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A macroeconomic model of liquidity crises*

Keiichiro Kobayashi[†] Tomoyuki Nakajima[‡]

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Abstract

We develop a macroeconomic model in which liquidity plays an essential role in the production process, because firms have a commitment problem regarding factor payments. A liquidity crisis occurs when firms fail to obtain sufficient liquidity, and may be caused either by self-fulfilling beliefs or by fundamental shocks. Our model is consistent with the observation that the decline in output during the Great Recession is mostly attributable to the deterioration in the labor wedge, rather than in productivity. The government's commitment to guarantee bank deposits reduces the possibility of a self-fulfilling crisis, but it increases that of a fundamental crisis.

Keywords: Liquidity crises; Systemic crises; Corporate liquidity demand; Limited commitment; Debt overhang.

JEL Classification numbers: E30, G01, G21.

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1 Introduction

The Great Recession, that is, the global recession in the late 2000s, was the deepest economic downturn since the 1930s. Lucas and Stokey (2011), among others, argue that just as in the Great Depression, the recession was made severer by a liquidity crisis.¹ A liquidity crisis is a sudden evaporation of the supply of liquidity that leads to a large drop in production and employment.² In addition, the decline in output in the Great Recession was mostly due to deterioration in the labor wedge, rather than in productivity, as emphasized by Arellano, Bai, and Kehoe (2012).³

To understand the mechanism behind such a crisis, we develop a macroeconomic model in which liquidity is an essential element in the process of production. Specifically, we assume that firms cannot make a credible commitment on factor payments after production. Thus, firms have an incentive to pay for their factors before production, for which they need liquidity. The amount of output that a firm can produce thus depends on the amount of liquidity it obtains. A liquidity crisis prevents firms from obtaining liquidity, restricts their ability to hire factors of production, and therefore, results in a severe recession.

Firms obtain liquidity from banks in the form of short-term loans. Banks, in turn, receive deposits from households.⁴ We make the following assumptions regarding loans and deposits. First, they take the form of risky debt. Second, their maturity periods are different, namely, “short-term” and “long-term.” Third, markets for loans and deposits are perfectly competitive, and all participants in those markets take the interest rates and recovery rates as given. With these assumptions, loans in our model could be interpreted as corporate bonds or commercial paper, rather than traditional bank loans. In this regard, we should point out that

¹Overviews of the crisis are given by Adrian and Shin (2010), Brunnermeier (2009), and Gorton (2010), among many others.

²See, for instance, Borio (2009).

³The labor wedge is the wedge between the marginal rate of substitution of consumption for leisure and the marginal product of labor. See, for instance, Chari, Kehoe, and McGrattan (2007) and Shimer (2009) for a more detailed discussion.

⁴We embed banks in the real business cycle framework following Gertler and Karadi (2011), Gertler and Kiyotaki (2013), Gertler, Kiyotaki, and Queralto (2012), and Occhino and Pescatori (2010), among others.

commercial paper did play a central role during the financial crisis of 2007–2009, as described, for instance, by Kacperczyk and Schnabl (2010).

We consider two types of liquidity crises: sunspot and fundamental. A sunspot crisis is caused by (self-fulfilling) beliefs that firms are unable to repay loans and would go bankrupt. Given such beliefs, the short-term interest rate for loans goes up. If this increase is large enough, firms indeed go bankrupt, justifying those beliefs. Furthermore, massive defaults by firms may induce depositors to believe that their banks would also go bankrupt. Thus, the interest rate on bank deposits also rises, thereby causing banks to default as well. A fundamental crisis occurs when a bad productivity shock makes firms insolvent. Again, bankruptcy of firms leads to that of banks.

As described by Arellano, Bai, and Kehoe (2012), the decline in output in the Great Recession is mostly attributable to the deterioration in the labor wedge, rather than in productivity. A fundamental crisis in our model, as well as a sunspot crisis, is consistent with such evidence. First, as illustrated in our numerical exercise, a relatively minor productivity shock can trigger a fundamental crisis. Second, during a crisis, the wage rate that firms can credibly offer becomes far smaller than the marginal product of labor, leading to deterioration in the labor wedge and stagnation of output.

In terms of government interventions, we consider the effects of a policy that guarantees bank deposits. We find that it has the following type of tradeoffs. On the one hand, if the government commits to guaranteeing bank deposits, the possibility of self-fulfilling crises is reduced. On the other hand, however, it raises the probability of fundamental crises. The overall welfare effect of the bailout policy would therefore depend on the relative likelihood of self-fulfilling and fundamental crises.

Our model is related to several strands of literature. The first of these is related to the theory of corporate liquidity demand, such as in Holmström and Tirole (2011). Second, it is related to the theory of bank runs by Bryant (1980) and Diamond and Dybvig (1983).⁵ In this theory, a crisis occurs when there is a run on existing deposits. In our model, the crisis occurs because of an evaporation of short-term loans. Arguably, both aspects are present in

⁵For more recent developments of this theory, see Allen and Gale (1998), Uhlig (2010), Ennis and Keister (2009), Keister (2012), Kato and Tsuruga (2012), and Gertler and Kiyotaki (2013), among many others.

actual liquidity crises and the two approaches are considered to be complementary. The third strand of literature related to our study is the literature on debt overhang, such as Myers (1977), Philippon (2010), and Occhino and Pescatori (2010). In our model, the possibility of bankruptcies of firms and banks arises because they are indebted with long-term debt at the beginning of each period. The crises in our model can thus be understood to be caused by debt overhang in short-term loans.

The rest of the paper is organized as follows. In the next section, we describe a simple example. The basic structure of the model economy is described in section 3. Then, liquidity crises caused by self-fulfilling beliefs are considered in section 4, and those due to fundamental shocks are in section 5. Some policy implications are discussed in section 6. Concluding remarks are given in section 7.

2 An example

In this section, we provide a simple, one-period example that illustrates the key mechanism of our model. Consider a bank, a firm, and a depositor. The depositor lends to the bank and the bank lends to the firm.

Initially, the firm owes 50 to the bank and the bank owes 50 to the depositor. These amounts represent the long-term debt of the firm and the bank, respectively. The centerpiece of our theory is the essentiality of liquidity in the production process. Here, we simply assume that the firm needs additional liquidity of 10. Its output depends on whether or not it obtains a new short-term loan of this amount from the bank. To provide liquidity to the firm, the bank, in turn, needs to collect an additional deposit from the depositor. The depositor is willing to put an additional deposit of 10 as long as its gross rate of return is greater than or equal to unity.

We assume that the same recovery rate applies for short-term and long-term debt. Let R^F and R^B denote the gross interest rates on the short-term loans and deposits, and ξ^F and ξ^B the recovery rates of loans and deposits, respectively. The firm, bank, and depositor are all price takers. Specifically, they take R^F , R^B , ξ^F , and ξ^B as given.⁶

⁶To justify this assumption, we may suppose that there are a large number of identical firms, banks, and

2.1 Self-fulfilling crisis

Let us start with a self-fulfilling crisis. Suppose that if the firm obtains the short-term loan of 10 from the bank, it produces 70; otherwise, it produces only 30. Then, there exist two equilibria: “normal” and “crisis.” The short-term loan is available in the normal equilibrium, but not in the crisis.

In the normal equilibrium, the short-term rates on loans and deposits are both unity, $R^F = R^B = 1$; the firm obtains the liquidity it needs; and neither the firm nor the bank defaults, that is, $\xi^F = \xi^B = 1$. To see that such an equilibrium exists, note that the depositor is willing to make the deposit of 10 given that $R^B = 1$ and $\xi^B = 1$. If the firm obtains the short-term loan of 10, its revenue becomes 70. This is greater than the total debt of $50 + R^F \times 10 = 60$. As a result, the firm is solvent and $\xi^F = 1$. If the bank provides the short-term loan to the firm, the bank’s revenue is $\xi^F(50 + R^F \cdot 10) = 60$. The amount it has to repay to the depositor is $50 + R^B \cdot 10 = 60$. Thus, the bank is solvent (breaks even) so that $\xi^B = 1$.

