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Yasuo Hirose

UTokyo Price Project
702 Faculty of Economics, The University of Tokyo,
7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan
Tel: +81-3-5841-5595
E-mail: watlab@e.u-tokyo.ac.jp
http://www.price.e.u-tokyo.ac.jp/english/

An Estimated DSGE Model with a Deflation Steady State*

Yasuo Hirose†

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Abstract

Benhabib, Schmitt-Grohé, and Uribe (2001) argue for the existence of a deflation steady state when the zero lower bound on the nominal interest rate is considered in a Taylor-type monetary policy rule. This paper estimates a medium-scale DSGE model with a deflation steady state for the Japanese economy during the period from 1999 to 2013, when the Bank of Japan conducted a zero interest rate policy and the inflation rate was almost always negative. Although the model exhibits equilibrium indeterminacy around the deflation steady state, a set of specific equilibria is selected by Bayesian methods. According to the estimated model, shocks to households' preferences, investment adjustment costs, and external demand do not necessarily have an inflationary effect, in contrast to a standard model with a targeted-inflation steady state. An economy in the deflation equilibrium could experience unexpected volatility because of sunspot fluctuations, but it turns out that the effect of sunspot shocks on Japan's business cycles is marginal and that macroeconomic stability during the period was a result of good luck.

Keywords: Deflation, Zero interest rate, Japanese economy, Indeterminacy, Bayesian estimation

JEL Classification: E31, E32, E52

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†Faculty of Economics, Keio University, 2-15-45 Mita, Minato-ku, Tokyo 108-8345, Japan. E-mail: yhirose@econ.keio.ac.jp

1 Introduction

Dynamic stochastic general equilibrium (DSGE) models have become a popular tool in macroeconomics. In particular, following the development of Bayesian estimation and evaluation techniques, an increased number of researchers have estimated DSGE models for empirical research as well as quantitative policy analysis. These models typically consist of optimizing behavior of households and firms, and a monetary policy rule, along the lines of King (2000) and Woodford (2003). In this class of models, a central bank follows an active monetary policy rule; that is, the nominal interest rate is adjusted more than one for one when inflation deviates from a given target, and the economy fluctuates around the steady state where actual inflation coincides with the targeted inflation. In addition to such a target-inflation steady state, Benhabib, Schmitt-Grohé, and Uribe (2001) argue that the combination of an active monetary policy rule and the zero lower bound on the nominal interest rate gives rise to another long-run equilibrium, called a deflation steady state, where the inflation rate is negative and the nominal interest rate is very close to zero.

The primary contribution of this paper is to build and estimate a DSGE model with a deflation steady state for the Japanese economy, whereas almost all the existing studies have estimated DSGE models with a targeted-inflation steady state.[1] The analysis in the present paper is motivated by Bullard (2010), who points out the possibility that the Japanese economy has been stuck in a deflation equilibrium. Figure 1 plots the nominal interest rate and inflation in Japan during the period from 1981Q1 to 2013Q1.[2] In the figure, following Bullard (2010), a nonlinear Taylor-type monetary policy rule (thick solid line) is fitted to the data. Moreover, two long-run Fisher relations are added: One (dotted line) is the Fisher equation where the real interest rate r is fixed at 3.30, which is the mean of the ex post real interest rate for the sample from 1981Q1 to 1998Q4. The other (dashed line) is the one where $r = 1.42$, which is the same mean for the sample from 1999Q1 to 2013Q1. During the latter sample period, the Bank of Japan conducted a virtually zero interest rate policy, with the exception of August 2000–March 2001 and July 2006–December 2008, and the inflation rate was almost always negative. As argued in Benhabib, Schmitt-Grohé, and Uribe (2001), two steady states emerge as the intersections of the nonlinear policy rule and the Fisher equations; that is, the steady states for the

[1] The only exception is Hirose (2007), who estimates a prototypical New Keynesian DSGE model for the Japanese economy around a deflation steady state with the nominal interest rate being exogenous.

[2] The overnight call rate, which is the monetary policy rate in Japan, is used for the nominal interest rate. Inflation is measured as the percentage change in the GDP deflator from one year earlier.

pre- and post-1999 period correspond to the targeted-inflation and deflation steady state, respectively. Therefore, Japan's macroeconomic fluctuations during the zero interest rate period are possibly well characterized as equilibrium dynamics near the deflation steady state.

Specifically, this paper estimates a medium-scale DSGE model, along the lines of Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2003, 2007), and Justiniano, Primiceri, and Tambalotti (2010), approximated around the deflation steady state, using data from 1999 to 2013 for Japan. The difficulty in estimating the model around the deflation steady state is that the equilibrium is indeterminate; i.e., there are an infinite number of equilibrium trajectories that converge to the deflation steady state, because of a passive monetary policy that is constrained by the zero lower bound on the nominal interest rate.[3] In this regard, following Lubik and Schorfheide (2004), a set of specific equilibrium paths is selected among an infinite number of equilibria using Bayesian methods.

Through the lens of the estimated model, the characteristics of the Japanese economy during the zero interest rate period are revealed. First, shocks to households' preferences, investment adjustment costs, and external demand do not necessarily have an inflationary effect, in contrast to a standard model with a targeted-inflation steady state. This finding about the inflation responses to these shocks provides a novel view about the flattening of the short-run Phillips curve in Japan, which has been examined in the literature by, for example, Nishizaki and Watanabe (2000) and De Veirman (2009). While the analyses in the previous studies are based on the estimation of reduced-form Phillips curves, our full-information-based estimation of the DSGE model offers a structural interpretation about their arguments. According to the estimated structural parameters, the slope of the Phillips curve itself does not become flat. Rather, the ambiguity of the inflation responses, as shown in the impulse response analysis, leads to a weak comovement between inflation and output. This weak comovement can be identified as a flattening of the Phillips curve in the estimation of reduced-form equations. Second, while an economy in the deflation equilibrium could be unexpectedly volatile because of sunspot shocks, which are nonfundamental disturbances, our estimation results show that the effect of sunspot shocks to Japan's business cycle fluctuations is quite marginal. On the contrary, the sunspot shocks contribute to stabilizing the economy over the business cycles. Therefore, macroeconomic stability during the zero interest period occurs as a result of good luck.

The most closely related paper is Aruoba, Cuba-Borda, and Schorfheide (2013). They consider Markov switching between the targeted-inflation and deflation steady state in a

[3] Cochrane (2013) emphasizes that a New Keynesian model in a liquidity trap exhibits completely different dynamics, depending on the choice of equilibrium.

small-scale New Keynesian DSGE model and estimate whether the US and Japan have been in either the targeted-inflation or deflation regime. Regarding the Japanese economy, they find that it shifted from a targeted-inflation regime into a deflation regime in 1999 and remained in the latter regime thereafter. Their finding validates our assumption that Japan has been stuck in a deflation equilibrium during our sample period; i.e., from 1999 to 2013. Their primary focus is on the estimation of the timing of the regime change, given the structural parameters pre-estimated around the targeted-inflation steady state for the sample from 1981 to 1994 in Japan. In contrast, the present paper estimates parameters in a richer DSGE model with a deflation steady state using data since 1999 and investigates the economic properties of this period.

The model estimated in this paper is the first benchmark model to investigate empirically a deflationary economy constrained by the zero lower bound on the nominal interest rate. In the literature, Sugo and Ueda (2007), Kaihatsu and Kurozumi (2010), Fueki, Fukunaga, Ichiue, and Shirota (2010), Hirakata, Sudo, and Ueda (2011), Iwata (2011), Hirose and Kurozumi (2012), and Ichiue, Kurozumi, and Sunakawa (2013) estimate medium-scale DSGE models using Japanese data. However, these authors either exclude the zero interest rate period from their samples or ignore the zero lower bound constraint in their estimation because of computational difficulties in the treatment of nonlinearities arising from the bound. Although the model in the present paper does not take account of the zero lower bound explicitly,[4] the effect of ignoring it is mitigated around the deflation steady state, where the slopes of the monetary policy rule with respect to inflation and output are very flat.

This paper also contributes to the literature on the estimation of DSGE models under equilibrium indeterminacy. With several exceptions such as Hirose (2007, 2008, 2013), Belaygorod and Dueker (2009), Bhattarai, Lee, and Park (2012a, 2012b), and Zheng and Guo (2013), there have been few papers that estimate indeterminate models using the methods developed by Lubik and Schorfheide (2004). This paper is the first empirical work that applies Lubik and Schorfheide's approach to the estimation of a medium-scale DSGE model under indeterminacy, whereas the previous studies estimate relatively small models. A technical contribution here is that our estimation procedure numerically computes a particular solution for the medium-scale DSGE model such that the impulse responses of the endogenous variables to fundamental shocks are continuous at the boundary between the determinacy and indeterminacy regions, while Lubik and Schorfheide (2004) analyt-

[4]Empirical studies that estimate DSGE models with the zero lower bound are still scarce. A remarkable exception is Gust, López-Salido, and Smith (2012), who estimate a nonlinear DSGE model in which the interest-rate lower bound is occasionally binding for the US economy.

ically characterize such a solution for a prototypical New Keynesian model. Moreover, our model is estimated based on another particular solution where the contributions of fundamental and sunspot shocks to endogenous forecast errors are orthogonal,[5] and we find from the Bayesian model comparison that the former solution is well fitted to the data.

The remainder of this paper proceeds as follows. Section 2 presents a DSGE model with a deflation steady state. Section 3 describes the solution and econometric strategy for estimating the model. Section 4 reports the estimation results. Section 5 concludes.

2 The Model

The model is a medium-scale DSGE model along the lines of Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2003, 2007), and Justiniano, Primiceri, and Tambalotti (2010) but differs from these models in the following respects. First, households' preferences are specified as in Erceg Guerrieri, and Gust (2006), which ensures the existence of the balanced growth path under the constant relative risk aversion (CRRA) utility function. Second, following Greenwood, Hercowitz, and Huffman (1988), the model assumes that a higher utilization rate of capital leads to a higher depreciation rate of capital. This assumption is supported by Sugo and Ueda (2007) who estimate a Christiano, Eichenbaum, and Evans type model under the same assumption for the Japanese economy and successfully replicate a negative correlation between capital utilization and rental cost observed in the data. Finally, the equilibrium conditions are approximated around the deflation steady state, which is the main difference between our model and those in the existing studies.

In the model economy, there is a continuum of households, a representative final-good firm, a continuum of intermediate-good firms, and a central bank. Their optimization problems and equilibrium conditions are presented below.

2.1 Households

There is a continuum of households $h \in [0, 1]$, each of which purchases consumption goods $C_t(h)$ and one-period riskless bonds $B_t(h)$, and supplies one kind of differentiated labor service $l_t(h)$ to intermediate-good firms. Each household's preferences are represented by

[5] This particular solution is directly obtained with the algorithm described in Sims (2002).

the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t e^{z_t^b} \left\{ \frac{(C_t(h) - \gamma C_{t-1}(h))^{1-\sigma}}{1-\sigma} - \frac{Z_t^{1-\sigma} e^{z_t^l} l_t(h)^{1+\chi}}{1+\chi} \right\},$$

where $\beta \in (0,1)$ is the subjective discount factor, $\sigma > 0$ measures the degree of risk aversion, $\gamma \in (0,1)$ represents the degree of habit persistence in consumption preferences, $\chi > 0$ is the inverse of the labor supply elasticity, Z_t represents the level of neutral technology, and z_t^b and z_t^l denote shocks relevant to the subjective discount factor and to labor supply, respectively. As in Erceg, Guerrieri, and Gust (2006), we assume the presence of $Z_t^{1-\sigma}$ in the labor disutility, which ensures the existence of the balanced growth path in the model economy.

At the beginning of each period, each household owns capital stock $K_{t-1}(h)$ and rents utilization-adjusted capital $u_t(h)K_{t-1}(h)$ to intermediate-good firms at the real price $R_t^k(h)$. Then, the capital utilization rate $u_t(h)$ and investment spending $I_t(h)$ are determined subject to the capital accumulation equation

$$K_t(h) = \{1 - \delta(u_t(h))\} K_{t-1}(h) + \left\{ 1 - S\left(\frac{I_t(h)}{I_{t-1}(h)} \frac{e^{z_t^i}}{z} \right) \right\} I_t(h). \tag{1}$$

Here, following Greenwood, Hercowitz, and Huffman (1988), the model assumes that a higher utilization rate of capital leads to a higher depreciation rate of capital. Hence, the depreciation rate function $\delta(\cdot)$ has the properties $\delta' > 0$, $\delta'' > 0$, $\delta(u) = \delta \in (0,1)$, and $\mu = \delta'(u)/\delta''(u) > 0$, where $u = 1$ is the steady-state capital utilization rate. The function $S(\cdot)$ represents the costs involved in changing investment spending, such as financial intermediation costs as analyzed by Carlstrom and Fuerst (1997), and takes the quadratic form of $S(x) = (x-1)^2/(2\zeta)$, where ζ is a positive constant. The variable z_t^i is a shock to the investment adjustment costs. The parameter $z > 1$ represents the gross balanced growth rate.

Each household's budget constraint is given by

$$C_t(h) + I_t(h) + \frac{B_t(h)}{P_t} = W_t(h)l_t(h) + R_t^k(h)u_t(h)K_{t-1}(h) + R_{t-1}^n \frac{B_{t-1}(h)}{P_t} + T_t(h),$$

where P_t is the price of final goods, $W_t(h)$ is the real wage, R_t^n is the gross nominal interest rate and $T_t(h)$ consists of a lump-sum public transfer and profits received from firms.

