alpha} \sum_{i=0}^{\infty} \beta^i \left(\rho_u \theta^e_{t+i} \widehat{l}_{t+i-1} - \beta \rho_u \theta^e_{t+i+1} \widehat{l}_{t+i} \right) \\
&\quad + u_c \alpha \kappa \bar{u} \sum_{i=0}^{\infty} \beta^i \theta^e_{t+i} \widehat{\theta}_{t+i} \\
&\quad + \beta \sigma u_c \bar{C} \left(\delta_2 \frac{\bar{L}}{\bar{C}} \right)^2 \sum_{i=0}^{\infty} \beta^i (-\beta l^e_{t+i+1} + l^e_{t+i} + \beta l^e_{t+i} - l^e_{t+i-1}) \widehat{l}_{t+i} + t.i.p.,
\end{aligned}
$$

where $\widehat{\theta}_t = \frac{1}{(1-\alpha)\rho}(\widehat{l}_t - \rho_u \widehat{l}_{t-1})$ is used. The two terms in the first summation on the RHS are again $t.i.p.$, and the last summation can be rewritten as

$$
\begin{aligned}
&\sum_{i=0}^{\infty} \beta^i (-\beta l^e_{t+i+1} + l^e_{t+i}) \widehat{l}_{t+i} + \sum_{i=0}^{\infty} \beta^i (\beta l^e_{t+i} - l^e_{t+i-1}) \widehat{l}_{t+i} \\
&= \beta^{-1} \sum_{i=0}^{\infty} \beta^i (-\beta l^e_{t+i} + l^e_{t+i-1}) \widehat{l}_{t+i-1} - \beta^{-1} (-\beta l^e_t + l^e_{t-1}) \widehat{l}_{t-1} + \sum_{i=0}^{\infty} \beta^i (\beta l^e_{t+i} - l^e_{t+i-1}) \widehat{l}_{t+i} \\
&= \beta^{-1} \sum_{i=0}^{\infty} \beta^i (-\beta l^e_{t+i} + l^e_{t+i-1}) \left(-\beta \widehat{l}_{t+i} + \widehat{l}_{t+i-1} \right) + t.i.p..
\end{aligned}
$$

We thus obtain

$$
\begin{aligned}
A_t &= \sigma u_c \bar{C} \left(\frac{\bar{L}}{\bar{C}} \delta_2 \right)^2 \sum_{i=0}^{\infty} \beta^i (-\beta l^e_{t+i} + l^e_{t+i-1}) \left(-\beta \widehat{l}_{t+i} + \widehat{l}_{t+i-1} \right) \\
&\quad + u_c \alpha \kappa \bar{u} \sum_{i=0}^{\infty} \beta^i \theta^e_{t+i} \widehat{\theta}_{t+i} + t.i.p..
\end{aligned}
$$

This expression for A_t is substituted into the policy objective function of equation (65), yielding

$$
\begin{aligned}
\sum_{i=0}^{\infty}\beta^i u(C_{t+i}) &= \sigma u_c\bar{C}\left(\frac{\bar{L}}{\bar{C}}\delta_2\right)^2\sum_{i=0}^{\infty}\beta^i(-\beta l^e_{t+i}+l^e_{t+i-1})\left(-\beta\widehat{l}_{t+i}+\widehat{l}_{t+i-1}\right)\\
&+ u_c\alpha\kappa\bar{u}\sum_{i=0}^{\infty}\beta^i\theta^e_{t+i}\widehat{\theta}_{t+i}\\
&- \frac{1}{2}\lambda_\pi\sum_{i=0}^{\infty}\beta^i\pi^2_{t+i}-\frac{1}{2}u_c\alpha\kappa\bar{u}\sum_{i=0}^{\infty}\beta^i\widehat{\theta}^2_{t+i}\\
&- \frac{1}{2}\sigma u_c\bar{C}\left(\frac{\bar{L}}{\bar{C}}\delta_2\right)^2\sum_{i=0}^{\infty}\beta^i\left(-\beta\widehat{l}_{t+i}+\widehat{l}_{t+i-1}\right)^2+t.i.p.\\
&= -\frac{1}{2}\lambda_\pi\sum_{i=0}^{\infty}\beta^i\pi^2_{t+i}\\
&- \frac{1}{2}\sigma u_c\bar{C}\left(\frac{\bar{L}}{\bar{C}}\delta_2\right)^2\sum_{i=0}^{\infty}\beta^i\left[\left(-\beta\widehat{l}_{t+i}+\widehat{l}_{t+i-1}\right)-\left(-\beta l^e_{t+i}+l^e_{t+i-1}\right)\right]^2\\
&- \frac{1}{2}u_c\alpha\kappa\bar{u}\sum_{i=0}^{\infty}\beta^i\left(\widehat{\theta}_{t+i}-\theta^e_{t+i}\right)^2+t.i.p.\\
&= -\frac{1}{2}\lambda_\pi\sum_{i=0}^{\infty}\beta^i\pi^2_{t+i}-\frac{1}{2}\sigma u_c\bar{C}\left(\frac{\bar{L}}{\bar{C}}\delta_2\right)^2\sum_{i=0}^{\infty}\beta^i\left(-\beta\widetilde{l}_{t+i}+\widetilde{l}_{t+i-1}\right)^2\\
&- \frac{1}{2}u_c\alpha\kappa\bar{u}\sum_{i=0}^{\infty}\beta^i\widetilde{\theta}^2_{t+i}+t.i.p.,
\end{aligned}
$$