In the crisis equilibrium, a liquidity crisis occurs. The firm fails to obtain liquidity; so, its output is low (30); and both the bank and the firm default. Because the firm’s debt is 50 and it earns only 30, the recovery rate of loans to the firm is $\xi^F = \frac{3}{5}$. Similarly, the recovery rate of deposits at the bank is $\xi^B = \frac{3}{5}$. The short-term interest rates that support the crisis equilibrium are not unique; any pair of (R^F, R^B) satisfying $R^B \leq \frac{5}{3}$ and $2 < R^F < \frac{10}{3} + R^B \cdot \frac{5}{3}$ can comprise the equilibrium interest rates.⁷ Because $\xi^B = \frac{3}{5}$ and $R^B \leq \frac{5}{3}$, the depositor (weakly) prefers not to make the short-term deposit (of 10). The firm also prefers not to obtain the short-term loan because if it did, its profit would be $70 - (50 + R^F \cdot 10) < 0$, given that $R^F > 2$. Finally, the bank does not have an incentive to make the short-term loan either—by making the loan, its profit would be $\xi^F(50 + R^F \cdot 10) - (50 + R^B \cdot 10) < 0$, given that $R^F < \frac{10}{3} + R^B \cdot \frac{5}{3}$. Thus, as long as the above restrictions on (R^F, R^B) hold, the supply of liquidity evaporates, resulting in a liquidity crisis.

Whether the normal or crisis equilibrium is realized depends on the expectations about

depositors.

⁷The non-uniqueness of the equilibrium interest rates follows from the fact that the crisis equilibrium is a “corner solution,” because the supply of short-term loans and deposits is zero.

interest rates (R^F, R^B) that are self-fulfilling. If the agents believe that $(R^F, R^B) = (1, 1)$, the normal equilibrium is realized: the firm and bank are both solvent, $\xi^F = \xi^B = 1$, and output is high (70). If, on the other hand, they believe that (R^F, R^B) with $R^B \leq \frac{5}{3}$ and $2 < R^F < \frac{10}{3} + R^B \cdot \frac{5}{3}$, then a liquidity crisis occurs: both the firm and the bank default, $\xi^F = \xi^B = \frac{3}{5}$, and output is depressed (30).

2.2 Fundamental crisis

In our framework, a fundamental crisis occurs when the level of productivity is so low that the firm necessarily goes bankrupt. To illustrate the idea, assume a bad productivity shock such that even with the short-term loan of 10, the firm can only produce 50, rather than 70. Without the short-term loan, the firm can only produce 30, just as before.

In this case, the “normal” equilibrium no longer exists. To see this, suppose that there were an equilibrium in which the firm and the bank obtain the short-term loan and deposit of 10, respectively, and neither the firm nor the bank defaults. Then, the bank’s profit would be $50 + R^F \cdot 10 - (50 + R^B \cdot 10)$. For this to be positive, $R^F \geq R^B \geq 1$. This, in turn, leads to the contradiction that the firm’s profit would be negative: $50 - (50 + R^F \cdot 10) < 0$.

Thus, a liquidity crisis necessarily occurs in equilibrium. Based on an argument similar to the previous example, any pair of (R^F, R^B) satisfying $R^B \leq \frac{5}{3}$ and $0 < R^F < \frac{10}{3} + R^B \cdot \frac{5}{3}$ can comprise the equilibrium interest rates. Given these rates, no short-term deposits and loans are provided, leading to a liquidity crisis where the firm’s production falls to 30.

3 The model economy

Time is discrete and continues to infinity: $t = 0, 1, 2, \dots$. We abstract from capital accumulation and assume that the total supply of capital is fixed at unity. There is a unit mass of identical and infinitely lived households who consume goods, save funds, and supply labor. In addition, in every period, a “firm” and a “bank” are born in each household, who are alive for two periods. We assume that all agents are price takers and all markets are perfectly competitive.

Financial intermediation is introduced within the representative household framework in

the standard way.⁸ Funds flow from households to banks and from banks to firms. We assume that firms cannot obtain funds directly from the household they belong to; they need to borrow from banks that are members of other households.⁹ Banks raise funds in the form of equity from the households they belong to. They also collect funds from other households in the form of deposits.

Regarding loans and deposits, the following three assumptions are important for our argument. First, both loans and deposits take the form of risky debt, where borrowers make a fixed repayment as long as they are solvent.¹⁰ Second, there are loans and deposits with different maturity periods. Specifically, we assume that firms need two types of loans: inter-period (“long-term”) and intra-period (“short-term”). Corresponding to these financial needs of firms, banks also collect short-term and long-term deposits. Third, all loans (deposits) have the same seniority, regardless of their maturity. Thus, the recovery rate for short-term and long-term loans (deposits) becomes identical.

Let $s_t \in \Omega$ denote the state of nature in period t . We divide Ω into Ω^n and Ω^b , where Ω^n is the set of “normal” states and Ω^b is the set of “crisis” states. A liquidity crisis occurs if and only if $s_t \in \Omega^b$. Note that $\Omega^n \cup \Omega^b = \Omega$ and $\Omega^n \cap \Omega^b = \emptyset$. For simplicity, we assume that s_t is independent and identically distributed (i.i.d.). Let $F(s)$ denote the probability distribution over Ω . The following two cases are mainly analyzed in this paper.

Example 1 (Sunspot Shock Economy): The first case we consider is the sunspot shock economy in which there are no fundamental shocks, but a liquidity crisis occurs as a result of self-fulfilling beliefs. In this case, $s_t \in \Omega$ denotes a sunspot shock, where $\Omega = \{n, b\}$, $\Omega^n = \{n\}$, and $\Omega^b = \{b\}$. A liquidity crisis occurs if and only if $s_t = b$. Let $\varepsilon \in [0, 1]$ denote the probability of the crisis: $F(s_t = n) = 1 - \varepsilon$ and $F(s_t = b) = \varepsilon$. The value of ε is exogenously given.

⁸See, for instance, Christiano, Motto, and Rostagno (2010), Gertler and Karadi (2011), and Gertler and Kiyotaki (2013).

⁹Theories that account for why some firms need to borrow from banks include delegated monitoring (Diamond 1984) and superior auditing technology of relationship banks (Diamond and Rajan 2000, 2001).

¹⁰As is well known, with asymmetric information and costly state verification, the optimal contract does take the form of risky debt (e.g., Townsend 1979, Gale and Hellwig 1985).

Example 2 (Fundamental Shock Economy): The second case we consider is the fundamental shock economy, where a liquidity crisis is caused by a fundamental productivity shock. In this case, s_t denotes the aggregate productivity shock, and $\Omega = [0, s_{\max}]$, $\Omega^b = [0, \underline{s}]$, and $\Omega^n = [\underline{s}, s_{\max}]$, where \underline{s} is the threshold between the normal and crisis states, and s_{\max} is the exogenous upper limit. The value of \underline{s} is determined endogenously.

3.1 Households

The flow budget constraint for the representative household is given by

$$c_t + d_t^L + d_t^S + e_t = \tilde{\xi}_t^B R_{t-1}^D d_{t-1}^L + \tilde{\xi}_t^B R_t^B d_t^S + w_t l_t + \tilde{R}_t^E e_{t-1} + \pi_t^F, \quad (1)$$

where c_t denotes the amount of consumption, l_t the amount of labor supplied to firms (in other households), and π_t^F the profits earned by member firms.