In the presence of complete insurance markets, the decisions are the same for all households, and hence the first-order conditions with respect to consumption, bond-holdings, investment, capital utilization, and capital stock are given by

$$\Lambda_t = e^{z_t^b} \left(C_t - \gamma C_{t-1} \right)^{-\sigma} - \beta\gamma E_t e^{z_{t+1}^b} \left(C_{t+1} - \gamma C_t \right)^{-\sigma}, \tag{2}$$

$$\Lambda_t = \beta E_t \Lambda_{t+1} \frac{R_t^n}{\pi_{t+1}}, \tag{3}$$

$$R_t^k = Q_t \delta'(u_t), \tag{4}$$

$$\begin{aligned} 1 = {} & Q_t \left\{ 1 - S\left(\frac{I_t}{I_{t-1}} \frac{e^{z_t^i}}{z} \right) - S'\left(\frac{I_t}{I_{t-1}} \frac{e^{z_t^i}}{z} \right) \frac{I_t}{I_{t-1}} \frac{e^{z_t^i}}{z} \right\} \\ & + \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} Q_{t+1} S'\left(\frac{I_{t+1}}{I_t} \frac{e^{z_{t+1}^i}}{z} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \frac{e^{z_{t+1}^i}}{z}, \end{aligned} \tag{5}$$

$$Q_t = E_t \, \beta \frac{\Lambda_{t+1}}{\Lambda_t} \left\{ R_{t+1}^k u_{t+1} + Q_{t+1} \left(1 - \delta(u_{t+1}) \right) \right\}, \tag{6}$$

where Λ_t is the Lagrange multiplier interpreted as the marginal utility of income, and $\pi_t = P_t/P_{t-1}$, Q_t is the real price of capital. In monopolistically competitive labor markets, nominal wages are set on a staggered basis à la Calvo (1983) in the face of the labor demand given by $l_t(h) = l_t(W_t(h)/W_t)^{-(1+\lambda_t^w)/\lambda_t^w}$, where $l_t = \left\{ \int_0^1 l_t(h)^{1/(1+\lambda_t^w)} dh \right\}^{1+\lambda_t^w}$, and $\lambda_t^w > 0$ is related to the substitution elasticity between differentiated labor services and represents the exogenous time-varying wage markup. In each period, a fraction $1 - \xi_w \in (0, 1)$ of wages is reoptimized, while the remaining fraction ξ_w is set by indexation to the balanced growth rate z as well as a weighted average of past inflation π_{t-1} and steady-state inflation π. Then, the reoptimized wages solve the following problem

$$\max_{W_t(h)} E_t \sum_{j=0}^{\infty} (\beta \xi_w)^j \left\{ \Lambda_{t+j} l_{t+j|t}(h) \frac{P_t W_t(h)}{P_{t+j}} \prod_{k=1}^{j} \left(z \pi_{t+k-1}^{\gamma_w} \pi^{1-\gamma_w} \right) - \frac{e^{z_{t+j}^b} Z_{t+j}^{1-\sigma} e^{z_{t+j}^l} l_{t+j|t}(h)^{1+\chi}}{1+\chi} \right\}$$

subject to

$$l_{t+j|t}(h) = l_{t+j} \left\{ \frac{P_t W_t(h)}{P_{t+j} W_{t+j}} \prod_{k=1}^{j} \left(z \pi_{t+k-1}^{\gamma_w} \pi^{1-\gamma_w} \right) \right\}^{-\frac{1+\lambda_{t+j}^w}{\lambda_{t+j}^w}},$$

where $\gamma_w \in [0, 1]$ is the weight of wage indexation to past inflation relative to steady-state inflation. The first-order condition for the reoptimized wage W_t^o is given by

$$E_t \sum_{j=0}^{\infty} \left[\begin{array}{l} (\beta\xi_w)^j \frac{\Lambda_{t+j}}{\lambda_{t+j}^w} l_{t+j} \left[\frac{z^j W_t^o}{W_{t+j}} \prod_{k=1}^j \left\{ \left(\frac{\pi_{t+k-1}}{\pi} \right)^{\gamma_w} \frac{\pi}{\pi_{t+k}} \right\} \right]^{-\frac{1+\lambda_{t+j}^w}{\lambda_{t+j}^w}} \\ \times \left\{ \begin{array}{l} z^j W_t^o \prod_{k=1}^j \left\{ \left(\frac{\pi_{t+k-1}}{\pi} \right)^{\gamma_w} \frac{\pi}{\pi_{t+k}} \right\} - (1+\lambda_{t+j}^w) \frac{e^{z_{t+j}^b} e^{z_{t+j}^l} Z_{t+j}^{1-\sigma}}{\Lambda_{t+j}} \\ \times \left(l_{t+j} \left[\frac{z^j W_t^o}{W_{t+j}} \prod_{k=1}^j \left\{ \left(\frac{\pi_{t+k-1}}{\pi} \right)^{\gamma_w} \frac{\pi}{\pi_{t+k}} \right\} \right]^{-\frac{1+\lambda_{t+j}^w}{\lambda_{t+j}^w}} \right)^{\chi} \end{array} \right\} \end{array} \right] = 0. \tag{7}$$

The aggregate wage equation $W_t = \left(\int_0^1 W_t(h)^{-1/\lambda_t^w} dh \right)^{-\lambda_t^w}$ can be expressed as

$$1 = (1 - \xi_w) \left(\left(\frac{W_t^o}{W_t} \right)^{-\frac{1}{\lambda_t^w}} + \sum_{j=1}^{\infty} \xi_w^j \left[\frac{z^j W_{t-j}^o}{W_t} \prod_{k=1}^{j} \left\{ \left(\frac{\pi_{t+k-1}}{\pi} \right)^{\gamma_w} \frac{\pi}{\pi_{t+k}} \right\} \right]^{-\frac{1}{\lambda_t^w}} \right). \tag{8}$$

2.2 Firms

2.2.1 Final-good firm

The final-good firm produces output Y_t by choosing a combination of intermediate inputs $\{Y_t(f)\}$, $f \in [0,1]$, so as to maximize its profit $P_tY_t - \int_0^1 P_t(f)Y_t(f)df$ subject to the production technology $Y_t = \left\{\int_0^1 Y_t(f)^{1/(1+\lambda_t^p)}df\right\}^{1+\lambda_t^p}$, where $P_t(f)$ is the price of intermediate good f, and $\lambda_t^p > 0$ is related to the substitution elasticity between differentiated goods and corresponds to the exogenous time-varying price markup.

The first-order condition for profit maximization yields the final-good firm's demand for each intermediate good given by $Y_t(f) = Y_t(P_t(f)/P_t)^{-(1+\lambda_t^p)/\lambda_t^p}$, while perfect competition in the final-good market leads to its price P_t, given by

$$P_t = \left(\int_0^1 P_t(f)^{-\frac{1}{\lambda_t^p}}df\right)^{-\lambda_t^p}. \tag{9}$$

The market clearing condition for the final good is

$$Y_t = C_t + I_t + gZ_te^{z_t^g}, \tag{10}$$

where the term $gZ_te^{z_t^g}$ represents an external demand component. z_t^g is an external demand shock, and g is a scale parameter.

2.2.2 Intermediate-good firms

Each intermediate-good firm f produces output $Y_t(f)$ by choosing a cost-minimizing pair of capital and labor services $\{u_tK_{t-1}(f), l_t(f)\}$, given their real rental prices (R_t^k, W_t) and the production function

$$Y_t(f) = (Z_tl_t(f))^{1-\alpha}(u_tK_{t-1}(f))^{\alpha} - \phi Z_t. \tag{11}$$

Here, Z_t represents the level of neutral technology and is assumed to follow the stochastic process

$$\log Z_t = \log z + \log Z_{t-1} + z_t^z, \tag{12}$$

where $z > 1$ is the steady-state gross rate of neutral technological changes, and z_t^z represents a shock to the rate of the changes. The parameter $\alpha \in (0,1)$ measures the capital elasticity of output. The last term in the production function (11), $-\phi Z_t$, is the fixed cost of producing intermediate goods, and ϕ is a positive constant.[6]

[6] The zero profit condition for intermediate-good firms at the steady state leads to $\phi = \lambda^p$, where λ^p is the steady-state price markup.

Combining cost-minimizing conditions with respect to capital and labor services shows that the real marginal cost is identical among intermediate-good firms and is given by

$$mc_t = \left(\frac{W_t}{(1-\alpha)Z_t}\right)^{1-\alpha}\left(\frac{R_t^k}{\alpha}\right)^{\alpha}. \tag{13}$$

Furthermore, combining the cost-minimizing conditions and aggregating the resulting equation over intermediate-good firms shows that the capital–labor ratio is identical among intermediate-good firms and is given by

$$\frac{u_t K_{t-1}}{l_t} = \frac{\alpha W_t}{(1-\alpha)R_t^k}, \tag{14}$$

where $K_t = \int_0^1 K_t(f)df$ and $l_t = \int_0^1 l_t(f)df$. Moreover, using this equation to aggregate the production function (11) over intermediate-good firms yields

$$Y_t d_t = (Z_t l_t)^{1-\alpha}(u_t K_{t-1})^{\alpha} - \phi Z_t, \tag{15}$$

where $d_t = \int_0^1 (P_t(f)/P_t)^{-(1+\lambda_t^p)/\lambda_t^p} df$ measures the intermediate-good price dispersion and is of second order under the staggered price setting presented below.

Facing the final-good firm's demand, each intermediate-good firm sets the price of its product on a staggered basis à la Calvo (1983). In each period, a fraction $1-\xi_p \in (0,1)$ of intermediate-good firms reoptimizes prices, while the remaining fraction ξ_p indexes prices to a weighted average of past inflation π_{t-1} and steady-state inflation π. Then, the firms that reoptimize prices in the current period solve the problem

$$\max_{P_t(f)} E_t \sum_{j=0}^{\infty} \xi_p^j \left(\beta^j \frac{\Lambda_{t+j}}{\Lambda_t}\right)\left\{\frac{P_t(f)}{P_{t+j}}\prod_{k=1}^{j}\left(\pi_{t+k-1}^{\gamma_p}\pi^{1-\gamma_p}\right) - mc_{t+j}\right\} Y_{t+j|t}(f)$$

subject to

$$Y_{t+j|t}(f) = Y_{t+j}\left\{\frac{P_t(f)}{P_{t+j}}\prod_{k=1}^{j}\left(\pi_{t+k-1}^{\gamma_p}\pi^{1-\gamma_p}\right)\right\}^{-\frac{1+\lambda_{t+j}^p}{\lambda_{t+j}^p}},$$

where $\gamma_p \in [0,1]$ is the weight of price indexation to past inflation relative to steady-state inflation. The first-order condition for the reoptimized price P_t^o is given by

$$E_t \sum_{j=0}^{\infty} \left(\begin{array}{c} (\beta\xi_p)^j \frac{\Lambda_{t+j}}{\Lambda_t \lambda_{t+j}^p} Y_{t+j} \left[\frac{P_t^o}{P_t}\prod_{k=1}^{j}\left\{\left(\frac{\pi_{t+k-1}}{\pi}\right)^{\gamma_p}\frac{\pi}{\pi_{t+k}}\right\}\right]^{-\frac{1+\lambda_{t+j}^p}{\lambda_{t+j}^p}} \\ \times\left[\frac{P_t^o}{P_t}\prod_{k=1}^{j}\left\{\left(\frac{\pi_{t+k-1}}{\pi}\right)^{\gamma_p}\frac{\pi}{\pi_{t+k}}\right\} - \left(1+\lambda_{t+j}^p\right) mc_{t+j}\right] \end{array} \right) = 0. \tag{16}$$

The final-good price equation (9) can be written as

$$1 = (1-\xi_p)\left(\left(\frac{P_t^o}{P_t}\right)^{-\frac{1}{\lambda_t^p}} + \sum_{j=1}^{\infty}(\xi_p)^j\left[\frac{P_{t-j}^o}{P_{t-j}}\prod_{k=1}^{j}\left\{\left(\frac{\pi_{t-k}}{\pi}\right)^{\gamma_p}\frac{\pi}{\pi_{t-k+1}}\right\}\right]^{-\frac{1}{\lambda_t^p}}\right). \tag{17}$$

2.3 Central bank

The central bank adjusts the nominal interest rate following a monetary policy rule of the form

$$R_t^n = R^n\left(\pi_t, \frac{Y_t}{Z_t}, R_{t-1}^n, z_t^r\right), \tag{18}$$

where z_t^r is a monetary policy shock that captures an unsystematic component of monetary policy.

Although the functional form of $R^n(\cdot)$ is not specified at this stage, three assumptions are made regarding this monetary policy rule, as in Benhabib, Schmitt-Grohé, and Uribe (2001). First, the nominal interest rate is nondecreasing in inflation, output, and the past interest rate; i.e., $\partial R^n/\partial \pi_t \geq 0$, $\partial R^n/\partial(Y_t/Z_t) \geq 0$, and $\partial R^n/\partial R_{t-1}^n \geq 0$. Second, there is the zero lower bound constraint on the nominal interest rate. Thus, $R^n(\cdot) > 1$ for all $\{\pi_t, Y_t/Z_t, R_{t-1}^n, z_t^r\}$. Third, around the inflation target, the monetary policy rule satisfies the so-called Taylor principle; that is, the nominal interest rate increases (decreases) more than one percent in response to a one-percent increase (decrease) in the inflation rate.

2.4 Steady states and approximated equilibrium conditions

The equations (1)$-$(8), (10), (12)$-$(18), are the equilibrium conditions for the model economy. In the model, the real variables are nonstationary because the level of neutral technology has unit roots with drift as shown in (12). Thus, we rewrite the equilibrium conditions in terms of stationary variables detrended by Z_t; i.e., $y_t = Y_t/Z_t$, $c_t = C_t/Z_t$, $w_t = W_t/Z_t$, $\lambda_t = \Lambda_t Z_t^\sigma$, $i_t = I_t/Z_t$, and $k_t = K_t/Z_t$, so that we can compute the steady states for the detrended variables.

As argued by Benhabib, Schmitt-Grohé, and Uribe (2001), combining the monetary policy rule (18) described above and the Fisher equation—i.e., $R_t^n = R_t E_t \pi_{t+1}$, where R_t is the gross real interest rate—yields two steady states, which we call the targeted-inflation steady state and the deflation steady state. Analogous to Figure 1, in the targeted-inflation steady state, the gross inflation and nominal interest rates are expressed as $\pi^* > 1$ and $R^{n*} = R\pi^*$, respectively. In the deflation steady state, they are denoted by π^D and R^{nD}. Notice that π^D is very close to (but not equal to) $1/R < 1$, and R^{nD} is very close to (but not equal to) unity.[7]

A remarkable feature of our analysis is that the model is approximated around the deflation steady state, whereas almost all the estimated DSGE models in the existing

[7] Hirose (2007) and Aruoba, Cuba-Borda, and Schorfheide (2013) consider monetary policy rules that are kinked at zero, and hence their steady-state values are given by $\pi^D = 1/R$ and $R^{nD} = 1$.

studies are approximated around the targeted-inflation steady state. Specifically, the monetary policy rule (18) is approximated as

$$\tilde{R}_t^n = \psi_r \tilde{R}_{t-1}^n + (1-\psi_r)(\psi_\pi \tilde{\pi}_t + \psi_y \tilde{y}_t) + z_t^r, \tag{19}$$

where $\psi_r \in [0,1)$ is the degree of policy rate smoothing, $\psi_\pi > 0$ and $\psi_y > 0$ are the degrees of policy responses to inflation and output respectively, and the variables with $\tilde{}$ represent log-deviations from steady-state values. This equation appears to be the same as a standard Taylor-type monetary policy rule around the targeted-inflation steady state, but it differs in the policy coefficients. In particular, the degrees of policy responses to inflation and output are very small because there is little room for lowering the nominal interest rate in response to a decrease in inflation and output because of the existence of the zero lower bound. As a consequence, the monetary policy rule approximated around the deflation steady state does not satisfy the Taylor principle.