which can be simplified as:

$$
\sum_{i=0}^{\infty}\beta^i u(C_{t+i}) = -\sum_{i=0}^{\infty}\beta^i\frac{1}{2}\left(\lambda_\pi\pi^2_{t+i}+\lambda_\theta\widetilde{\theta}^2_{t+i}+\lambda_c\widetilde{c}^2_{t+i}\right).
$$

We therefore confirm that the form of the utility-based policy objective function remains the same even when we introduce the productivity shock. In addition, we can easily see that all the relevant structural equations, (25)–(29), can be written identically if we replace $\widehat{x}$ by $\widetilde{x}$ for all the variables of X_t.

C Proof of Inequalities (23) and (24)

The first step for the proof is to express

$$
\frac{\lambda_\theta}{\lambda_c}=\frac{\alpha}{\sigma}\frac{\kappa\bar{u}}{\bar{C}} \tag{66}
$$

in terms of $\bar{\theta}$. By evaluating equations (8), (10), (13) and (17) at the efficient steady-state equilibrium, we obtain

$$\bar{u} = \frac{\rho\bar{\theta}}{\rho + (1-\rho)\chi\bar{\theta}^{1-\alpha}} L^* \tag{67}$$

and

$$\bar{C} = \frac{Z\chi\bar{\theta}^{1-\alpha} - \kappa\rho\bar{\theta}}{\rho + (1-\rho)\chi\bar{\theta}^{1-\alpha}} L^*. \tag{68}$$

From equations (66)–(68), we have

$$\frac{\lambda_\theta}{\lambda_c} = \frac{\alpha}{\sigma} \frac{\rho\frac{\kappa}{\chi}\bar{\theta}^\alpha}{Z - \rho\frac{\kappa}{\chi}\bar{\theta}^\alpha}.$$

By taking the partial derivative with respect to κ, we obtain

$$\begin{aligned}\frac{\partial}{\partial\kappa}\left(\frac{\lambda_\theta}{\lambda_c}\right) &= \frac{\alpha}{\sigma}\frac{Z}{(Z-\rho\frac{\kappa}{\chi}\bar{\theta}^\alpha)^2}\frac{\partial}{\partial\kappa}\left(\rho\frac{\kappa}{\chi}\bar{\theta}^\alpha\right) \\ &= \frac{\alpha}{\sigma}\frac{Z}{(Z-\rho\frac{\kappa}{\chi}\bar{\theta}^\alpha)^2}\frac{\rho}{\chi}\left(\kappa\alpha\bar{\theta}^{\alpha-1}\frac{\partial\bar{\theta}}{\partial\kappa} + \bar{\theta}^\alpha\right).\end{aligned} \tag{69}$$

On the other hand, taking the partial derivative of equation (19) with respect to κ yields the following expression for $\partial\bar{\theta}/\partial\kappa$:

$$\begin{aligned}&\left[\frac{\kappa}{\chi}\alpha\bar{\theta}^{\alpha-1}\left(1-\beta(1-\rho)\right) + \alpha\beta(1-\rho)\kappa\right]\frac{\partial\bar{\theta}}{\partial\kappa} \\ &\qquad + \frac{1}{\chi}\bar{\theta}^\alpha\left(1-\beta(1-\rho)\right) + \alpha\beta(1-\rho)\bar{\theta} = 0.\end{aligned} \tag{70}$$

Finally, by eliminating $\partial\bar{\theta}/\partial\kappa$ from equations (69) and (70), we prove the inequality (23). The other inequality (24) can be shown in a similar way.