The household provides funds to banks in two ways. First, it provides equity, e_t , to its member banks. As shown below, a moral hazard problem of banks requires them to hold some equity. The realized rate of return on equity is \tilde{R}_t^E . Second, each household puts deposits in banks that belong to other households. Deposits are of two types: long-term (inter-period), d_t^L , and short-term (intra-period), d_t^S . Their rates of interest are R_t^D and R_t^B , respectively.¹¹

If $s_t \in \Omega^b$, all banks go bankrupt in period t . In such a case, the depositors recover only a fraction $\xi_t^B \in [0, 1]$ of their claims. Let $\tilde{\xi}_t^B$ denote the stochastic recovery rate of depositors in period t .

$$\tilde{\xi}_t^B = \begin{cases} 1, & \text{if } s_t \in \Omega^n, \\ \xi_t^B, & \text{if } s_t \in \Omega^b. \end{cases}$$

Taking stochastic processes $(\tilde{\xi}_t^B, R_{t-1}^D, R_t^B, \tilde{R}_t^E, w_t, \pi_t^F)$ as given, the household maximizes its lifetime utility as follows:

$$\max_{(c_t, d_t^L, d_t^S, e_t, l_t, h_t) \geq 0} E_0 \sum_{t=0}^{\infty} \beta^t [\ln c_t + \gamma \ln(1 - l_t - h_t)], \quad (2)$$

¹¹The long-term deposit rate between periods $t-1$ and t , R_{t-1}^D , is a predetermined variable in period t .

subject to the sequence of the flow budget constraint (1). The stochastic discount factor $\lambda_{t-1,t}$ is then defined as

$$\lambda_{t-1,t} = \beta \frac{c_{t-1}}{c_t},$$

and the first-order conditions for d_t^L and e_t are

$$1 = E_t \left[\lambda_{t,t+1} \tilde{\xi}_{t+1}^B R_t^D \right] = E_t \left[\lambda_{t,t+1} \tilde{R}_{t+1}^E \right]. \quad (3)$$

For a bounded solution for d_t^S to exist, $\tilde{\xi}_t^B$ and R_t^B must satisfy

$$\tilde{\xi}_t^B R_t^B \leq 1.$$

3.2 Firms

Firms produce a single homogeneous good according to the following production technology:

$$y_t = A_t m_t^\nu \kappa_t^{\alpha-\nu} l_t^{1-\alpha}, \quad (4)$$

where κ_t denotes the capital input, l_t the labor input, and m_t the managerial input. Each firm supplies one unit of managerial input. This supply is inelastic, so that $m_t = 1$ in equilibrium. The firm cannot obtain the other inputs, κ_t and l_t , directly from the household it belongs to. Instead, it has to purchase them at the market. On a related note, the household cannot directly consume what its member firms produce. Thus, firms have to sell their products to other households in the market. The earnings of a firm are transferred back to the household it belongs to.

As documented, for instance, by Reinhart and Rogoff (2009, 2014), a financial crisis tends to trigger a severe decline in output. This suggests that liquidity plays an essential role in the production process. To incorporate this idea, we assume that firms have a limited ability to commit to pay for the factors of production. Specifically, each firm has the option to pay for its factors either before or after output is produced. If it chooses to pay before production, it needs to borrow from banks to make the payments. If it does not, the firm can credibly commit to using only a fraction θ of its revenue for factor payments. This assumption creates demand for liquidity (short-term loans) by firms and implies that failing to obtain it results in a reduction in employment and output.

To be specific, consider the profit maximization problem of a firm that is born in period $t - 1$. It purchases physical capital in the first year of its life, and produces output in the second. The household does not provide any funds to its member firms. Thus, the firm needs to borrow from banks to purchase capital of amount k_{t-1} . Let q_{t-1} denote the price of capital in period $t - 1$. Thus, the amount the firm needs to borrow is given by $L_{t-1} = q_{t-1}k_{t-1}$. This is an inter-period (long-term) loan with a gross interest rate of R_{t-1}^L . In period t , after s_t is realized, the firm determines its capital input, κ_t , and labor input, l_t . There is a rental market for capital with rental price x_t ; so, the actual capital input κ_t can be different from the amount of capital purchased in the last period, k_{t-1} . The wage rate is w_t .

The amount that the firm needs to pay for its factors in period t is $w_t l_t + x_t(\kappa_t - k_{t-1})$. It can pay this amount either before or after production. To pay before production, it needs to borrow $W_t \geq w_t l_t + x_t(\kappa_t - k_{t-1})$. Here, W_t is a short-term (intra-period) loan, interpreted as working capital. We let R_t^F denote the gross interest rate on short-term loans. If it chooses to pay after production, the factor payment is bounded from above by θy_t . After production, the firm sells the capital at the price q_t .

The firm takes stochastic processes $(\lambda_{t-1,t}, q_{t-1}, q_t, w_t, x_t, R_{t-1}^L, R_t^F)$ as given and chooses $(k_{t-1}, \kappa_t, l_t, W_t)$ in order to solve the following profit maximization problem:

$$\begin{aligned} \max_{k_{t-1} \geq 0} \quad & E_{t-1} \left[\lambda_{t-1,t} \left\{ \max_{(\kappa_t, l_t, W_t) \geq 0} \pi_t^F(k_{t-1}, \kappa_t, l_t, W_t) \right\} \right], \\ \text{s.t.} \quad & w_t l_t + x_t(\kappa_t - k_{t-1}) \leq \begin{cases} W_t, & \text{if } W_t > 0, \\ \theta A_t \kappa_t^{\alpha-\nu} l_t^{1-\alpha}, & \text{otherwise.} \end{cases} \end{aligned} \quad (5)$$

Here,

$$\pi_t^F(k, \kappa, l, W) = \max \left\{ A_t \kappa^{\alpha-\nu} l^{1-\alpha} + q_t k - R_{t-1}^L q_{t-1} k - \sigma_t, 0 \right\}, \quad (6)$$

where σ_t is the payment for factors in period t , that is,

$$\sigma_t = \begin{cases} R_t^F W_t, & \text{if } W_t > 0, \\ w_t l_t + x_t(\kappa_t - k_{t-1}), & \text{otherwise,} \end{cases} \quad (7)$$

and we have used the fact that the firm chooses $m_t = 1$ in (4).

In what follows, we restrict our attention to the type of equilibria where if $s_t \in \Omega^n$, all firms are solvent and make the factor payments before production; and if $s_t \in \Omega^b$, all firms go

bankrupt and make factor payments after production. In such an equilibrium, when $s_t \in \Omega^b$, $\max_{(\kappa_t, l_t, W_t)} \pi_t^F(k_{t-1}, \kappa_t, l_t, W_t) = 0$. Thus, firms would be indifferent about (κ_t, l_t, W_t) . To determine the equilibrium allocation, we assume that even when firms default, they choose (κ_t, l_t, W_t) to maximize $A_t \kappa_t^{\alpha-\nu} l_t^{1-\alpha} - R_t^F W_t$, subject to the constraint in (5).

3.3 Banks

Consider a bank of a household that is born in period $t-1$. The household provides the bank with funds e_{t-1} as equity. In period $t-1$, the bank collects inter-period deposits d_{t-1}^L (from other households) and makes inter-period loans L_{t-1} to firms (in other households), where $L_{t-1} = d_{t-1}^L + e_{t-1}$. In period t , it collects intra-period deposits d_t^S and makes intra-period loans W_t to firms, so that $d_t^S = W_t$. If firms are solvent, the bank receives the scheduled amount, $R_{t-1}^L L_{t-1} + R_t^F W_t$, from them. If they go bankrupt, however, the bank can only acquire a fraction $\xi_t^F \in [0, 1]$ of that amount. Let $\tilde{\xi}_t^F$ denote the recovery rate of loans to firms. Then,

$$\tilde{\xi}_t^F = \begin{cases} 1, & \text{if } s_t \in \Omega^n, \\ \xi_t^F. & \text{if } s_t \in \Omega^b. \end{cases}$$

Taking into account the possibility that the bank may default, the bank's profit in period t is given by

$$\pi_t^B(e_{t-1}, L_{t-1}, W_t) = \max \left\{ \tilde{\xi}_t^F (R_{t-1}^L L_{t-1} + R_t^F W_t) - R_t^B W_t - R_{t-1}^D (L_{t-1} - e_{t-1}), 0 \right\}. \quad (8)$$

To take into account frictions associated with financial intermediation, we assume that banks are subject to a moral hazard problem similar to the one considered by Gertler and Karadi (2011).¹² As a result, only a fraction of the bank's revenue can be pledged to its depositors. To simplify the analysis, we assume that the moral hazard problem is associated only with the short-term loans W_t . Specifically, suppose that the bank can divert a fraction ψ of the revenue from short-term loans $\tilde{\xi}_t^F R_t^F W_t$, so that the amount of the bank's revenue that can be pledged becomes

$$\tilde{\xi}_t^F [R_{t-1}^L L_{t-1} + (1 - \psi) R_t^F W_t].$$

¹²We make this assumption for a technical reason as well. Without such an assumption, the bank's size would become infinite in this model.