The monetary policy rule in the present model does not take account of the zero lower bound constraint explicitly in the sense that the nominal interest rate suggested by (19) can take a negative value. However, the effect of ignoring the constraint should be marginal near the deflation steady state, where the slopes of the monetary policy rule with respect to inflation and output are very flat. In such a circumstance, the hypothetical nominal interest rate will not be substantially negative even if large negative shocks against inflation and output occur in the economy.[8]

Log-linearizing the other equilibrium conditions represented in terms of the detrended variables and rearranging the resulting equations with the steady-state conditions leads to

$$\begin{aligned}\left(1-\frac{\beta\gamma}{z^\sigma}\right)\tilde{\lambda}_t = &-\frac{\sigma z}{z-\gamma}\left\{\tilde{c}_t - \frac{\gamma}{z}\left(\tilde{c}_{t-1} - z_t^z\right)\right\} + z_t^b \\ &+ \frac{\beta\gamma}{z^\sigma}\left\{\frac{\sigma z}{z-\gamma}\left(E_t\tilde{c}_{t+1} + E_t z_{t+1}^z - \frac{\gamma}{z}\tilde{c}_t\right) - E_t z_{t+1}^b\right\},\end{aligned} \tag{20}$$

$$\tilde{\lambda}_t = E_t\tilde{\lambda}_{t+1} - \sigma E_t z_{t+1}^z + \tilde{R}_t^n - E_t\tilde{\pi}_{t+1}, \tag{21}$$

$$\tilde{u}_t = \mu\left(\tilde{r}_t^k - \tilde{q}_t\right), \tag{22}$$

$$\frac{1}{\zeta}\left(\tilde{\imath}_t - \tilde{\imath}_{t-1} + z_t^z + z_t^i\right) = \tilde{q}_t + \frac{\beta z^{1-\sigma}}{\zeta}\left(E_t\tilde{\imath}_{t+1} - \tilde{\imath}_t + E_t z_{t+1}^z + E_t z_{t+1}^i\right), \tag{23}$$

$$\tilde{q}_t = E_t\tilde{\lambda}_{t+1} - \tilde{\lambda}_t - \sigma E_t z_{t+1}^z + \frac{\beta}{z^\sigma}\left\{R^k E_t\tilde{R}_{t+1}^k + (1-\delta)E_t\tilde{q}_{t+1}\right\}, \tag{24}$$

[8] If the effect of omitting the zero lower bound constraint were nonnegligible, the estimated standard deviation of the monetary policy shock would be somewhat large. However, it turns out to be very small according to the posterior estimates presented in Section 4 (Table 2).

$$\tilde{k}_t = \frac{1-\delta}{z}\left(\tilde{k}_{t-1} - z_t^z\right) + \frac{R^k}{z}\tilde{u}_t + \left(1 - \frac{1-\delta}{z}\right)\tilde{\imath}_t, \tag{25}$$

$$\tilde{w}_t - \tilde{w}_{t-1} + \tilde{\pi}_t - \gamma_w \tilde{\pi}_{t-1} - z_t^z = \beta z^{1-\sigma}\left(E_t \tilde{w}_{t+1} - \tilde{w}_t + E_t \tilde{\pi}_{t+1} - \gamma_w \tilde{\pi}_t + E_t z_{t+1}^z\right)$$
$$+ \frac{(1-\xi_w)(1-\xi_w \beta z^{1-\sigma})\lambda^w}{\xi_w\{\lambda^w + \chi(1+\lambda^w)\}}\left(\chi \tilde{l}_t - \tilde{\lambda}_t - \tilde{w}_t + z_t^b\right) + z_t^w, \tag{26}$$

$$\tilde{y}_t = \frac{c}{y}\tilde{c}_t + \frac{i}{y}\tilde{\imath}_t + \frac{g}{y} z_t^g, \tag{27}$$

$$\tilde{mc}_t = (1-\alpha)\tilde{w}_t + \alpha \tilde{R}_t^k, \tag{28}$$

$$\tilde{w}_t - \tilde{R}_t^k = \tilde{u}_t + \tilde{k}_{t-1} - \tilde{l}_t - z_t^z, \tag{29}$$

$$\tilde{y}_t = (1+\lambda^p)\left\{(1-\alpha)\tilde{l}_t + \alpha\left(\tilde{u}_t + \tilde{k}_{t-1} - z_t^z\right)\right\}, \tag{30}$$

$$\tilde{\pi}_t - \gamma_p \tilde{\pi}_{t-1} = \beta z^{1-\sigma}\left(E_t \tilde{\pi}_{t+1} - \gamma_p \tilde{\pi}_t\right) + \frac{(1-\xi_p)(1-\xi_p \beta z^{1-\sigma})}{\xi_p}\tilde{mc}_t + z_t^p, \tag{31}$$

where $z_t^w = \frac{(1-\xi_w)\left(1-\beta\xi_w z^{1-\sigma}\right)\lambda^w}{\xi_w\{\lambda^w+\chi(1+\lambda^w)\}}(\tilde{\lambda}_t^w + z_t^l)$ is a composite shock relevant to the labor disutility and the wage markup (hereafter called a wage markup shock), $z_t^p = \frac{(1-\xi_p)\left(1-\beta\xi_p z^{1-\sigma}\right)}{\xi_p}\tilde{\lambda}_t^p$ is a shock associated with the price markup, and the variables without time subscripts represent their steady-state values. The steady-state relationships are given by

$$\beta = \frac{z^\sigma}{R},$$
$$R^k = R - 1 + \delta,$$
$$w = (1-\alpha)\left(\frac{1}{1+\lambda^p}\right)^{\frac{1}{1-\alpha}}\left(\frac{R^k}{\alpha}\right)^{-\frac{\alpha}{1-\alpha}},$$
$$\frac{k}{l} = \frac{\alpha z w}{(1-\alpha)R^k},$$
$$\phi = \lambda^p,$$
$$\frac{k}{y} = (1+\phi) z^\alpha \left(\frac{k}{l}\right)^{1-\alpha},$$
$$\frac{i}{y} = \left(1 - \frac{1-\delta}{z}\right)\frac{k}{y},$$
$$\frac{c}{y} = 1 - \frac{i}{y} - \frac{g}{y}.$$

2.5 Fundamental shock processes

The model contains seven fundamental shocks; i.e., technology z_t^z, preference z_t^b, investment adjustment cost z_t^i, external demand z_t^g, wage markup z_t^w, price markup z_t^p, and monetary policy z_t^r shocks. Each of these follows a stationary first-order autoregressive process

$$z_t^x = \rho_x z_{t-1}^x + \varepsilon_t^x,\ \ \varepsilon_t^x \sim i.i.d.N(0, \sigma_x^2), \tag{32}$$

where $\rho_x \in [0, 1)$ and $x \in \{z, b, i, g, w, p, r\}$.

3 Model Solution and Econometric Methodology

The equations $(19)-(31)$, together with the stochastic processes of fundamental shocks (32), constitute a linear rational expectation system. It is well known that sticky price monetary DSGE models have multiple equilibria, often referred to as indeterminacy, if the Taylor principle is not satisfied.[9] As addressed in the preceding section, the monetary policy rule approximated around the deflation steady state does not satisfy the Taylor principle, and hence the present system exhibits equilibrium indeterminacy. In this section, we describe how to solve and estimate the model under indeterminacy.

3.1 Solution under indeterminacy

In solving the rational expectations system under indeterminacy, we follow the approach of Lubik and Schorfheide (2003), who provide a full set of nonunique solutions for linear rational expectations models by extending the solution algorithm developed by Sims (2002).[10] In their approach, the log-linearized system is written in the following canonical form

$$\Gamma_0(\theta)\, s_t = \Gamma_1(\theta)\, s_{t-1} + \Psi_0(\theta)\, \varepsilon_t + \Pi_0(\theta)\, \eta_t, \tag{33}$$

where $\Gamma_0(\theta)$, $\Gamma_1(\theta)$, $\Psi_0(\theta)$ and $\Pi_0(\theta)$ are the conformable matrices of coefficients that depend on the structural parameters θ, s_t is a vector of endogenous variables including those expected at t, and ε_t is a vector of disturbances to fundamental shocks. η_t is a vector of endogenous forecast errors, defined as

$$\eta_t = s_t^E - E_{t-1} s_t^E,$$

[9] See, for instance, Bullard and Mitra (2002) or Woodford (2003).

[10] Sims' solution method generalizes the technique in Blanchard and Kahn (1980) and characterizes one particular solution in the case of indeterminacy.

where s_t^E is a subvector of s_t that contains the expected variables. In the present model, $s_t^E = [\tilde{c}_t, \tilde{\lambda}_t, \tilde{\pi}_t, \tilde{\imath}_t, \tilde{R}_t^k, \tilde{q}_t, \tilde{w}_t, z_t^z, z_t^b, z_t^i]'$.

According to Lubik and Schorfheide (2003), the full set of rational expectations solutions is of the form

$$s_t = \Phi_1(\theta) s_{t-1} + \Phi_\varepsilon(\theta, \tilde{M})\varepsilon_t + \Phi_\zeta(\theta)\zeta_t, \tag{34}$$

where $\Phi_1(\theta)$, $\Phi_\varepsilon(\theta, \tilde{M})$, and $\Phi_\zeta(\theta)$ are the coefficient matrices, $\tilde{M}$ is an arbitrary matrix, and ζ_t is a reduced-form sunspot shock,[11] which is a nonfundamental stochastic disturbance.[12] For estimation, it is assumed that $\zeta_t \sim i.i.d.N(0, \sigma_\zeta^2)$. The solution (34) has two important features under indeterminacy. First, business cycle fluctuations are generated not only by fundamental shocks but also by sunspot shocks. Second, the equilibrium representation cannot be unique because of the arbitrary matrix $\tilde{M}$; that is, the model has multiple solutions, and different solutions may exhibit different propagation of shocks. Therefore, in order to specify the law of motion for the endogenous variables, we need to pin down $\tilde{M}$; otherwise, any path can be considered as the equilibrium of the model.

In this paper, following Lubik and Schorfheide (2004), the components of the arbitrary matrix $\tilde{M}$ are estimated using Bayesian methods. To this end, we construct a prior distribution that is centered on a particular solution $M^*(\theta)$. Specifically, we replace $\tilde{M}$ with $M^*(\theta) + M$ and set the prior mean for M equal to zero.

In the subsequent empirical analysis, two particular solutions are considered. One is the particular solution employed in Lubik and Schorfheide (2004), referred to as the "continuity solution," where $M^*(\theta)$ is chosen such that the contemporaneous impulse responses of endogenous variables to fundamental shocks—i.e., $\partial s_t/\partial \varepsilon_t$—are continuous at the boundary between the determinacy and indeterminacy region. More specifically, for each θ, we construct a vector $\tilde{\theta} = a(\theta)$ that lies on the boundary of the determinacy region and select $M^*(\theta)$ that minimizes the discrepancy between $\partial s_t/\partial \varepsilon_t(\theta, \tilde{M})$ and $\partial s_t/\partial \varepsilon_t(\tilde{\theta})$. While Lubik and Schorfheide (2004) analytically characterize the function $a(\theta)$ for a prototypical New Keynesian model, we numerically find $\tilde{\theta}$ for our medium-scale DSGE model by perturbing the parameter ψ_π in the monetary policy rule (19).

The other particular solution is obtained by setting $M^*(\theta) = 0$, called the "orthogonality solution," where the contributions of fundamental shocks ε_t and sunspot shocks ζ_t

[11] Lubik and Schorfheide (2003) express the last term in (34) as $\Phi_\zeta(\theta, M_\zeta)\zeta_t$, where M_ζ is another arbitrary matrix, and ζ_t is a vector of sunspot shocks. For identification, Lubik and Schorfheide (2004) impose normalization such that $M_\zeta = 1$ with the dimension of the sunspot shock vector being unity. Such a normalized shock is referred to as a reduced-form sunspot shock in the sense that it contains beliefs associated with all the expectational variables in a model.

[12] If the equilibrium is determinate, the solution (34) is reduced to $s_t = \Phi_1^D(\theta) s_{t-1} + \Phi_\varepsilon^D(\theta)\varepsilon_t$.

to the forecast errors η_t are orthogonal. This particular solution is often used in the literature because it can be directly obtained with the algorithm described in Sims (2002).[13] In what follows, this paper conducts Bayesian model comparison to investigate which particular solution is well fitted to the data.

3.2 Bayesian inference

The model is estimated using Bayesian methods. Seven quarterly time series of Japan's economy are used as observable variables: the log difference of real GDP, real consumption, real investment and real wage, the log of hours worked, the log difference of the GDP deflator, and the overnight call rate. Real GDP, real consumption, and real investment are on a per capita basis, divided by the population over 15 years old. The real series of consumption and investment are obtained respectively by dividing the nominal private consumption expenditure and gross private domestic investment expenditure series by the GDP deflator. The series of real wage and hours worked are constructed following Sugo and Ueda (2008).

The data are related to model-implied variables by the following measurement equations

$$\begin{bmatrix} 100\Delta\log Y_t \\ 100\Delta\log C_t \\ 100\Delta\log I_t \\ 100\Delta\log W_t \\ 100\log l_t \\ 100\Delta\log P_t \\ 100\log R_t^n \end{bmatrix} = \begin{bmatrix} \bar{z} \\ \bar{z} \\ \bar{z} \\ \bar{z} \\ \bar{l} \\ \bar{\pi} \\ \bar{r}+\bar{\pi} \end{bmatrix} + \begin{bmatrix} \tilde{y}_t - \tilde{y}_{t-1} + z_t^z \\ \tilde{c}_t - \tilde{c}_{t-1} + z_t^z \\ \tilde{\imath}_t - \tilde{\imath}_{t-1} + z_t^z \\ \tilde{w}_t - \tilde{w}_{t-1} + z_t^z \\ \tilde{l}_t \\ \tilde{\pi}_t \\ \tilde{R}_t^n \end{bmatrix},$$

where $\bar{z} = 100\log z$, $\bar{l} = 100\log l$, $\bar{\pi} = 100\log\pi$, and $\bar{r} = 100\log R$.