D Derivation of Equations (37) and (42)

To derive (37), we first eliminate $\widehat{\theta}_t$ and $\widehat{c}_t$ from equations (33) and (34) by using equations (28), (29), and (36). This leads to

$$\varphi_{3t} = \frac{\delta}{Z}\frac{\alpha}{1-\alpha}\frac{\kappa}{\chi}\bar{\theta}^\alpha(h_t - \rho_u h_{t-1}) \tag{71}$$

and

$$\varphi_{4t} = \frac{\delta\delta_2\sigma}{Z}(-\beta h_t + h_{t-1}), \tag{72}$$

where

$$h_t \equiv \varphi_{1t} - \frac{\lambda_\theta Z\chi}{\delta\alpha\rho\kappa\bar{\theta}^\alpha}\widehat{l}_t. \tag{73}$$

Substituting equations (71) and (72) into equation (35), we obtain a second-order recurrence equation for h_t. By examining the eigenvalues of this equation, it is shown that $h_t = 0$ for all t, or by using equation (73),

$$\varphi_{1t} = \frac{\lambda_\theta Z \chi}{\delta \alpha \rho \kappa \bar{\theta}^\alpha} \widehat{l}_t. \tag{74}$$

Equation (37) is finally obtained by substituting equation (74) into (32).

To derive (42), note that in this environment, equation (74) again follows from the argument above. Equation (37) is thus obtained by substituting equation (74) into (39).

E Policy Objective Function when ρ is a Choice Variable

In this section, we show the derivation of equations (47) and (48).

The efficient steady-state condition is obtained from the social planner's following maximization problem:

$$\max_{C,L,v,\theta,\rho} \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \{ \frac{C_{t+i}^{1-\sigma}}{1-\sigma} + \phi_{t+i}[f(\rho_{t+i-1})L_{t+i} - \kappa\theta_{t+i}v_{t+i} - C_{t+i}]$$
$$+ \psi_{t+i}[(1-\rho_{t+i-1})L_{t+i-1} + \chi\theta_{t+i}^{1-\alpha}v_{t+i} - L_{t+i}] + s_{t+i}[v_{t+i} - L^* + (1-\rho_{t+i-1})L_{t+i-1}]\}.$$

Note that unlike in the problem in Section 3.1, ρ_t is a choice variable.

By taking the first-order conditions and rearranging the equations, we obtain

$$f(\rho_{t-1}) - \frac{1}{1-\alpha}\frac{\kappa}{\chi}\theta_t^\alpha = -\beta \mathrm{E}_t \frac{\lambda_{t+1}}{\lambda_t}(1-\rho_t)\frac{1}{1-\alpha}\left(\frac{\kappa}{\chi}\theta_{t+1}^\alpha - \alpha\kappa\theta_{t+1}\right)$$

and

$$f'(\rho_{t-1})\frac{L_t}{L_{t-1}} = \frac{1}{1-\alpha}\left(\frac{\kappa}{\chi}\theta_t^\alpha - \alpha\kappa\theta_t\right).$$

At the efficient steady-state equilibrium, the former condition becomes identical to equation (20), while the latter can be rearranged as condition (47).

Note that the latter equation, when linearized around the efficient steady-state equilibrium, is written as

$$\bar{f}_2\bar{\rho}\widehat{\rho}_{t-1} + \bar{f}_1\left(\widehat{l}_t - \widehat{l}_{t-1}\right) = \frac{\alpha}{1-\alpha}\frac{\kappa}{\chi}\bar{\theta}^\alpha\left(1-\chi\bar{\theta}^{1-\alpha}\right)\widehat{\theta}_t,$$

which could be obtained from the optimal policy in the main text, if there were no inflation. This point can be easily confirmed by substituting equation (33) into equation (52) to eliminate φ_{3t} and by remembering that φ_{1t} is zero without inflation from equation (32).