Thus, for the bank to make short-term loans, this amount must exceed the amount of debt the bank owes to its depositors. That is,

$$\tilde{\xi}_t^F [R_{t-1}^L L_{t-1} + (1 - \psi) R_t^F W_t] \geq R_t^B W_t + R_{t-1}^D (L_{t-1} - e_{t-1}). \quad (9)$$

It follows that the set of feasible values of W_t and $\Gamma_t(e_{t-1}, L_{t-1})$, is defined as

$$\Gamma_t(e_{t-1}, L_{t-1}) \equiv \{0\} \cup \{W \geq 0 : \tilde{\xi}_t^F [R_{t-1}^L L_{t-1} + (1 - \psi) R_t^F W] \geq R_t^B W + R_{t-1}^D (L_{t-1} - e_{t-1})\}. \quad (10)$$

The bank takes stochastic processes $(\lambda_{t-1,t}, e_{t-1}, \tilde{\xi}_t^F, R_{t-1}^D, R_{t-1}^L, R_t^B, R_t^F)$ as given and chooses (L_{t-1}, W_t) to maximize profit as follows:

$$\max_{L_{t-1} \geq 0} E_{t-1} \left[\lambda_{t-1,t} \left\{ \max_{W_t \in \Gamma_t(e_{t-1}, L_{t-1})} \pi_t^B(e_{t-1}, L_{t-1}, W_t) \right\} \right], \quad (11)$$

where the function π_t^B is defined in (8) and the correspondence Γ_t is in (10).

Note that the moral hazard constraint (9) is rewritten as

$$\tilde{\xi}_t^F (R_{t-1}^L L_{t-1} + R_t^F W_t) - R_t^B W_t - R_{t-1}^D (L_{t-1} - e_{t-1}) \geq \tilde{\xi}_t^F \psi R_t^F W_t.$$

It follows that whenever banks default (i.e., when $\max_{W_t \in \Gamma_t} \pi_t^B = 0$), short-term loans cannot be provided; that is, $\Gamma_t = \{0\}$. Hence, all firms default as well. Note also that the definition of π_t^B in (8) implies that solvent banks (i.e., those with $\max_{W_t \in \Gamma_t} \pi_t^B > 0$) provide a positive amount of short-term loans, $W_t > 0$, only if $\tilde{\xi}_t^F R_t^F \geq R_t^B$.

It is shown later (equation 20) that π^B is linear in e_{t-1} in equilibrium. The realized return to the bank equity, \tilde{R}_t^E , is therefore defined by

$$\tilde{R}_t^E e_{t-1} = \pi_t^B(e_{t-1}, L_{t-1}, W_t).$$

3.4 Equilibrium

Remember that s_t is i.i.d. We thus restrict our attention to equilibria where all endogenous variables are written as functions of the current state of nature $s_t \in \Omega$, and both banks and firms go bankrupt in period t if and only if $s_t \in \Omega^b$. In what follows, we often use $s \in \Omega$ to denote the state in the ‘‘current period,’’ $s_- \in \Omega$ in the ‘‘previous period,’’ and $s' \in \Omega$ in the ‘‘next period.’’

The market clearing conditions for the capital stock, managerial inputs, loans, deposits, and consumption are that for all $s \in \Omega$,

$$k(s) = \kappa(s) = m(s) = 1, \quad L(s) = q(s) = d^L(s) + e(s), \quad W(s) = d^S(s),$$

and

$$c(s) = A(s)l(s)^{1-\alpha}. \quad (12)$$

Observe that the inter-period rates of loans and deposits must be equal in equilibrium; that is,

$$R^L(s) = R^D(s) \equiv R(s). \quad (13)$$

A formal proof is provided in the Appendix. Intuitively, because of the limited liability in (8), banks choose $L(s_{t-1})$ by examining its effect on their profits only in the normal states $s_t \in \Omega^n$. Thus, if $R^L(s_{t-1}) > R^D(s_{t-1})$, then the bank would choose $L(s_{t-1}) = \infty$; and if $R^L(s_{t-1}) < R^D(s_{t-1})$, then $L(s_{t-1}) = 0$ would be the choice. In either case, the equilibrium condition $L(s_{t-1}) = q(s_{t-1}) > 0$ would be violated.

Given (12), utility maximization of the representative household implies that for all $s \in \Omega$,

$$w(s) = \frac{\gamma A(s)l(s)^{1-\alpha}}{1 - l(s)}. \quad (14)$$

Let us define $r_t \equiv R_t^F x_t$ and consider the profit maximization problem of firms (5). We restrict our parameter values so that firms choose to make the factor payments before production if and only if $s_t \in \Omega^n$,¹³ and the constraint in (5) binds for all $s_t \in \Omega$. Then, the first-order conditions with respect to l_t and κ_t can be rewritten as follows:

$$R^F(s)w(s) = \begin{cases} (1 - \alpha)A(s)l(s)^{-\alpha}, & s \in \Omega^n, \\ R^F(s)\theta A(s)l(s)^{-\alpha}, & s \in \Omega^b, \end{cases} \quad (15)$$

$$r(s) = \begin{cases} (\alpha - \nu)A(s)l(s)^{1-\alpha}, & s \in \Omega^n, \\ R^F(s)\frac{\theta}{1-\alpha}(\alpha - \nu)A(s)l(s)^{1-\alpha}, & s \in \Omega^b. \end{cases} \quad (16)$$

The first-order condition with respect to k_{t-1} is rewritten as¹⁴

$$\int_{s' \in \Omega^n} \frac{1}{c(s')} dF(s') R(s) q(s) = \int_{s' \in \Omega^n} \frac{1}{c(s')} \{r(s') + q(s')\} dF(s'), \quad s \in \Omega. \quad (17)$$

¹³This is the case if $\theta < \frac{1-\alpha}{R^F(s)}$ for all $s \in \Omega^n$.

¹⁴See the Appendix for the derivation.

This implies that the product $R(s)q(s)$ is independent of $s \in \Omega$.

The equilibrium short-term rates, $R^B(s)$ and $R^F(s)$, are determined as follows. First, consider a normal state $s \in \Omega^n$. The opportunity cost of a household for providing short-term deposits is unity, so that the equilibrium short-term deposit rate is also unity: $R^B(s) = 1$ for $s \in \Omega^n$. On the other hand, the short-term loan rate, $R^F(s)$, is generally greater than one, because of the enforcement constraint of banks. A bank meets the demand for short-term loans, $W(s) = w(s)l(s) > 0$, as long as $R^F(s) \geq R^B(s)$ and (9) are satisfied. It follows that the equilibrium short-term loan rate is given by

$$R^F(s) = \begin{cases} \frac{1}{1-\psi} \left[1 - \frac{R(s_-)e(s_-)}{w(s)l(s)} \right], & \text{if (9) binds,} \\ 1, & \text{otherwise,} \end{cases}$$

for all $s_- \in \Omega$ and $s \in \Omega^n$, where s_- denotes the state in the previous period. In what follows, we restrict our parameter values so that the enforcement constraint (9) always binds in normal states Ω^n and $R^F(s) > 1$ for all $s \in \Omega^n$.