The sample period is from 1999Q1 to 2013Q1, when the Bank of Japan conducted the virtually zero interest rate policy, with the exception of August 2000–March 2001 and July 2006–December 2008, and the inflation rate was almost always negative. These observations are consistent with a deflation equilibrium argued by Benhabib, Schmitt-Grohé, and Uribe (2001). Moreover, the choice of the sample period is supported by Aruoba, Cuba-Borda, and Schorfheide (2013). They consider Markov switching between the targeted-inflation and deflation steady state in a New Keynesian model and find that the Japanese economy shifted from the targeted-inflation regime into the deflation regime

[13] Hirose (2007, 2008, 2013) estimates indeterminate DSGE models based on the orthogonality solution.

in 1999 and has remained there ever since.[14]

Before estimation, some parameters are fixed to avoid identification issues. Following Sugo and Ueda (2008), we set the steady-state depreciation rate at $\delta = 0.06/4$, the capital elasticity of output at $\alpha = 0.37$, and the steady-state wage markup at $\lambda^w = 0.2$. The steady-state ratios of external demand to output are set at the sample mean; i.e., $g/y = 0.248$.

Table 1 summarizes the prior distributions of the parameters. Most of the priors for the parameters that characterize the private-sector behavior $(\sigma, \gamma, \chi, 1/\zeta, \mu, \gamma_w, \xi_w, \gamma_p, \xi_p, \lambda^p)$ are taken from Justiniano, Primiceri, and Tambalotti (2010), except for the parameters that determine the degree of relative risk aversion σ and the inverse elasticity of the utilization adjustment costs μ. The priors for σ and μ are set according to Smets and Wouters (2007) and Sugo and Ueda (2008), respectively.

The prior distributions of the monetary policy parameters $(\psi_r, \psi_\pi, \psi_y)$ are distinctive in our analysis. During the sample period, the adjustment of the nominal interest rate was severely constrained by the zero lower bound, and the Bank of Japan kept the nominal interest rate almost zero for most of the period. Thus, a very small value of 0.2 is assigned to the prior means for the policy response parameters ψ_π and ψ_y, whereas a large value of 0.9 is set for the policy smoothing parameter ψ_r.

The priors for the steady-state values for the balanced growth rate, the hours worked, the inflation rate, and the real interest rate $(\bar{z}, \bar{l}, \bar{\pi}, \bar{r})$ are set using a normal distribution with a mean based on the sample average of the corresponding data. For these parameters, relatively tight priors are imposed to ensure the stationarity of the system. Notice that the prior mean for $\bar{\pi}$ is negative, which is consistent with the deflation steady state considered in the present model.

The priors for the shock persistence parameters $(\rho_x, x \in \{z, b, i, g, w, p, r\})$ are set using the beta distribution with a mean of 0.5 and a standard deviation of 0.15, and the priors for the standard deviations of the shock innovations $(\sigma_x,\ x \in \{z, b, i, g, w, p, r, \zeta\})$ are set using the inverse gamma distribution with a mean of 0.5 and a standard deviation of infinity. Fairly wide intervals are set for the components of the arbitrary matrix M. For each component, we assign the normal distribution with mean zero and a standard deviation of 0.5.

The likelihood function is evaluated using the Kalman filter. Draws from the posterior distribution of the model parameters are generated with the Metropolis–Hastings algo-

[14] However, there is uncertainty about the timing of the regime shift. Concerning this issue, we have estimated the model for three alternative samples—i.e., from 1997Q1, 1998Q1, and 2000Q1—and confirmed that the results are almost the same.

rithm.[15] Based on the posterior draws, we make inferences on the parameters, impulse response functions, and variance decompositions.

4 Empirical Results

This section presents the estimation results. Based on the estimates of the parameters, impulse response functions, and shock decompositions, we identify some remarkable features of the Japanese economy during the zero interest rate period.

4.1 Model selection

For the post-1999 sample, the model is estimated based on two particular solutions—i.e., the continuity solution and the orthogonality solution—depending on the choice of $M^*(\theta)$. We investigate which particular solution is empirically more plausible by computing marginal data densities.

Let $\mathcal{M}_c$ and $\mathcal{M}_o$ denote the model based on the continuity solution and the orthogonality solution, respectively. The resulting log marginal data densities are $\log p\left(\mathcal{Y}^T|\mathcal{M}_c\right) = -371.8$ and $\log p\left(\mathcal{Y}^T|\mathcal{M}_o\right) = -373.4$, where $\mathcal{Y}^T$ is the sample of observations.[16] Then, the Bayes factor is given by

$$\frac{p\left(\mathcal{Y}^T|\mathcal{M}_c\right)}{p\left(\mathcal{Y}^T|\mathcal{M}_o\right)} = 4.648.$$

According to Jeffreys (1961), this value is interpreted as "substantial" evidence in favor of the continuity solution. Therefore, in the subsequent analysis, the results based on the continuity solution are considered as baseline estimates for the Japanese economy during the post-1999 period.

4.2 Parameter estimates

Table 2 reports the posterior means and 90-percent credible intervals for the model parameters in three cases. The second to fifth columns compare the parameter estimates based on the continuity solution with those based on the orthogonality solution. Both estimates are by and large in line with each other. Below, we focus on the former estimates as a baseline.

[15] In the estimation procedure, 200,000 draws are generated, and the first 20,000 draws are discarded. The scale factor for the jumping distribution in the Metropolis–Hastings algorithm is adjusted so that the acceptance rate is about 25 percent.

[16] The log marginal data densities are approximated using the harmonic mean estimator proposed by Geweke (1999).

For comparison, the last two columns in the table show the posterior estimates when a similar model is estimated for the period from 1983Q2 to 1998Q4. During the period, the economy was in a normal state in the sense that the Bank of Japan was able to adjust the nominal interest rate to achieve an implicit inflation target. Taking account of such a situation, the model in this case differs from the baseline in two respects. First, the model is approximated around the targeted-inflation steady state, as is standard in the literature.[17] Second, the monetary policy rule satisfies the Taylor principle.[18] Then, the equilibrium is determinate, and hence the sunspot shock ζ_t and the arbitrary matrix M no longer affect the equilibrium dynamics.

The households' preference parameters for the post-1999 sample are substantially different from those for the pre-1999 sample. The estimates regarding relative risk aversion σ and labor supply elasticity χ, which are still controversial in the literature, are much smaller; in particular, the 90-percent credible interval for σ is less than one for the post-1999 sample. The habit persistence parameter γ is also smaller, implying less internal persistence in the consumption dynamics during the period.

The elasticity parameter regarding investment adjustment costs $1/\zeta$ is almost the same, while the elasticity of capital utilization adjustment costs μ is smaller for the post-1999 sample. The parameters related to wage and price setting behavior $(\gamma_w, \xi_w, \gamma_p, \xi_p, \lambda^p)$ are not much different, although the wage and price indexation parameters are slightly lower.

As for the monetary policy parameters, the policy smoothing parameter ψ_r is large, and the policy reaction parameters (ψ_π, ψ_y) are very small, reflecting the fact that the Bank of Japan kept the nominal interest rate at virtually zero for most of the post-1999 period.

The mean estimates for the steady-state balanced growth rate, the hours worked, the inflation rate, and the real interest rate $(\bar{z}, \bar{l}, \bar{\pi}, \bar{r})$ are almost the same as the prior means because tight priors are set on these parameters.

Some of the shocks' persistence parameters $(\rho_b, \rho_i, \rho_g, \rho_p)$ are lower for the post-1999 sample than for the pre-1999 sample. A straightforward explanation for this result is

[17] The prior mean for steady-state inflation is set at $\bar{\pi} = 0.214$, which is the sample mean for the pre-1999 period. The prior means for the steady-state balanced growth rate, the hours worked, and the real interest rate $(\bar{z}, \bar{l}, \bar{R})$ are also set at their corresponding sample means; i.e., 0.476, 1.172, 0.778, respectively.

[18] The priors for the policy response parameters, ψ_π and ψ_y, are the gamma distributions with means of 1.5 and 0.125, and standard deviations of 0.15 and 0.1, respectively. These mean values follow from the coefficients in the original Taylor (1993) rule, adapted to a quarterly frequency. For the policy smoothing parameter ψ_r, the beta distribution is set with a mean of 0.5 and a standard deviation of 0.15.

that the observed data have exhibited less persistent dynamics during the zero interest rate period. Another possible explanation is that, as addressed in Lubik and Schorfheide (2003, 2004), Farmer and Buyer (2007), and Fujiwara and Hirose (2014), the model under indeterminacy can internally generate persistent dynamics as observed in the data without relying on the persistency of exogenous shocks.

The standard deviations of the shocks ($\sigma_x,\ x \in \{z, b, i, g, w, p, r, \zeta\}$) are not much different for both periods, except for that of the preference shock σ_b. The small standard deviation of the preference shock implies that the consumption Euler equation in the present model can accurately capture the consumption dynamics without any wedges for the post-1999 sample. On the contrary, the large standard deviation for the pre-1999 sample can be explained by the increased volatility in consumption because of the consumption tax increases in 1989 and 1997.

As for the components of the arbitrary matrix M, some of them (M_z, M_w, M_p) are far different from zero, indicating that the propagation of the fundamental shocks can be altered compared with the one in the continuity solution. With the posterior estimates of M, we can specify a set of particular solutions and characterize equilibrium dynamics under indeterminacy.

4.3 Impulse responses

Figures 2–9 present the Bayesian impulse responses of output, consumption, investment, wage, hours worked, inflation, and the nominal interest rate to one-standard-deviation shocks in technology, preferences, investment adjustment costs, external demand, wage markup, price markup, monetary policy, and sunspot. In each panel, the solid line and dashed lines respectively show the posterior mean and 90-percent credible interval for the estimated responses to each one-standard-deviation shock, in terms of percentage deviation from the steady state. As in the previous subsection, each figure compares the responses estimated for the post-1999 sample around the deflation steady state with those for the pre-1999 sample around the targeted-inflation steady state.

Remarkable changes are found in the estimated impulse responses to the shocks about preferences (Figure 3), investment adjustment costs (Figure 4), and external demand (Figure 5). According to the pre-1999 estimates, these shocks have an inflationary effect.[19]

[19]Positive shocks to preferences and external demand lead to typical demand-pull inflation. While a positive shock to investment adjustment costs decreases investment and puts downward pressure on inflation, a rise in the rental rate of capital because of a reduction in capital has an inflationary effect through an increase in real marginal cost. The latter effect dominates the former, given the estimated parameters.

For the post-1999 sample, however, the effect on inflation is ambiguous; that is, the 90-percent intervals are quite large and contain both positive and negative values. This result comes from the estimated arbitrary matrix M and its parameter uncertainty, which is an inherent feature under indeterminacy. Why can inflation both decrease and increase in response to these shocks? Initially, these shocks would have a positive effect on inflation. In response to an increase in inflation, the central bank would raise the nominal interest rate following a monetary policy rule. Around the deflation steady state, however, the degree of policy response to inflation is very small because of the existence of the zero lower bound, and hence the monetary policy rule does not satisfy the Taylor principle. Consequently, the real interest rate would decrease. The low real interest rate would stimulate demand for goods and lead to higher inflation. Such a loop can make the inflation trajectory explosive, which cannot be an equilibrium. Therefore, inflation must decrease in this case. If the initial inflationary effect is moderate, the loop does not necessarily give rise to an explosive path. In such a case, an increase in inflation can be an equilibrium.

This finding about the inflation responses to these shocks for the post-1999 period provides a novel view about the flattening of Japan's short-run Phillips curve. In the literature, Nishizaki and Watanabe (2000) show that the slope of Japan's Phillips curve became flatter as the inflation rate approached zero. De Veirman (2009) also provides evidence of a gradual flattening of the Phillips curve since the late 1990s and examines the reason why the large negative output gap did not accelerate deflation during the period. While their analyses are based on the estimates of reduced-form equations, our analysis provides a structural interpretation for their arguments. According to the Phillips curve (31) that relates inflation to real marginal cost, its slope is expressed as $(1-\xi_p)(1-\xi_p\beta z^{1-\sigma})/\xi_p$. Evaluating this slope based on the posterior mean estimates in Table 2 gives 0.028 for the post-1999 sample and 0.017 for the pre-1999 sample. Thus, the slope itself did not become flat. Rather, the ambiguity of the inflation responses to the shocks leads to a weak comovement between inflation and output, which can be identified as a flattening of the Phillips curve in the estimation of reduced-form equations.

As for the propagation of technology (Figure 2), wage markup (Figure 6), price markup (Figure 7), and monetary policy shocks (Figure 8), the impulse responses estimated for the post-1999 sample are similar to those for the pre-1999 sample. The responses to these shocks are qualitatively the same as those in Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2003, 2007), and Justiniano, Primiceri, and Tambalotti (2010) because our model shares many similarities with theirs.

The sunspot shock affects equilibrium dynamics only for the post-1999 period. The identified sunspot shock has positive effects on all the observables presented in Figure 9. The sunspot shock in the present model is constructed in a reduced-form manner, following Lubik and Schorfheide (2004), and hence has positive effects on expectational variables irrelevant to fundamentals. Such nonfundamental beliefs are self-fulfilling under indeterminacy and affect the actual variables in the same direction.

4.4 Variance decompositions

Table 3 presents the mean estimates of the asymptotic forecast error variance decompositions of output growth, consumption growth, investment growth, wage growth, hours worked, inflation, and the nominal interest rate, for the post- and pre-1999 samples.[20] Each number shows the relative contribution of technology, preferences, investment adjustment costs, external demand, wage markup, price markup, monetary policy, and sunspot shocks, in percentage terms.

The business cycle characteristics during the zero interest rate period are analyzed by focusing on the decomposition of output growth for the post-1999 sample. The primary source of output fluctuations is the technology shock, which accounts for almost half of output volatility. This finding is consistent with the conventional wisdom in the business cycle research for the US economy (e.g., King and Rebelo, 1999) and the results in the existing studies on Japan's business cycles in the 1980s and 1990s (e.g., Hayashi and Prescott, 2002; Sugo and Ueda, 2008; Hirose and Kurozumi, 2012). The second largest contribution to output fluctuations is the investment adjustment cost shock. The same result is obtained by Hirose and Kurozumi (2012) for the Japanese economy before 1999. The external demand shock also plays a substantial role in explaining output volatility, which is compatible with a common view that Japanese economic expansion in the mid-2000s was largely dependent on export demand. A distinctive feature of our analysis is that we can assess the extent to which sunspot shocks affect the macroeconomic fluctuations. However, the estimated contribution of sunspot shocks to output growth turns out to be very small. Thus, Japan's output fluctuations in the zero interest period are mainly driven by fundamental shocks rather than nonfundamental changes in expectations.