As for the second-order expansion of the household's utility function, both the time-dependence of the separation rate ρ_{t-1} and productivity Z_t make calculation slightly complicated, although the derivation is straightforward. For example, productivity is factored into the expansion of consumption, and the equation corresponding to equation (56) becomes:

$$\widehat{c}_t + \frac{1}{2}\widehat{c}_t^2 = \bar{Z}\frac{\bar{L}}{\bar{C}}(\widehat{z}_t + \frac{1}{2}\widehat{z}_t^2 + \widehat{z}_t\widehat{l}_t - \widehat{q}_t) + \bar{Z}\frac{\bar{L}}{\bar{C}}\left(\widehat{l}_t + \frac{1}{2}\widehat{l}_t^2\right) - \frac{\kappa\bar{u}}{\bar{C}}\left(\widehat{u}_t + \frac{1}{2}\widehat{u}_t^2\right).$$

On the other hand, the expansion of the number of credit seeker firms (57) is modified by the time-dependence of the separation rate as:

$$\begin{aligned}\widehat{u}_t + \frac{1}{2}\widehat{u}_t^2 = & \frac{1}{\bar{\rho}(1-\alpha)}\left(\widehat{l}_t + \frac{1}{2}\widehat{l}_t^2\right) - \frac{\bar{L}}{\kappa\bar{u}}\delta_2\left(\widehat{l}_{t-1} + \frac{1}{2}\widehat{l}_{t-1}^2\right) \\ & + \frac{1}{2}\frac{\alpha}{\bar{\rho}^2(1-\alpha)^2}\left(\widehat{l}_t - \rho_u\widehat{l}_{t-1} + \bar{\rho}\left(1-\chi\bar{\theta}^{1-\alpha}\right)\widehat{\rho}_{t-1}\right)^2 \\ & + \frac{1-\alpha\chi\bar{\theta}^{1-\alpha}}{1-\alpha}\left(\widehat{\rho}_{t-1} + \frac{1}{2}\widehat{\rho}_{t-1}^2 + \widehat{\rho}_{t-1}\widehat{l}_{t-1}\right).\end{aligned}$$

By using these two equations, we obtain the following expansion of the utility function:

$$\begin{aligned}u(C_t) = & u(\bar{C}) + u_c\bar{Z}\bar{L}\left(\widehat{z}_t + \frac{1}{2}\widehat{z}_t^2 + \widehat{z}_t\widehat{l}_t - \widehat{q}_t\right) \\ & - u_c\frac{\kappa\bar{u}}{1-\alpha}\left(1-\alpha\chi\bar{\theta}^{1-\alpha}\right)\left(\widehat{\rho}_{t-1} + \frac{1}{2}\widehat{\rho}_{t-1}^2 + \widehat{\rho}_{t-1}\widehat{l}_{t-1}\right) \\ & + u_c\bar{C}\left[\frac{\bar{L}}{\bar{C}}\left(\delta_1\widehat{l}_t + \delta_2\widehat{l}_{t-1}\right) + \frac{\bar{L}}{2\bar{C}}\left(\delta_1\widehat{l}_t^2 + \delta_2\widehat{l}_{t-1}^2\right) - \frac{\kappa\bar{u}}{\bar{C}}\frac{\alpha}{2(1-\alpha)^2\rho^2}\left(\widehat{l}_t - \rho_u\widehat{l}_{t-1} + \bar{\rho}\left(1-\chi\bar{\theta}^{1-\alpha}\right)\widehat{\rho}_{t-1}\right)^2\right] \\ & - \frac{1}{2}\sigma u_c\bar{C}\left(\frac{\bar{L}}{\bar{C}}\right)^2\left[\bar{Z}\widehat{z}_t + \delta_1\widehat{l}_t + \delta_2\widehat{l}_{t-1} - \frac{\kappa\bar{u}}{(1-\alpha)\bar{L}}\left(1-\alpha\chi\bar{\theta}^{1-\alpha}\right)\widehat{\rho}_{t-1}\right]^2. \qquad (75)\end{aligned}$$

At this stage, by using equations (46) and (47), we observe that the first-order difference between $\widehat{z}_t$ and $\widehat{\rho}_t$ is cancelled, and we finally obtain equation (48).