In a crisis state, $s \in \Omega^b$, the short-term rates are not uniquely determined, as illustrated in Section 2. To pin down those rates, we assume that the deposit rate, $R^B(s)$, is determined at the level where households are indifferent to providing short-term deposits. Given the recovery rate $\xi^B(s)$, this implies that $R^B(s) = 1/\xi^B(s)$ for $s \in \Omega^b$. Similarly, we assume that the loan rate, $R^F(s)$, is determined so that banks are indifferent to the amount of loans $W(s)$. From (8), this rate is given by $R^F(s) = 1/(\xi^F(s)R^B(s))$.

To summarize, the short-term interest rates become

$$R^B(s) = \begin{cases} 1, & \text{for } s \in \Omega^n, \\ \frac{1}{\xi^B(s)}, & \text{for } s \in \Omega^b, \end{cases} \quad (18)$$

$$R^F(s) = \begin{cases} \frac{1}{1-\psi} \left[1 - \frac{R(s_-)e(s_-)}{w(s)l(s)} \right], & \text{for } s \in \Omega^n, \\ \frac{1}{\xi^F(s)\xi^B(s)}, & \text{for } s \in \Omega^b. \end{cases} \quad (19)$$

It follows from equation (19) that the product $R(s_-)e(s_-)$ is a constant that is independent of $s_- \in \Omega$.

Given that $\tilde{\xi}^F(s_t) = 1$ for $s_t \in \Omega^n$ and $R^L(s_{t-1}) = R^D(s_{t-1}) = R(s_{t-1})$, the realized profit of the bank is expressed as

$$\begin{aligned}
\pi_t^B &= \begin{cases} R^F(s_t)W(s_t) - W(s_t) + R(s_{t-1})e(s_{t-1}), & \text{for } s_t \in \Omega^n, \\ 0, & \text{for } s_t \in \Omega^b, \end{cases} \\
&= \begin{cases} \frac{\psi R^F(s_t)}{1 - (1 - \psi)R^F(s_t)} R(s_{t-1})e(s_{t-1}), & \text{for } s_t \in \Omega^n, \\ 0, & \text{for } s_t \in \Omega^b. \end{cases} \tag{20}
\end{aligned}$$

The expected profit of the bank can then be written as

$$E_{t-1}[\lambda_{t-1,t}\pi_t^B] = \left\{ \int_{s_t \in \Omega^n} \beta \frac{c(s_{t-1})}{c(s_t)} \frac{\psi R^F(s_t)}{1 - (1 - \psi)R^F(s_t)} dF(s_t) \right\} R(s_{t-1})e(s_{t-1}).$$

It follows that

$$E_{t-1}[\lambda_{t-1,t}\tilde{R}_t^E] = \left\{ \int_{s_t \in \Omega^n} \beta \frac{c(s_{t-1})}{c(s_t)} \frac{\psi R^F(s_t)}{1 - (1 - \psi)R^F(s_t)} dF(s_t) \right\} R(s_{t-1}). \tag{21}$$

The first-order conditions (3) of the household's utility maximization problem imply that the expected returns on bank equity and bank deposits should be equal in equilibrium. That is,

$$E_{t-1}[\lambda_{t-1,t}\tilde{\xi}_t^B R_{t-1}^D] = E_{t-1}[\lambda_{t-1,t}\tilde{R}_t^E] = 1. \tag{22}$$

The left-hand side of this equation is

$$E_{t-1}[\lambda_{t-1,t}\tilde{\xi}_t^B R_{t-1}^D] = \beta \left\{ \int_{s_t \in \Omega^n} \frac{c(s_{t-1})}{c(s_t)} dF(s_t) + \int_{s_t \in \Omega^b} \frac{c(s_{t-1})}{c(s_t)} \xi^B(s_t) dF(s_t) \right\} R(s_{t-1}). \tag{23}$$

It follows from (21) and (23) that the first equation in (22) can be rewritten as

$$\int_{s \in \Omega^n} \frac{1}{c(s)} \frac{\psi R^F(s)}{1 - (1 - \psi)R^F(s)} dF(s) = \int_{s \in \Omega^n} \frac{1}{c(s)} dF(s) + \int_{s \in \Omega^b} \frac{1}{c(s)} \xi^B(s) dF(s). \tag{24}$$

Note that if the probability of a liquidity crisis is zero, that is, $F(\Omega^b) = 0$, then $R^F(s) = 1$ for $s \in \Omega^n$, similar to $R^B(s)$. Otherwise, however, $R^F(s) > 1$ for $s \in \Omega^n$. This is because during a crisis, bank deposits pay a fraction ξ^B , but the value of the equity of the bank reduces to zero. To compensate for this difference, the bank equity needs to yield a higher return in normal states $s \in \Omega^n$ by making $R^F(s) > 1$.

The second equation in (22) becomes

$$\beta \left\{ \int_{s' \in \Omega^n} \frac{1}{c(s')} dF(s') + \int_{s' \in \Omega^b} \frac{1}{c(s')} \xi^B(s') dF(s') \right\} c(s)R(s) = 1, \quad s \in \Omega. \tag{25}$$

Thus, $c(s)R(s)$ is a constant that does not depend on $s \in \Omega$.

When $s_t \in \Omega^n$, both banks and firms are solvent so that the equilibrium recovery rates of their debt are unity; that is, $\tilde{\xi}^F(s_t) = \tilde{\xi}^B(s_t) = 1$ for $s_t \in \Omega^n$.

When $s_t \in \Omega^b$, both banks and firms go bankrupt, and they do not obtain short-term loans/deposits; that is, $W_t = 0$. Because firms make the factor payments after production and $\kappa_t = k_{t-1} = 1$ in equilibrium, equation (7) implies

$$\sigma_t = w_t l_t = \theta A_t l_t^{1-\alpha}.$$

It follows that the recovery rate of the loans to firms, ξ_t^F , is determined by

$$(1 - \theta)A_t l_t^{1-\alpha} + q_t - \xi_t^F R_{t-1} q_{t-1} = 0.$$

Here, remember that the product $R_{t-1} q_{t-1}$ is a constant, as shown in (17). It follows that

$$\xi^F(s) = \frac{1}{R(s_-)q(s_-)} \left\{ (1 - \theta)A(s)l(s)^{1-\alpha} + q(s) \right\}, \quad s \in \Omega^b. \quad (26)$$

Similarly, the recovery rate of bank deposits in the crisis, ξ_t^B , is determined by

$$\xi_t^F R_{t-1} L_{t-1} - \xi_t^B R_{t-1} [L_{t-1} - e_{t-1}] = 0.$$

Because $L_{t-1} = q_{t-1}$, this equation implies that

$$\xi^B(s) = \xi^F(s) \frac{R(s_-)q(s_-)}{R(s_-)q(s_-) - R(s_-)e(s_-)}, \quad s \in \Omega^b. \quad (27)$$

Here, note that $R(s_-)q(s_-)$ and $R(s_-)e(s_-)$ are again independent of $s_- \in \Omega$. Note also that (27) implies that $\xi^B(s) > \xi^F(s)$ for all $s \in \Omega^b$. The model parameters must be such that $\xi^B(s)$ defined in (27) is less than one.

When we consider the sunspot shock economy, $F(s = b) = \varepsilon$ is exogenously given. On the other hand, when we consider the fundamental shock economy, the threshold value \underline{s} is determined endogenously by the break-even condition as follows:

$$\begin{aligned} & A(\underline{s})l(\underline{s})^{1-\alpha} - R^F(\underline{s})w(\underline{s})l(\underline{s}) + q(\underline{s}) \\ & - \frac{1}{\int_{\underline{s}}^{\infty} c(s)^{-1} dF(s)} \int_{\underline{s}}^{\infty} c(s)^{-1} \{r(s) + q(s)\} dF(s) = 0. \end{aligned} \quad (28)$$

If s_t is below this threshold, all firms go bankrupt.