These findings are confirmed by the historical decomposition. Figure 10 shows the historical decomposition of the output growth rate in terms of percentage deviation from the steady state, evaluated at the posterior mean estimates of the parameters. Consistent with the results in the variance decomposition, the shocks to technology, investment

[20] The asymptotic variances are computed by solving a discrete Lyapunov equation for the system of log-linearized equilibrium conditions.

adjustment costs, and external demand are the main driving force of output fluctuations. In particular, the technology shocks contribute positively to the fluctuations in output growth for most of the sample period. In contrast, the sunspot shocks contribute negatively to output fluctuations; that is, they cause output to move in the opposite direction. Therefore, the sunspot shocks contribute to macroeconomic stability during the period, although an economy in the deflation equilibrium could be unexpectedly volatile because of sunspot fluctuations.

As for the variance decompositions of the other observed variables, the price markup shock has a substantial effect in addition to the technology shock. This finding is interpreted as follows. Even if the deflation steady state is taken into account, inflation fluctuations are largely explained by the exogenous shocks rather than endogenous feedback mechanism in the model. The effects of this shock are broadly transmitted to the other macroeconomic variables because, around the deflation steady state, the monetary policy is not able to react sufficiently to the movements of inflation and output because of the interest-rate lower bound.

In the pre-1999 period (the lower half of Table 3), the preference shocks have larger effects on all the observed variables, compared with those in the post-1999 period. This result is explained by the change in the estimate of its standard deviation, as mentioned in Section 4.2. The contributions of the price markup shock become smaller for most of the variables for the pre-1999 sample. Our last finding is that the central bank's adjustment of the nominal interest rate is not constrained around the targeted-inflation steady state, which differs from the explanation in the previous paragraph.

5 Concluding Remarks

In this paper, we estimated a medium-scale DSGE model with a deflation steady state for the Japanese economy during Japan's zero interest rate period. Although the equilibrium of the model is indeterminate because of the zero lower bound constraint, a specific equilibrium path is selected by extending the Bayesian methods developed by Lubik and Schorfheide (2004). The estimated model differs from a standard model with a targeted-inflation steady state in that the preference, investment adjustment cost, and external demand shocks do not necessarily have an inflationary effect. This finding provides a structural interpretation about the flattening of the short-run Phillips curve observed in Japan. According to the estimates of the variance decompositions, Japan's business cycle fluctuations are mainly driven by the shocks about technology, investment adjustment costs, and external demand. In contrast, the effect of the sunspot shock on macroeconomic volatilities is very small. Rather, the sunspot shocks helped to stabilize the economy during the period.

Our analysis assumed that Japan has been stuck in a deflation equilibrium since the Bank of Japan adopted its zero interest rate policy in 1999. However, the Japanese economy will possibly return to the targeted-inflation steady state at some time in the future. In order to consider such a steady-state change, regime switching between the two steady states, as in Aruoba, Cuba-Borda, and Schorfheide (2013), must be incorporated into the present model. Estimating such a regime switching DSGE model with indeterminate equilibria is a fruitful avenue for future research.[21]

[21] Bianchi (2013) estimates a DSGE model switching between determinacy and indeterminacy regimes using a particular solution proposed by Farmer, Waggoner, and Zha (2011). Farmer, Waggoner, and Zha (2009) provide a sunspot solution for indeterminate equilibria in Markov switching rational expectations models.

References

Aruoba, S. Borağan, Pablo Cuba-Borda, and Frank Schorfheide, 2013. "Macroeconomic Dynamics Near the ZLB: A Tale of Two Countries." NBER Working Papers 19248, National Bureau of Economic Research.

Belaygorod, Anatoliy, and Michael Dueker, 2009. "Indeterminacy, Change Points and the Price Puzzle in an Estimated DSGE Model." *Journal of Economic Dynamics and Control*, 33, 624–648.

Benhabib, Jess, Stephanie Schmitt-Grohé and Martin Uribe, 2001. "The Perils of Taylor Rules." *Journal of Economic Theory*, 96, 40-69.

Beyer, Andreas, and Roger E. A. Farmer, 2007. "Testing for Indeterminacy: An Application to U.S. Monetary Policy: Comment." *American Economic Review*, 97, 524–529.

Bhattarai, Saroj, Jae Won Lee, and Woong Yong Park, 2012a. "Policy Regimes, Policy Shifts, and U.S. Business Cycles." Globalization and Monetary Policy Institute Working Paper, 109, Federal Reserve Bank of Dallas.

Bhattarai, Saroj, Jae Won Lee, and Woong Yong Park, 2012b. "Monetary-Fiscal Policy Interactions and Indeterminacy in Postwar U.S. Data." *American Economic Review: Papers & Proceedings*, 102, 173–178.

Bianchi, Francesco, 2013. "Regime Switches, Agents' Beliefs, and Post-World War II U.S. Macroeconomic Dynamics." *Review of Economic Studies*, 2013, 80, 463–490.

Blanchard, Oliver J., and Charles M. Kahn, 1980. "The Solution of Linear Difference Models under Rational Expectations." *Econometrica*, 48, 1305–1311.

Bullard, James, 2010. "Seven Faces of 'The Peril'." *Federal Reserve Bank of St. Louis Review*, 92, 339–352.

Bullard, James and Kaushik Mitra, 2002. "Learning about Monetary Policy Rules." *Journal of Monetary Economics*, 49, 1105–1129.

Calvo, Guillermo A., 1983. "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics*, 12, 383–398.

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans, 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." *Journal of Political Economy*, 113, 1–45.

Cochrane, John H., 2013. "The New-Keynesian Liquidity Trap." NBER Working Papers 19476, National Bureau of Economic Research.

De Veirman, Emmanuel, 2009. "What Makes the Output-Inflation Trade-Off Change? The Absence of Accelerating Deflation in Japan," *Journal of Money, Credit and Banking*, 41, 1117–1140.

Erceg, Christopher J., Luca Guerrieri, and Christopher Gust, 2006. "SIGMA: A New Open Economy Model for Policy Analysis." *International Journal of Central Banking*, 2, 1–50.

Farmer, Roger E. A., Daniel F. Waggoner, and Tao Zha, 2009. "Indeterminacy in a Forward-Looking Regime Switching Model." *International Journal of Economic Theory*, 5, 69–84.

Farmer, Roger E. A., Daniel F. Waggoner, and Tao Zha, 2011. "Minimal State Variable Solutions to Markov-Switching Rational Expectations Models." *Journal of Economic Dynamics and Control*, 35, 2150–2166.

Fueki, Takuji, Ichiro Fukunaga, Hibiki Ichiue, and Toyoichiro Shirota, 2010. "Measuring Potential Growth with an Estimated DSGE Model of Japan's Economy." Bank of Japan Working Paper Series, 10-E-13.

Fujiawra, Ippei, and Yasuo Hirose, 2014. "Indeterminacy and Forecastability." *Journal of Money, Credit and Banking*, 46, 243–251.

Geweke, John F., 1999. "Using Simulation Methods for Bayesian Econometric Models: Inference, Development and Communication." *Econometric Reviews*, 18, 1–73.

Greenwood, Jeremy, Zvi Hercowitz, and Gregory W. Huffman, 1988. "Investment, Capacity Utilization, and the Real Business Cycle." *American Economic Review*, 78, 402–417.

Gust, Christopher, David López-Salido, and Matthew E. Smith, 2012. "The Empirical Implications of the Interest-Rate Lower Bound." Finance and Economics Discussion Series 2012-83, Board of Governors of the Federal Reserve System.

Hayashi, Fumio, and Edward C. Prescott, 2002. "The 1990s in Japan: A Lost Decade." *Review of Economic Dynamics*, 5, 206–235.

Hirakata, Naohisa, Nao Sudo, and Kozo Ueda (2011) "Japan's Banking Crisis and Lost Decades." mimeo, Institute for Monetary and Economic Studies, Bank of Japan.

Hirose, Yasuo, 2007. "Sunspot Fluctuations under Zero Nominal Interest Rates." *Economics Letters*, 97, 39–45.

Hirose, Yasuo, 2008. "Equilibrium Indeterminacy and Asset Price Fluctuation in Japan: A Bayesian Investigation." *Journal of Money, Credit and Banking*, 40, 967–999.

Hirose, Yasuo, 2013. "Monetary Policy and Sunspot Fluctuations in the United States and the Euro Area." *Macroeconomic Dynamics*, 17, 1–28.

Hirose, Yasuo, and Takushi Kurozumi, 2012. "Do Investment-Specific Technological Changes Matter For Business Fluctuations? Evidence From Japan." *Pacific Economic Review*, 17, 208–230.

Ichiue, Hibiki, Takushi Kurozumi, and Takeki Sunakawa, 2013. "Inflation Dynamics and Labor Market Specifications: A Bayesian Dynamic Stochastic General Equilibrium Approach for Japan's Economy." *Economic Inquiry*, 51, 273–287.

Iwata, Yasuharu, 2011. "The Government Spending Multiplier and Fiscal Financing: Insights from Japan," *International Finance*, 14, 231–264.

Kaihatsu, Sohei, and Takushi Kurozumi, 2010. "Sources of Business Fluctuations: Financial or Technology Shocks?" Bank of Japan Working Paper Series, 10-E-12.

Jeffreys, Harold, 1961. *The Theory of Probability*. Oxford University Press, Oxford.

Justiniano, Alejandro, Giorgio E. Primiceri, and Andrea Tambalotti, 2010. "Investment Shocks and Business Cycles." *Journal of Monetary Economics*, 57, 132–145.

King, Robert G., 2000. "The New IS-LM Model : Language, Logic, and Limits." *Federal Reserve Bank of Richmond Economic Quarterly*, 86/3, 45–103.

King, Robert G., and Sergio T. Rebelo, 1999. "Resuscitating Real Business Cycles." In: John B. Taylor and Michael Woodford (eds.), *Handbook of Macroeconomics Volume 1B*, 927–1007. Elsevier, Amsterdam.

Lubik, Thomas A., and Frank Schorfheide, 2003. "Computing Sunspot Equilibria in Linear Rational Expectations Models." *Journal of Economic Dynamics and Control*, 28, 273–285.

Lubik, Thomas A., and Frank Schorfheide, 2004. "Testing for Indeterminacy: An Application to U.S. Monetary Policy." *American Economic Review*, 94, 190–217.

Nishizaki, Kenji, and Tsutomu Watanabe, 2000. "Output-Inflation Trade-Off at Near-Zero Inflation Rates," *Journal of the Japanese and International Economies*, 14, 304–326.

Sims, Christopher A., 2002. "Solving Linear Rational Expectations Models." *Computational Economics*, 20, 1–20.

Smets, Frank, and Rafael Wouters, 2003. "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area." *Journal of the European Economic Association*, 1, 1123–1175.

Smets, Frank, and Rafael Wouters, 2007. "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach." *American Economic Review*, 97, 586–606.

Sugo, Tomohiro, and Kozo Ueda, 2008. "Estimating a Dynamic Stochastic General Equilibrium Model for Japan." *Journal of the Japanese and International Economies*, 22, 476–502.

Taylor, John B., 1993. "Discretion Versus Policy Rules in Practice." *Carnegie-Rochester Conference Series on Public Policy*, 39, 195–214.

Woodford, Michael, 2003. *Interest and Prices: Foundations of a Theory of Monetary Policy.* Princeton University Press, Princeton, NJ.

Zheng, Tingguo, and Huiming Guo, 2013. "Estimating a Small Open Economy DSGE Model with Indeterminacy: Evidence from China." *Economic Modelling*, 31, 642–652.

Table 1: Prior distributions of parameters

Parameter		Distribution	Mean	S.D.
σ	Relative risk aversion	Gamma	1.500	0.375
γ	Habit persistence	Beta	0.500	0.100
χ	Inverse elasticity of labor supply	Gamma	2.000	0.750
$1/\zeta$	Elasticity of the investment adj. cost	Gamma	4.000	1.000
μ	Inverse elasticity of the utilization rate adj. cost	Gamma	1.000	0.500
γ_w	Wage indexation	Beta	0.500	0.150
ξ_w	Wage stickiness	Beta	0.660	0.100
γ_p	Price indexation	Beta	0.500	0.150
ξ_p	Price stickiness	Beta	0.660	0.100
λ_p	Steady-state price markup	Gamma	0.150	0.050
ψ_r	Interest rate smoothing	Beta	0.900	0.100
ψ_π	Policy response to inflation	Gamma	0.200	0.100
ψ_y	Policy response to output	Gamma	0.200	0.100
$\overline{z}$	Steady-state output growth rate	Normal	0.145	0.025
$\bar{l}$	Steady-state hours worked	Normal	0.000	0.050
$\overline{\pi}$	Steady-state inflation rate	Normal	-0.332	0.050
$\overline{r}$	Steady-state real interest rate	Normal	0.361	0.050
ρ_z	Persistence of technology shock	Beta	0.500	0.150
ρ_b	Persistence of preference shock	Beta	0.500	0.150
ρ_i	Persistence of investment adj. cost shock	Beta	0.500	0.150
ρ_g	Persistence of external demand shock	Beta	0.500	0.150
ρ_w	Persistence of wage markup shock	Beta	0.500	0.150
ρ_p	Persistence of price markup shock	Beta	0.500	0.150
ρ_r	Persistence of monetary policy shock	Beta	0.500	0.150
σ_z	Standard deviation of technology shock	Inverse gamma	0.500	∞
σ_b	Standard deviation of preference shock	Inverse gamma	0.500	∞
σ_i	Standard deviation of investment adj. cost shock	Inverse gamma	0.500	∞
σ_g	Standard deviation of external demand shock	Inverse gamma	0.500	∞
σ_w	Standard deviation of wage markup shock	Inverse gamma	0.500	∞
σ_p	Standard deviation of price markup shock	Inverse gamma	0.500	∞
σ_r	Standard deviation of monetary policy shock	Inverse gamma	0.500	∞
σ_ζ	Standard deviation of sunspot shock	Inverse gamma	0.500	∞
M_z	Arbitrary parameter on technology shock	Normal	0.000	0.500
M_b	Arbitrary parameter on preference shock	Normal	0.000	0.500
M_i	Arbitrary parameter on investment adj. cost shock	Normal	0.000	0.500
M_g	Arbitrary parameter on external demand shock	Normal	0.000	0.500
M_w	Arbitrary parameter on wage markup shock	Normal	0.000	0.500
M_p	Arbitrary parameter on price markup shock	Normal	0.000	0.500
M_r	Arbitrary parameter on monetary policy shock	Normal	0.000	0.500

Note: The inverse gamma priors are of the form $p(\sigma|\nu, s) \propto \sigma^{-\nu-1} e^{-\nu s^2/2\sigma^2}$, where $\nu = 2$ and $s = 0.282$.

Table 2: Posterior distributions of parameters

	Post-1999 (Continuity)		Post-1999 (Orthogonality)		Pre-1999	
Parameter	Mean	90% interval	Mean	90% interval	Mean	90% interval
σ	0.736	[0.528, 0.940]	0.853	[0.650, 1.050]	1.833	[1.232, 2.410]
γ	0.351	[0.244, 0.461]	0.360	[0.255, 0.468]	0.620	[0.494, 0.752]
χ	1.889	[0.790, 2.923]	2.191	[0.911, 3.327]	3.006	[1.743, 4.249]
$1/\zeta$	4.873	[3.106, 6.527]	5.210	[3.508, 6.879]	4.587	[2.925, 6.235]
μ	2.430	[1.227, 3.544]	2.448	[1.340, 3.508]	1.599	[0.997, 2.223]
γ_w	0.286	[0.136, 0.430]	0.297	[0.146, 0.444]	0.327	[0.168, 0.487]
ξ_w	0.732	[0.636, 0.829]	0.679	[0.570, 0.789]	0.857	[0.808, 0.902]
γ_p	0.234	[0.070, 0.387]	0.308	[0.111, 0.502]	0.377	[0.120, 0.648]
ξ_p	0.846	[0.783, 0.910]	0.851	[0.790, 0.914]	0.881	[0.817, 0.946]
λ_p	0.204	[0.098, 0.304]	0.213	[0.101, 0.320]	0.165	[0.089, 0.245]
ψ_r	0.824	[0.726, 0.921]	0.961	[0.921, 1.000]	0.897	[0.861, 0.932]
ψ_π	0.089	[0.020, 0.155]	0.164	[0.039, 0.285]	1.298	[1.075, 1.514]
ψ_y	0.066	[0.014, 0.117]	0.171	[0.042, 0.295]	0.444	[0.236, 0.649]
$\overline{z}$	0.136	[0.097, 0.175]	0.136	[0.096, 0.175]	0.462	[0.424, 0.500]
$\bar{l}$	-0.004	[-0.084, 0.079]	-0.004	[-0.086, 0.074]	1.168	[1.085, 1.249]
$\overline{\pi}$	-0.312	[-0.386, -0.237]	-0.332	[-0.412, -0.249]	0.195	[0.118, 0.270]
$\overline{r}$	0.423	[0.352, 0.493]	0.429	[0.355, 0.504]	0.837	[0.758, 0.913]
ρ_z	0.359	[0.229, 0.483]	0.417	[0.254, 0.570]	0.321	[0.182, 0.459]
ρ_b	0.448	[0.217, 0.683]	0.468	[0.218, 0.709]	0.576	[0.378, 0.773]
ρ_i	0.368	[0.205, 0.525]	0.291	[0.143, 0.426]	0.507	[0.398, 0.613]
ρ_g	0.856	[0.794, 0.921]	0.893	[0.838, 0.951]	0.937	[0.908, 0.969]
ρ_w	0.228	[0.083, 0.359]	0.249	[0.097, 0.396]	0.169	[0.058, 0.276]
ρ_p	0.294	[0.111, 0.468]	0.217	[0.074, 0.350]	0.470	[0.243, 0.702]
ρ_r	0.393	[0.195, 0.585]	0.458	[0.236, 0.682]	0.326	[0.178, 0.479]
σ_z	1.662	[1.346, 1.948]	1.604	[1.294, 1.901]	1.805	[1.459, 2.164]
σ_b	0.339	[0.157, 0.528]	0.454	[0.142, 0.744]	5.977	[3.673, 8.251]
σ_i	4.155	[3.411, 4.894]	4.185	[3.476, 4.878]	5.853	[4.448, 7.234]
σ_g	3.509	[2.936, 4.073]	3.454	[2.919, 4.002]	3.095	[2.588, 3.597]
σ_w	0.333	[0.267, 0.400]	0.335	[0.266, 0.406]	0.397	[0.329, 0.467]
σ_p	0.434	[0.325, 0.543]	0.479	[0.363, 0.595]	0.345	[0.220, 0.472]
σ_r	0.057	[0.047, 0.065]	0.055	[0.047, 0.063]	0.120	[0.100, 0.141]
σ_ζ	0.403	[0.165, 0.628]	0.434	[0.159, 0.691]	-	-
M_z	-0.665	[-1.085, -0.200]	-0.570	[-0.776, -0.377]	-	-
M_b	0.012	[-0.685, 0.720]	0.048	[-0.544, 0.743]	-	-
M_i	0.011	[-0.122, 0.141]	-0.019	[-0.152, 0.114]	-	-
M_g	-0.077	[-0.156, 0.006]	-0.188	[-0.298, -0.075]	-	-
M_w	-0.546	[-1.025, -0.078]	-0.329	[-0.812, 0.177]	-	-
M_p	-0.599	[-1.042, -0.118]	-1.698	[-2.137, -1.252]	-	-
M_r	0.032	[-0.775, 0.866]	-0.041	[-0.839, 0.808]	-	-

Table 3: Variance decompositions

	$\Delta \log Y_t$	$\Delta \log C_t$	$\Delta \log I_t$	$\Delta \log W_t$	$\log l_t$	$\Delta \log P_t$	$\log R_t^n$
Post-1999							
Technology	47.9	56.1	16.5	50.9	58.7	18.5	16.0
Preference	0.6	5.7	0.1	0.0	0.3	0.3	0.4
Investment adj. cost	25.5	8.7	76.8	0.3	14.5	2.0	16.3
External demand	17.0	4.9	1.0	0.1	3.8	0.9	2.1
Wage markup	0.3	0.4	0.3	14.6	3.5	1.3	3.6
Price markup	6.0	14.6	4.1	33.8	13.4	73.9	20.0
Monetary policy	2.1	7.6	0.9	0.2	3.8	0.6	38.4
Sunspot	0.6	1.9	0.4	0.1	2.1	2.6	3.2
Pre-1999							
Technology	50.7	38.6	16.9	35.8	79.2	23.8	40.5
Preference	6.3	54.5	1.0	0.1	1.8	0.3	3.7
Investment adj. cost	29.2	4.6	76.1	0.2	8.3	1.6	24.4
External demand	10.8	1.3	2.1	0.1	4.1	1.4	6.5
Wage markup	0.0	0.0	0.1	22.6	0.6	3.3	2.1
Price markup	1.7	0.4	2.4	41.3	2.4	69.5	11.3
Monetary policy	1.3	0.6	1.5	0.0	3.6	0.1	11.5

Notes: The table shows the mean estimates of the asymptotic forecast error variance decompositions of output growth, consumption growth, investment growth, wage growth, hours worked, inflation, and the nominal interest rate.

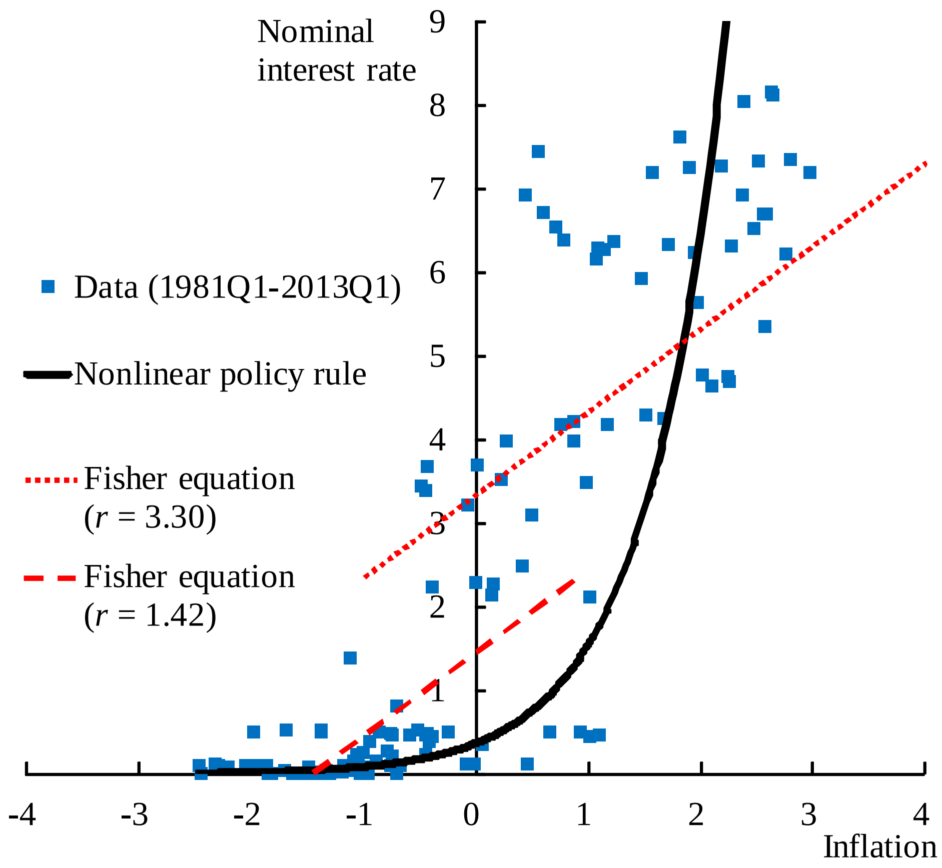


Figure 1: Interest rate and inflation in Japan

Notes: This figure plots the overnight call rate and the percentage change in the GDP deflator from one year earlier for the sample period from 1981Q1 to 2013Q1. The thick solid line is a nonlinear monetary policy rule fitted to the data. The dotted and dashed lines represent long-run Fisher relations with the real interest rates fixed at 3.30 and 1.42, respectively.

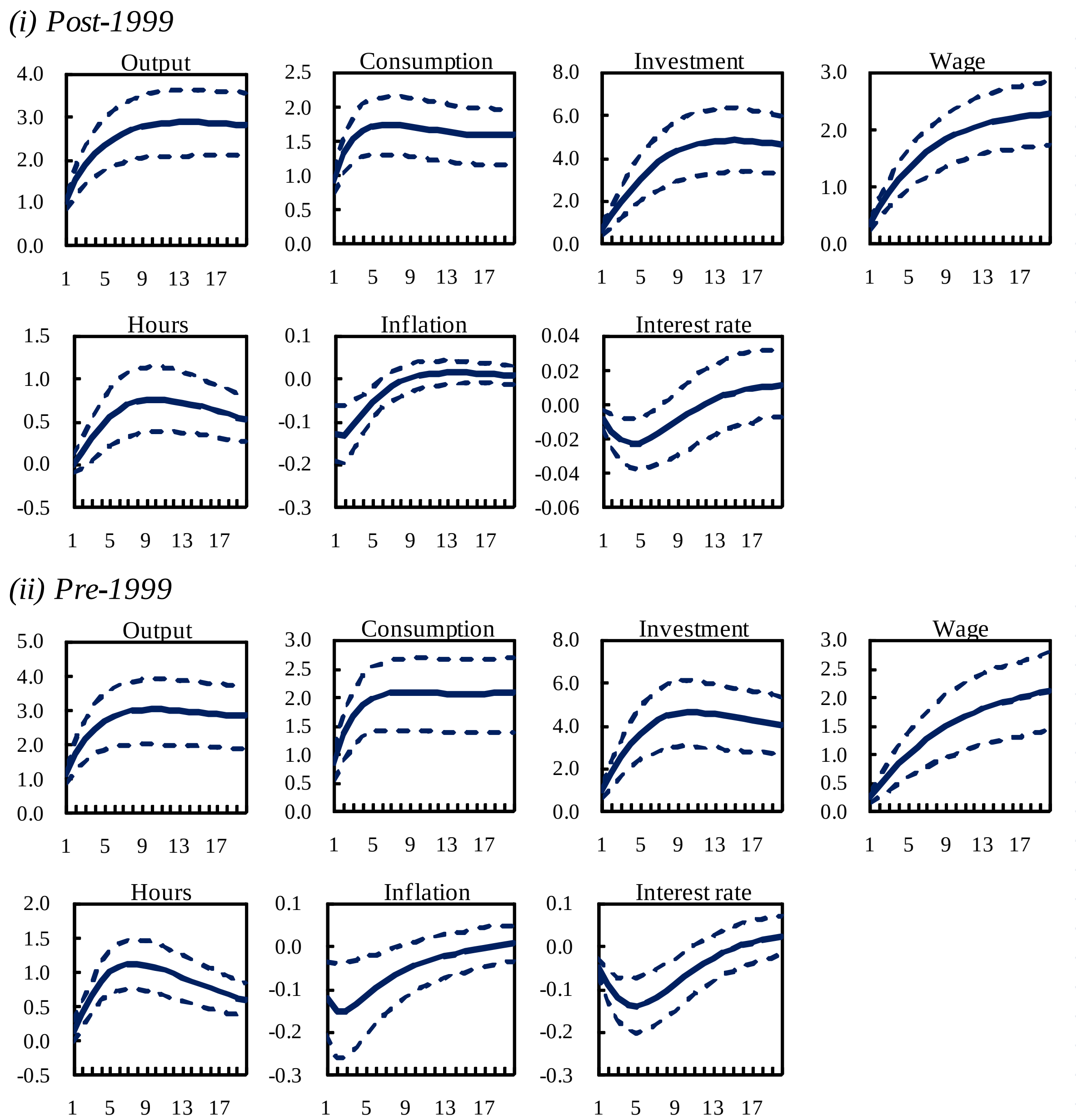


Figure 2: Impulse responses to technology shock

Notes: Each panel depicts the posterior mean (solid line) and 90-percent credible interval (dashed lines) for the impulse response to a one-standard-deviation technology shock in terms of percentage deviation from the steady state.