A competitive equilibrium is given by a collection of (positive) functions, namely, $\{c(s), l(s), w(s), e(s), r(s), q(s), R(s), R^B(s), R^F(s), \xi^F(s), \xi^B(s), \underline{s}\}$ that satisfies (12), (14), (15), (16), (17), (18), (19), (24), (25), (26), (27), and (28). (For the sunspot shock economy, remove \underline{s} from the definition of the equilibrium.)

4 Equilibrium in the sunspot shock economy

In this section, we consider the sunspot shock economy. There are two states in this economy, n and b , and $\Omega = \{n, b\}$, $\Omega^n = \{n\}$, $\Omega^b = \{b\}$, $\Pr(s_t = n) = 1 - \varepsilon$, and $\Pr(s_t = b) = \varepsilon$. The level of productivity is constant, that is, $A(s) = A$ for all $s \in \Omega$. Because the state of the economy is represented by a sunspot variable, the crisis is caused purely by self-fulfilling beliefs. The value of each variable in states n and b is denoted by subscripts n and b , respectively. The set of equilibrium conditions for the sunspot economy are given in the Appendix.

When $s_t = n$, firms and banks born in period $t - 1$ are solvent in period t . These firms obtain short-term loans $W_n = w_n l_n$, hire labor $l_t = l_n$, use capital $\kappa_t = k_{t-1} = 1$, and produce output $y_t = Al_n^{1-\alpha} = c_n$. Because of the banks' enforcement constraint, the short-term rate of loans, R_n^F , is greater than unity. This implies that the level of output is lower than the first-best level even in the normal state. The firms' profit is given by

$$\pi_t^F = \pi_n^F \equiv Al_n^{1-\alpha} + q_n - R(s_-)q(s_-) - R_n^F W_n > 0,$$

where s_- is the state in the previous period, and $R(s_-)q(s_-)$ is independent of s_- , as discussed in the previous section (see equation (17)). The firms' revenue consists of sales of output and capital: $Al_n^{1-\alpha} + q_t$. The cost $R_{t-1}q_{t-1} + R_t^F W_t$ represents the repayment of the inter-period loan q_{t-1} and the intra-period loan W_t . The former is used to purchase capital $k_{t-1} = 1$ in period $t - 1$; the latter is used to pay the wage bill, $W_t = w_t l_t$, in period t .

Firms obtain those funds from banks. The banks' profit in the normal state is

$$\pi_t^B = \pi_n^B \equiv R(s_-)q(s_-) + R_n^F W_n - R_n^B W_n - R(s_-)(q(s_-) - e(s_-)) > 0,$$

where $R(s_-)e(s_-)$ is also independent of s_- (see equation (19)). Here, $R(s_-)q(s_-) + R_n^F W_n$ is the repayment from the firms and $R_n^B W_n + R(s_-)(q(s_-) - e(s_-))$ is the payment to depositors.

Notice that in period t , firms and banks are indebted with inter-period loans, $R_{t-1}q_{t-1}$ and $R_{t-1}(q_{t-1} - e_{t-1})$, respectively, that constitute their fixed costs. In the normal state, banks and firms earn positive profits because the short-term rates, R_t^F and R_t^B , are sufficiently low. Otherwise, they would default and a crisis would occur.

Now consider the crisis state, $s_t = b$. In this state, everyone believes that $R_t^F = R_b^F \equiv 1/(\xi^F \xi^B)$ and $R_t^B = R_b^B \equiv 1/\xi^B$, where $\xi^F < \xi^B < 1$ are defined in (26) and (27), respectively. Later, we verify that these beliefs are rational. We assume parameter values such that

$$\max_{l \geq 0} \left\{ Al^{1-\alpha} + q_b - R(s_-)q(s_-) - R_b^F w_b l \right\} < 0, \quad (29)$$

$$\xi^F R(s_-)q(s_-) - R(s_-)[q(s_-) - e(s_-)] < 0, \quad (30)$$

where s_- denotes the state in the previous period. These assumptions guarantee that firms and banks go bankrupt when $s_t = b$. That is, Condition (29) implies that, given the short-term loan rate R_b^F and inter-temporal debt $R(s_-)q(s_-)$, it is impossible for firms to make a positive profit. When firms go bankrupt, banks can recover only a fraction ξ^F of their inter-temporal loans $R(s_-)q(s_-)$. Condition (30) then guarantees that banks also become insolvent when $s_t = b$.

With $R_t^F = R_b^F$ and $R_t^B = R_b^B$, firms' demand for intra-temporal loans is zero; and banks and households are indifferent about the amount of intra-temporal loans and deposits. Thus, markets are cleared with these prices.

The sunspot shock economy exhibits equilibrium fluctuations in the following manner. In the normal state that occurs with probability $1 - \varepsilon$, the short-term interest rates are low, short-term funds $W_n = w_n l_n$ flow from households to banks and from banks to firms, and the level of economic activity is high, that is, $y_n = Al_n^{1-\alpha}$. A liquidity crisis occurs with probability ε , where short-term rates rise to R_b^F and R_b^B , the supply of short-term liquidity diminishes, that is, $W_b = 0$, and market activity is depressed.

Figure 1 numerically illustrates what happens during a liquidity crisis in the sunspot shock economy. The parameter values are chosen so that $\varepsilon = 0.01$, $\beta = 0.95$, $\nu = 0.098$, $\alpha = 0.3$, $A = 1$, $\gamma = 1.63$, $\theta = 0.4$, and $\psi = 0.1$. Thus, a crisis occurs with probability 0.01 in each period. The figure plots the time paths of output (c), interest rates (R , R^F , R^B), short-term loans (W), profits (π^F , π^B), and the price of capital (q) in the case where a liquidity crisis occurs in period 0, that is, $s_t = n$ for $t \neq 0$ and $s_0 = b$.

5 Equilibrium in the fundamental shock economy

In this section, we consider the fundamental shock economy, where a crisis occurs owing to a productivity shock. The state of nature in each period denotes the level of aggregate productivity, that is, $A(s_t) = s_t$. The productivity shock s_t is i.i.d. across periods with the probability distribution function $F(s_t)$. The state space $\Omega = [0, s_{\max}]$ is divided into $\Omega^b = [0, \underline{s})$ and $\Omega^n = [\underline{s}, s_{\max}]$, where the threshold value, \underline{s} , is determined endogenously. We continue to restrict our attention to the case where the net worth constraint (9) binds for all $s_t \in \Omega^n$.

Here, a crisis occurs only when the productivity is too low for firms and banks to earn positive profits. When $s \in \Omega^n = [\underline{s}, s_{\max}]$, firms earn positive profits as follows:

$$\pi^F(s) = A(s)l(s)^{1-\alpha} + q(s) - R(s_-)q(s_-) - R^F(s)w(s)l(s) \geq 0.$$

The threshold value, \underline{s} , is determined by $\pi^F(\underline{s}) = 0$.

When the productivity level falls below the threshold level, that is, $s < \underline{s}$, a liquidity crisis occurs. The events during the crisis are similar to those in the sunspot economy: firms and banks go bankrupt; the supply of short-term loans evaporates, $W(s) = W_b = 0$; the short-term interest rates rise sharply, $R^F(s) = R_b^F \equiv 1/(\xi^F \xi^B)$ and $R^B(s) = R_b^B \equiv 1/\xi^B$; and the level of output declines.

Now consider a numerical example, where the parameter values are set as $\beta = 0.95$, $\nu = 0.098$, $\alpha = 0.3$, $\gamma = 1.63$, $\theta = 0.4$, and $\psi = 0.1$. The productivity shock $\ln(s)$ is assumed to follow a normal distribution with mean 0 and standard deviation 0.01. Then, as shown in Table 1, $\underline{s} = 0.98$, and $F(\underline{s}) = 0.0094$. Thus, on average, a liquidity crisis occurs about once in a hundred years. The equilibrium dynamics is illustrated in Figure 2, where the horizontal axis for each panel is the time index. In the simulation, the productivity level, $A_t = s_t$, is realized as shown in the top-left panel. Here, s_t is greater than \underline{s} , except for $t = 0$. Thus, a liquidity crisis occurs (only) in period 0. The figure illustrates the key features of the crisis discussed above (high interest rates and low loans and output). In addition, notice that a liquidity crisis works as a magnifying mechanism for productivity shocks. Indeed, the productivity level declines only slightly from period -1 to period 0. Nevertheless, such a small decline in the productivity level results in a huge reduction in economic activity.