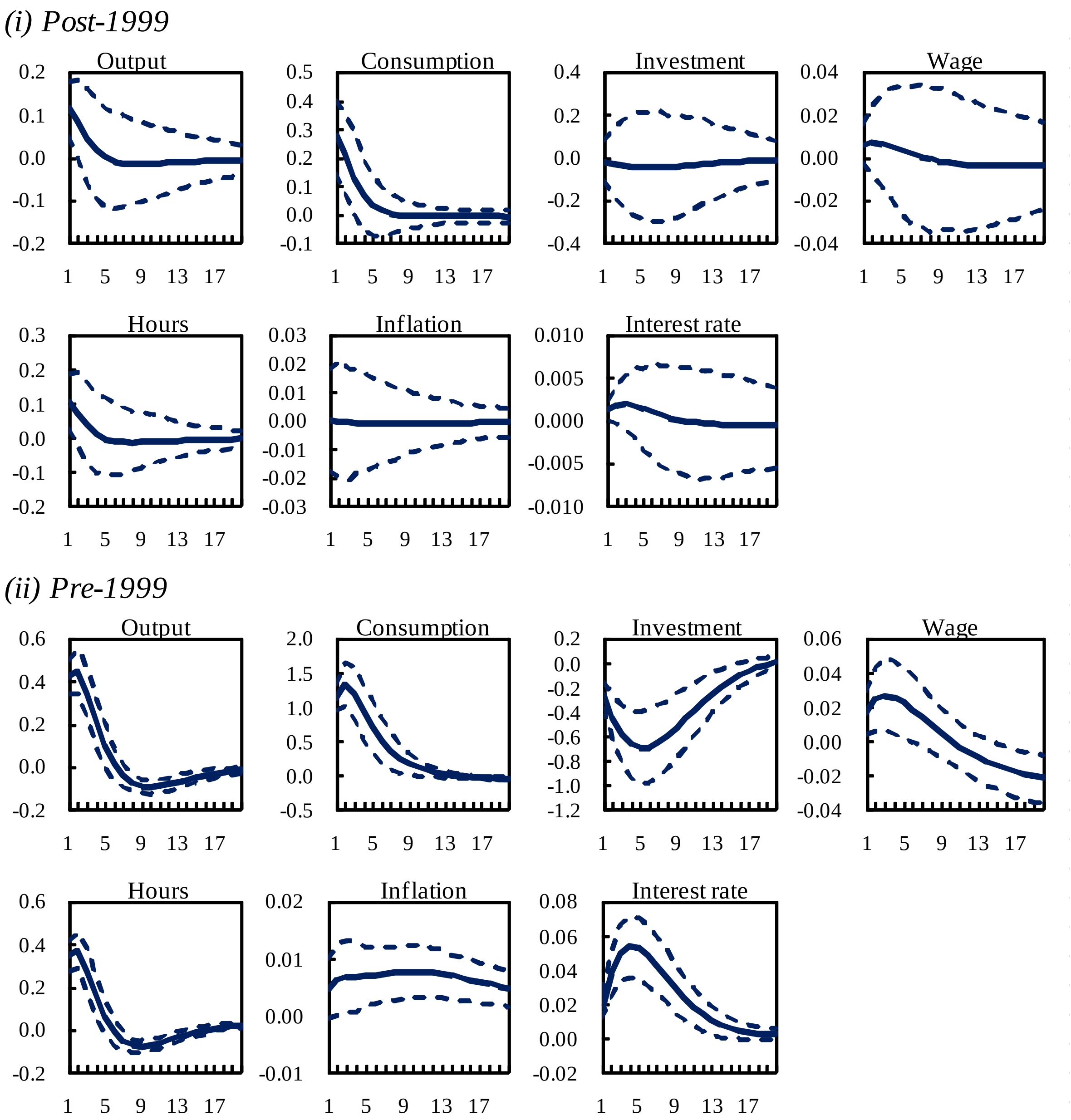


Figure 3: Impulse responses to preference shock

Notes: Each panel depicts the posterior mean (solid line) and 90-percent credible interval (dashed lines) for the impulse response to a one-standard-deviation preference shock in terms of percentage deviation from the steady state.

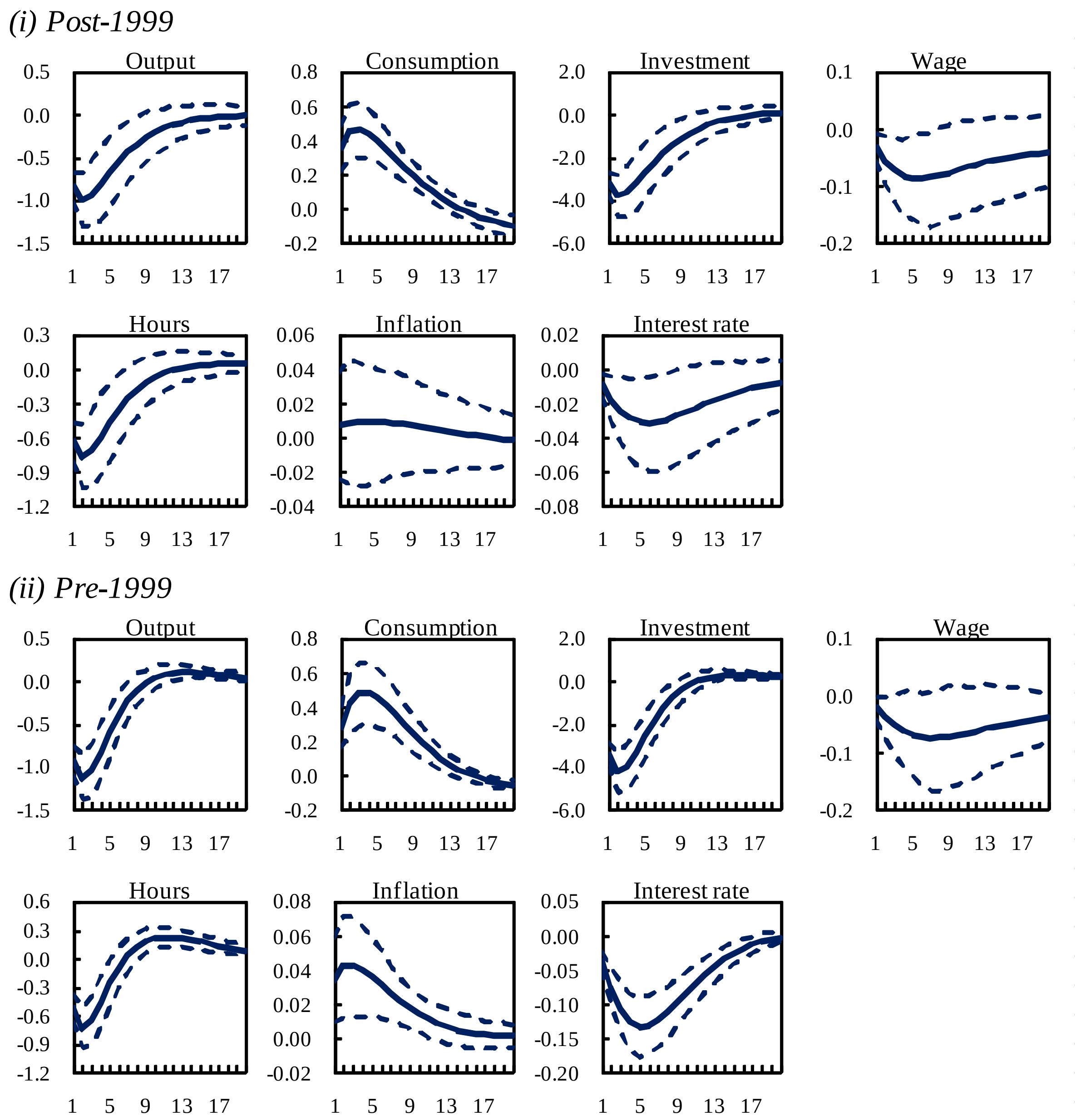


Figure 4: Impulse responses to investment adjustment cost shock

Notes: Each panel depicts the posterior mean (solid line) and 90-percent credible interval (dashed lines) for the impulse response to a one-standard-deviation investment adjustment cost shock in terms of percentage deviation from the steady state.

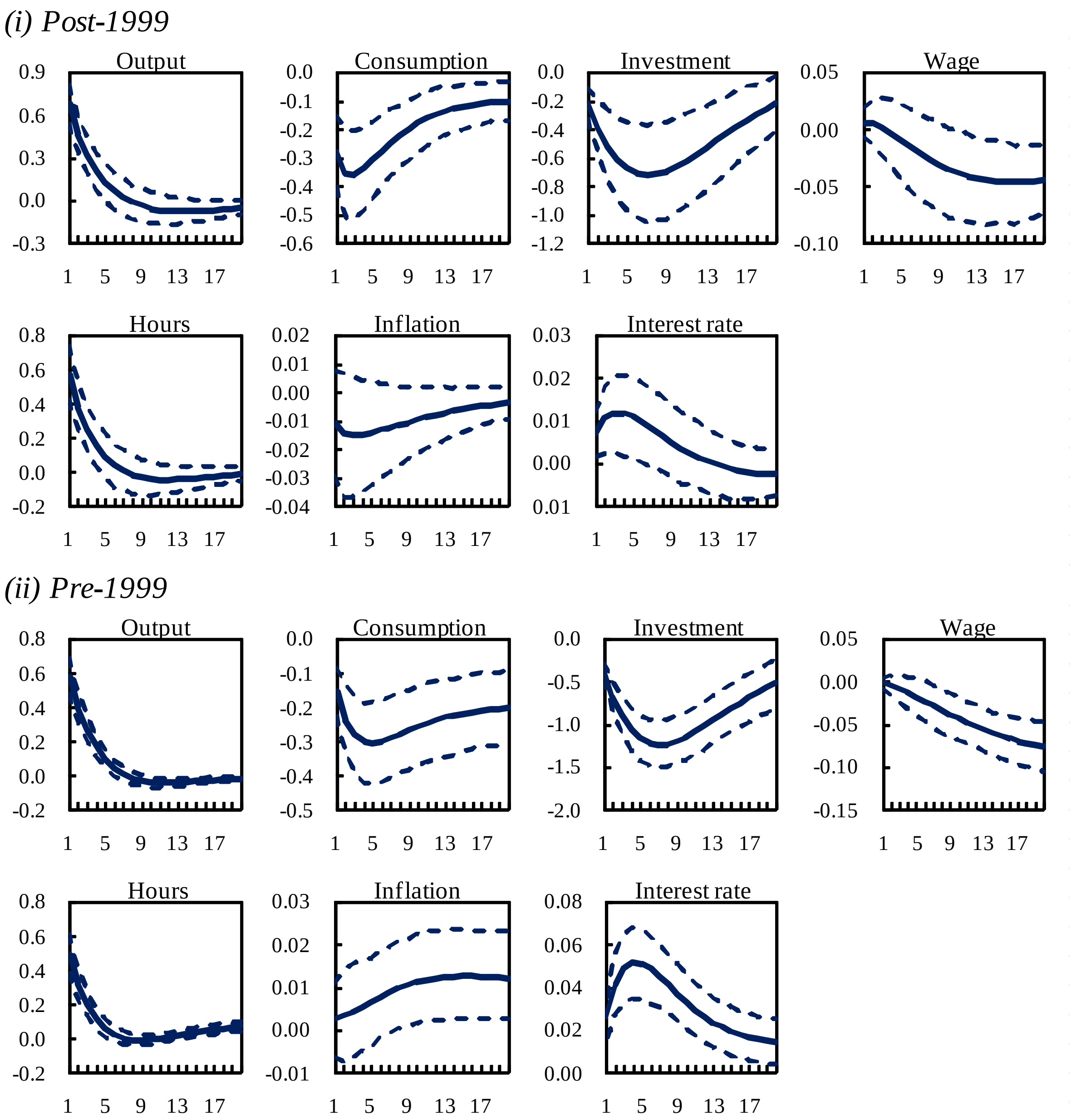


Figure 5: Impulse responses to external demand shock

Notes: Each panel depicts the posterior mean (solid line) and 90-percent credible interval (dashed lines) for the impulse response to a one-standard-deviation external demand shock in terms of percentage deviation from the steady state.

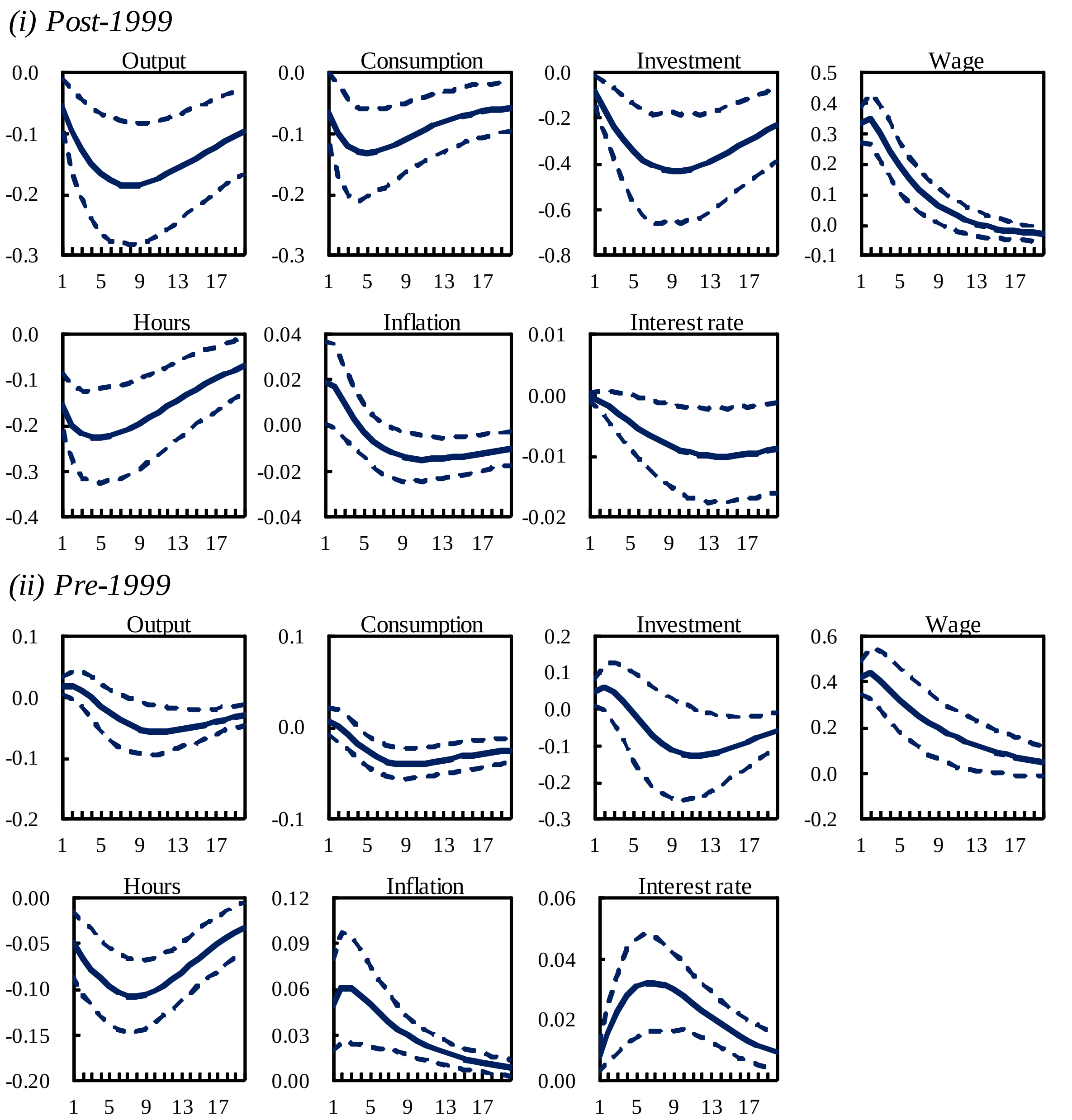
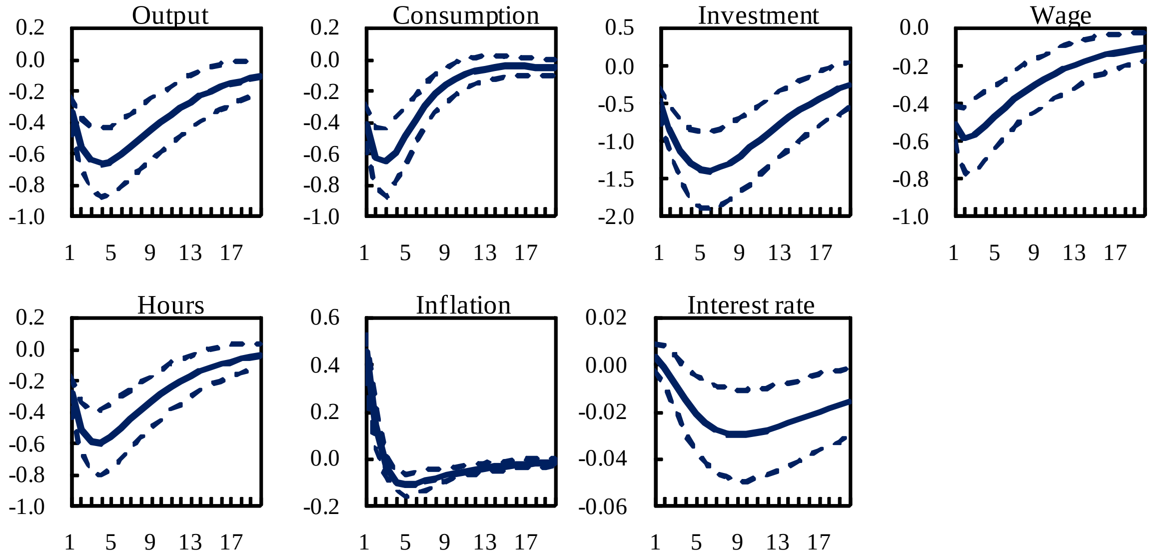