6 Policy analysis

So far, we have restricted our attention to the case without government intervention. The most typical form of government intervention during a financial crisis is probably to subsidize banks in some way. As an example, in this section, we examine the effects of a policy that guarantees bank deposits. Specifically, we suppose the following:

- The government gives a subsidy to banks if and only if $s \in \Omega^b$. The amount of the subsidy is determined such that in equilibrium, $\tilde{\xi}^B(s) = 1$ for both $s \in \Omega^n$ and $s \in \Omega^b$, and the return on the bank equity is zero when $s \in \Omega^b$;
- Firms do not receive any subsidy from the government; and
- The fund for the subsidy is raised through lump-sum taxes on households.

Note that the government does not save firms or holders of bank equity. Here, it only saves depositors.

When deposits are guaranteed, (24), (25), and (27) should be replaced by the following:

$$\begin{aligned} \int_{s \in \Omega^n} \frac{1}{c(s)} \frac{\psi R^F(s)}{1 - (1 - \psi)R^F(s)} dF(s) &= \int_{s \in \Omega} \frac{1}{c(s)} dF(s), \\ \beta \left\{ \int_{s' \in \Omega} \frac{1}{c(s')} dF(s') \right\} c(s)R(s) &= 1, \quad s \in \Omega, \\ \xi^B(s) &= 1, \quad s \in \Omega. \end{aligned}$$

6.1 Policy intervention in the sunspot shock economy

Remember that the set of parameters for the sunspot shock economy are $\{\beta, \nu, \alpha, A, \gamma, \theta, \psi, \varepsilon\}$. Given these values, a sunspot equilibrium may or may not exist. Thus, given the values of $\{\beta, \nu, \alpha, A, \gamma, \theta, \psi\}$ and a policy regime, let $B^\varepsilon \subset [0, 1]$ denote the set of values of crisis probabilities ε , such that a sunspot equilibrium exists. It is numerically confirmed that B^ε takes the form of an interval $[0, \bar{\varepsilon}]$, where $\bar{\varepsilon}$ is the upper bound of the crisis probability. We examine the effect of the deposit-guaranteeing policy on the sunspot crisis by considering how it affects $\bar{\varepsilon}$.

Figure 3 plots $\bar{\varepsilon}$ against θ , where the other parameter values are fixed at their benchmark values given in Section 4. The solid line in the figure represents $\bar{\varepsilon}$ under the laissez-faire

policy, whereas the dashed line corresponds to the deposit guarantee policy. Figure 4 plots $\bar{\varepsilon}$ under the two policies as a function of ψ , that is, the strength of the banks' moral hazard. Under the laissez-faire policy, the set of ε s consistent with the sunspot equilibrium is large: $\bar{\varepsilon} = 1$ for most values considered for θ and ψ . As illustrated in these figures, the deposit guarantee policy is very effective in eliminating the sunspot crisis, because $\bar{\varepsilon} = 0$ for all values of θ and ψ considered here.

Under the laissez-faire policy, if a crisis occurs, the short-term rates would rise to $R_b^B = 1/\xi^B$ and $R_b^F = 1/(\xi^B \xi^F)$. On the other hand, if bank deposits are guaranteed, a crisis would not affect the short-term rate on bank deposits, that is, $R_b^B = 1$. As a result, the interest rate on short-term loans during a crisis would be $R_b^F = 1/\xi^F$. Thus, R_b^F is smaller under the deposit guarantee policy than under the laissez-faire policy. This is why guaranteeing bank deposits reduces the likelihood of sunspot crises.

6.2 Policy intervention in the fundamental shock economy

Next, we consider how the fundamental crisis is affected by the deposit guarantee policy. Table 1 shows the value of \underline{s} under both the laissez-faire and deposit guarantee policies for the parameter values given in Section 5. Guaranteeing bank deposits increases \underline{s} from 0.9768 to 0.9796, and the probability of a fundamental crisis rises from about 1 to 2 percent. Thus, such a policy doubles the likelihood of a fundamental crisis from about once in a hundred years to about once in fifty years. Therefore, the economy becomes more susceptible to financial crises if the government commits to protect those who lend to the banks.

There is a simple intuition behind this result. If the government is expected to guarantee bank deposits when $s \in \Omega^b$, the expected return on bank deposits goes up, because $\xi^B(s) = 1$ even when $s \in \Omega^b$. The higher return on deposits tends to reduce the supply of bank equity. This tightens the moral hazard constraint (9) and increases the short-term interest rate on corporate loans, $R^F(s)$. Higher $R^F(s)$, in turn, squeezes the firms' profits, leading to an increase in the threshold value of productivity \underline{s} .

This result can also be interpreted as the "overleverage" induced by the government's bailout policy, as emphasized, for instance, by Bianchi (2012) and Keister (2012). With guaranteed deposits, banks pursue higher leverage that reduces firms' profits and increases

the risk of financial crisis. So, as far as the fundamental crisis is concerned, the government's commitment to guarantee bank deposits strictly worsens social welfare.

The results in this section illustrate the importance of distinguishing the type of crisis in order to design effective policy interventions. Taking the deposit guarantee policy as an example, it is shown that whereas this policy is effective in reducing the likelihood of sunspot crises; it has the side-effect of increasing the probability of a fundamental crisis.

7 Conclusion

We have proposed a new mechanism of how systemic financial crises occur, based on debt overhang in short-term loans. A crisis can be caused either by self-fulfilling beliefs or by a fundamental shock to the economy. During the crisis, the supply of short-term loans drops sharply; the short-term interest rate rises; the labor wedge deteriorates; and production activities are depressed. Our model roughly captures some of the key features observed during actual financial crises.

We have also examined the effects of guaranteeing bank deposits during a crisis. Such a policy has the following type of tradeoffs. On the one hand, it reduces the possibility of self-fulfilling crises, but on the other, it raises the probability of fundamental crises. The overall welfare effect of such a policy would depend on the probabilities of sunspot and fundamental crises. Ideally, policy intervention should be contingent on the type of crisis.

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A Appendix

A.1 Proof of equation (13)

Consider a bank born in period $t - 1$ and an arbitrary state in period $t - 1$, $s_{t-1} = s_- \in \Omega$.

We write s_t as $s \in \Omega$. Given $(e_{t-1}, L_{t-1}) = (e, L)$, define the following:

$$\begin{aligned}\Gamma(e, L, s, s_-) &\equiv \{0\} \cup \{W \geq 0 : \xi^F(s)[R^L(s_-)L + (1 - \psi)R^F(s)W] \geq R^B(s)W(s) + R^D(s_-)(L - e)\}, \\ \pi^B(e, L, W, s, s_-) &\equiv \max \{\xi^F(s)(R^L(s_-)L + R^F(s)W) - R^B(s)W(s) - R^D(s_-)(L - e), 0\}, \\ \Omega^{B+}(e, L, s_-) &\equiv \left\{ s \in \Omega : \max_{W \in \Gamma(e, L, s, s_-)} \pi^B(e, L, W, s, s_-) > 0 \right\}.\end{aligned}$$

Then, the maximand in (11) can be expressed as

$$\begin{aligned}\tilde{\pi}^B(e, L|s_-) &\equiv E \left[\lambda(s_-, s) \max_{W \in \Gamma(e, L, s, s_-)} \pi^B(e, L, W, s, s_-) \middle| s_- \right] \\ &= \int_{\Omega^{B+}(e, L, s_-)} \lambda(s_-, s) \max_{W \in \Gamma(e, L, s, s_-)} \pi^B(e, L, W, s, s_-) dF(s).\end{aligned}$$

Now, consider profit maximization with respect to L . Note first that in equilibrium, $\Omega^{B+}(e, L, s_-) = \Omega^n$. Thus, $\xi^F(s) = 1$ for all $s \in \Omega^{B+}(e, L, s_-)$. Second, on the boundary of $\Omega^{B+}(e, L, s_-)$, $\max_{W \in \Gamma(e, L, s, s_-)} \pi^B(e, L, W, s, s_-) = 0$. It then follows that

$$\frac{\partial \tilde{\pi}^B}{\partial L}(e, L|s_-) = [R^L(s_-) - R^D(s_-)] \int_{\Omega^n} \lambda(s_-, s) dF(s).$$

Because $\int_{\Omega^n} \lambda(s_-, s) dF > 0$, we must have $R^L(s_-) = R^D(s_-)$ for all s_- .