Figure 6: Impulse responses to wage markup shock

Notes: Each panel depicts the posterior mean (solid line) and 90-percent credible interval (dashed lines) for the impulse response to a one-standard-deviation wage markup shock in terms of percentage deviation from the steady state.

(i) Post-1999



(ii) Pre-1999

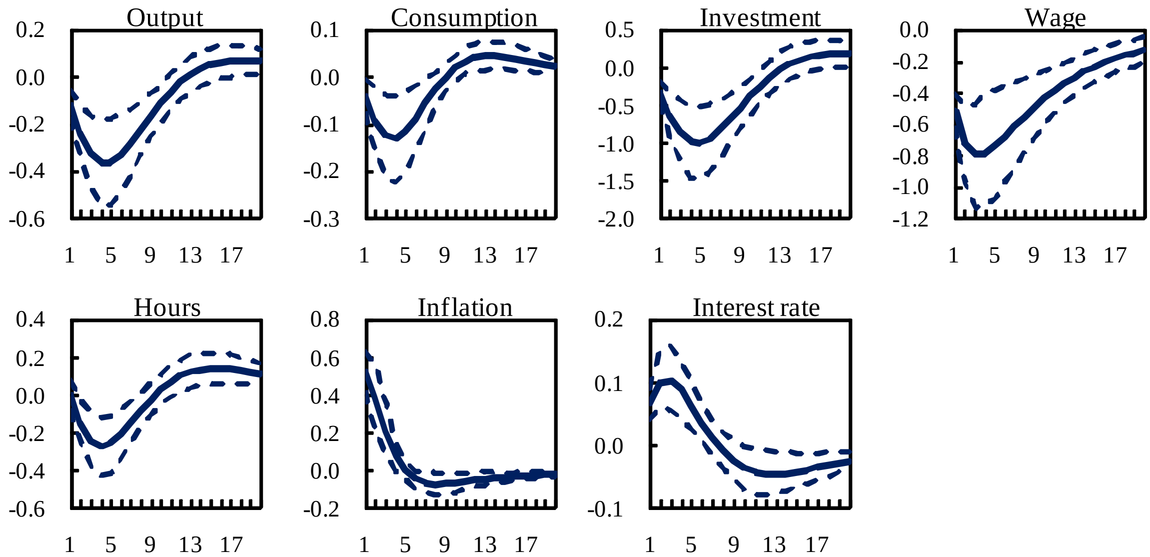


Figure 7: Impulse responses to price markup shock

Notes: Each panel depicts the posterior mean (solid line) and 90-percent credible interval (dashed lines) for the impulse response to a one-standard-deviation price markup shock in terms of percentage deviation from the steady state.

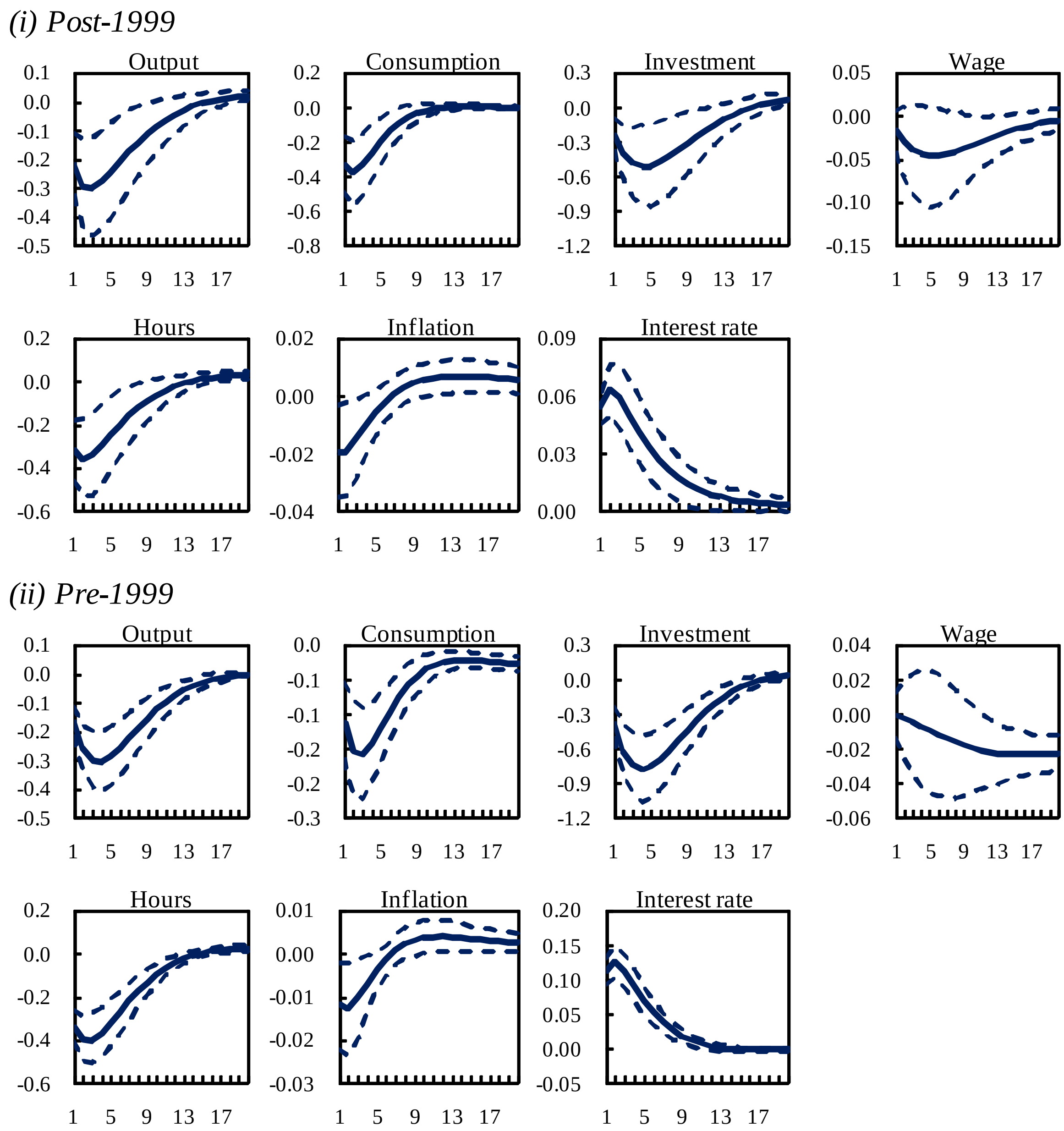


Figure 8: Impulse responses to monetary policy shock

Notes: Each panel depicts the posterior mean (solid line) and 90-percent credible interval (dashed lines) for the impulse response to a one-standard-deviation monetary policy shock in terms of percentage deviation from the steady state.

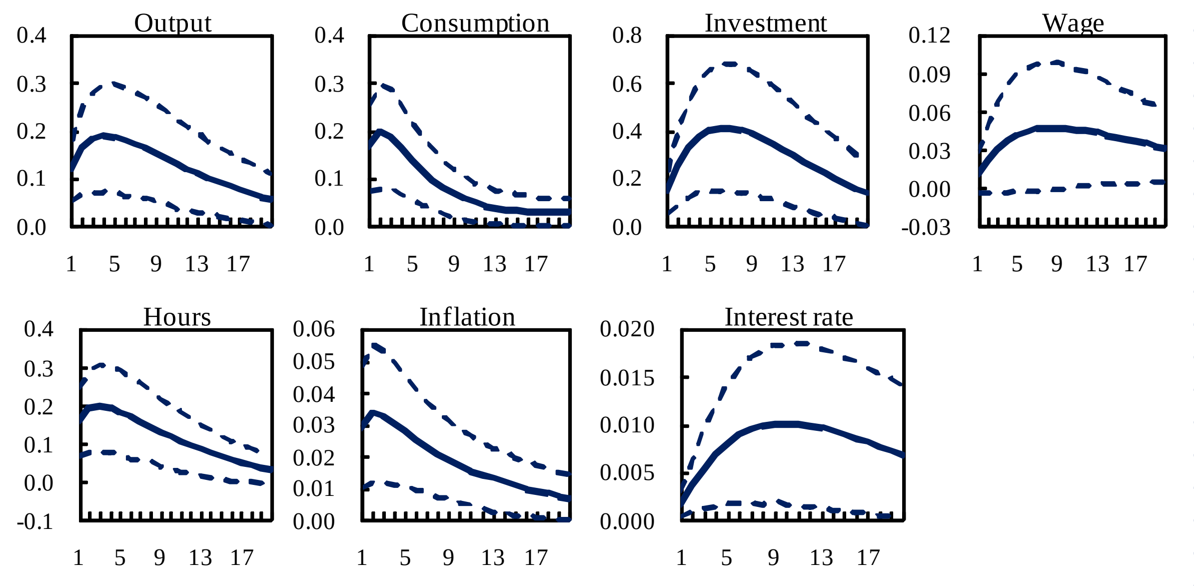


Figure 9: Impulse responses to sunspot shock

Notes: Each panel depicts the posterior mean (solid line) and 90-percent credible interval (dashed lines) for the impulse response to a one-standard-deviation sunspot shock in terms of percentage deviation from the steady state.

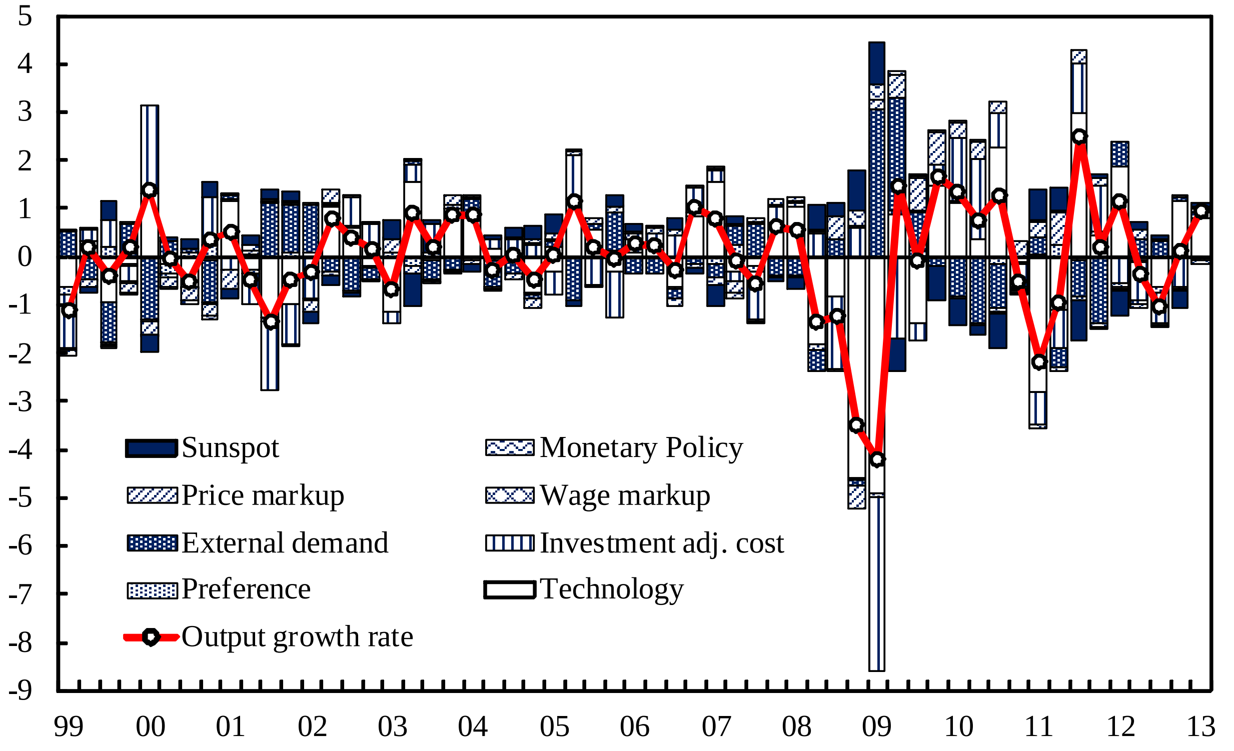


Figure 10: Historical decomposition of output growth

Notes: The figure depicts the output growth rate in terms of percentage deviation from the steady state and the contribution of each shock, evaluated at the posterior mean estimates of the parameters.