A.2 Derivation of equation (17)

Consider a firm born in period $t - 1$ and an arbitrary state in period $t - 1$, $s_{t-1} = s \in \Omega$. We use $s' \in \Omega$ to denote s_t . Given $k_{t-1} = k$, define the following:

$$\begin{aligned}\pi^F(k, s', s) &\equiv \max_{(\kappa, l, W) \geq 0} \left\{ A(s')\kappa^{\alpha-\nu}l^{1-\alpha} + q(s')k - R^L(s)q(s)k - R^F(s')W, 0 \right\}, \\ \text{s.t. } R^F(s')w(s')l + r(s')(\kappa - k) &\leq \begin{cases} R^F(s')W, & \text{if } W > 0, \\ R^F(s')\theta A(s')\kappa^{\alpha-\nu}l^{1-\alpha}, & \text{otherwise,} \end{cases}\end{aligned}$$

and

$$\Omega^{F+}(k, s) \equiv \{s' \in \Omega : \pi^F(k, s', s) > 0\}.$$

Then the maximand in (5) is written as

$$\tilde{\pi}^F(k|s) \equiv E \left[\lambda(s, s') \pi^F(k, s', s) \middle| s \right] = \int_{\Omega^{F^+}(k, s)} \lambda(s, s') \pi^F(k, s', s) dF(s').$$

Now, consider profit maximization with respect to k . Suppose that because $\pi^F(k, s', s) = 0$ on the boundary of $\Omega^{F^+}(k, s)$, the first-order condition with respect to k is written as

$$\frac{\partial \tilde{\pi}^F}{\partial k}(k|s) = \int_{\Omega^{F^+}(k, s)} \lambda(s, s') \frac{\partial \pi^F(k, s', s)}{\partial k} dF(s') = 0.$$

In equilibrium, $\Omega^{F^+}(k, s) = \Omega^n$ and firms make the factor payment before production in period t if and only if $s_t \in \Omega^n$. It follows that

$$\frac{\partial \pi^F(k, s', s)}{\partial k} = \begin{cases} [q(s') + r(s')] - R^L(s)q(s), & \text{for } s \in \Omega^n, \\ 0, & \text{for } s \in \Omega^b. \end{cases}$$

Using $\lambda(s, s') = \beta c(s)/c(s')$, the FOC with respect to k is rewritten as

$$0 = \int_{s' \in \Omega^n} \frac{\beta c(s)}{c(s')} \left\{ [q(s') + r(s')] - R^L(s)q(s) \right\} dF(s')$$

that leads to (17).

A.3 The equilibrium conditions for the sunspot economy

Consider the profit maximization problem (5) of a firm born in period $t - 1$. Given the state in that period, s_{t-1} , the first-order condition with respect to k leads to

$$r_n + q_n = R(s_{t-1})q(s_{t-1}).$$

Equation (24) implies that

$$(1 - \varepsilon) \frac{\psi R_n^F}{1 - (1 - \psi)R_n^F} = 1 - \varepsilon + \varepsilon \xi^B \frac{c_n}{c_b},$$

which is rewritten as

$$R_n^F = 1 + \frac{\psi \varepsilon \xi^B \frac{c_n}{c_b}}{1 - \varepsilon + (1 - \psi) \varepsilon \xi^B \frac{c_n}{c_b}}. \quad (31)$$

The firm's and bank's profit maximization and the household's utility maximization imply

$$R_n^F w_n = (1 - \alpha) A l_n^{-\alpha}, \quad (32)$$

$$w_b l_b = \theta A l_b^{1-\alpha}, \quad (33)$$

$$c_s = A l_s^{1-\alpha}, \quad s = n, b, \quad (34)$$

$$r_n = (\alpha - \nu) A l_n^{1-\alpha}, \quad (35)$$

$$R_n = \left[\beta \left\{ 1 - \varepsilon + \varepsilon \frac{c_n}{c_b} \xi^B \right\} \right]^{-1}, \quad (36)$$

$$R_b = \frac{c_n}{c_b} R_n, \quad (37)$$

$$q_n = \frac{1}{R_n} [r_n + q_n], \quad (38)$$

$$q_b = \frac{1}{R_b} [r_n + q_n], \quad (39)$$

$$w_s = \frac{\gamma A l_s^{1-\alpha}}{1 - l_s}, \quad s = n, b, \quad (40)$$

$$W_n = w_n l_n, \quad (41)$$

$$W_n = \frac{R(s_{t-1})e(s_{t-1})}{1 - (1 - \psi)R_n^F} = \frac{R_n e_n}{1 - (1 - \psi)R_n^F} = \frac{R_b e_b}{1 - (1 - \psi)R_n^F}. \quad (42)$$

By definition, the recovery rates during a crisis, ξ^F and ξ^B , are given by

$$\xi^F = \frac{(1 - \theta) A l_b^{1-\alpha} + q_b}{R_n q_n}, \quad (43)$$

$$\xi^B = \xi^F \frac{R_n q_n}{R_n q_n - R_n e_n}. \quad (44)$$

The solution to the system of equations (31)–(44) provides a candidate for the sunspot equilibrium, where a liquidity crisis occurs with probability ε . It is indeed an equilibrium if it satisfies some consistency conditions. First, the short-term loan rate in state b , $R_b^F = 1/(\xi^F \xi^b)$, must be sufficiently high, so that firms do not make a profit by borrowing short-term loans in that state. Second, both ξ^F and ξ^B are less than unity in (43)–(44). Third, firms have an incentive to make the factor payments before production in state n , that is, $\frac{1-\alpha}{R_n^F} > \theta$. Fourth, all variables are non-negative. If these four conditions are satisfied, the solution to (31)–(44) constitutes a sunspot equilibrium with the probability of a crisis equal to ε .

A.3.1 Deposit guarantee policy

Now suppose that bank deposits are guaranteed by the government. Because the profits of banks and firms are zero when $s = b$, their profit maximization problem is of the same form as in (5) and (11). Remember that the conditions for the laissez-faire equilibrium are given by (31)–(44). With $\tilde{\xi}^B(s) = 1$ for all $s \in \Omega$, conditions (31), (36), and (44) should be replaced by the following equations:

$$R_n^F = 1 + \frac{\psi \varepsilon \frac{c_n}{c_b}}{1 - \varepsilon + (1 - \psi) \varepsilon \frac{c_n}{c_b}}, \quad (45)$$

$$R_n = \left[\beta \left\{ 1 - \varepsilon + \varepsilon \frac{c_n}{c_b} \right\} \right]^{-1}, \quad (46)$$

$$\xi^B = 1. \quad (47)$$

Conditions (32)–(35), (37)–(43), and (45)–(47) provide a sunspot equilibrium with crisis probability ε if the consistency conditions discussed above are also satisfied.

Table 1: Threshold value \underline{s}

	\underline{s}	$F(\underline{s})$
(1) laissez faire	0.9768	0.94%
(2) bank bailout	0.9796	1.95%

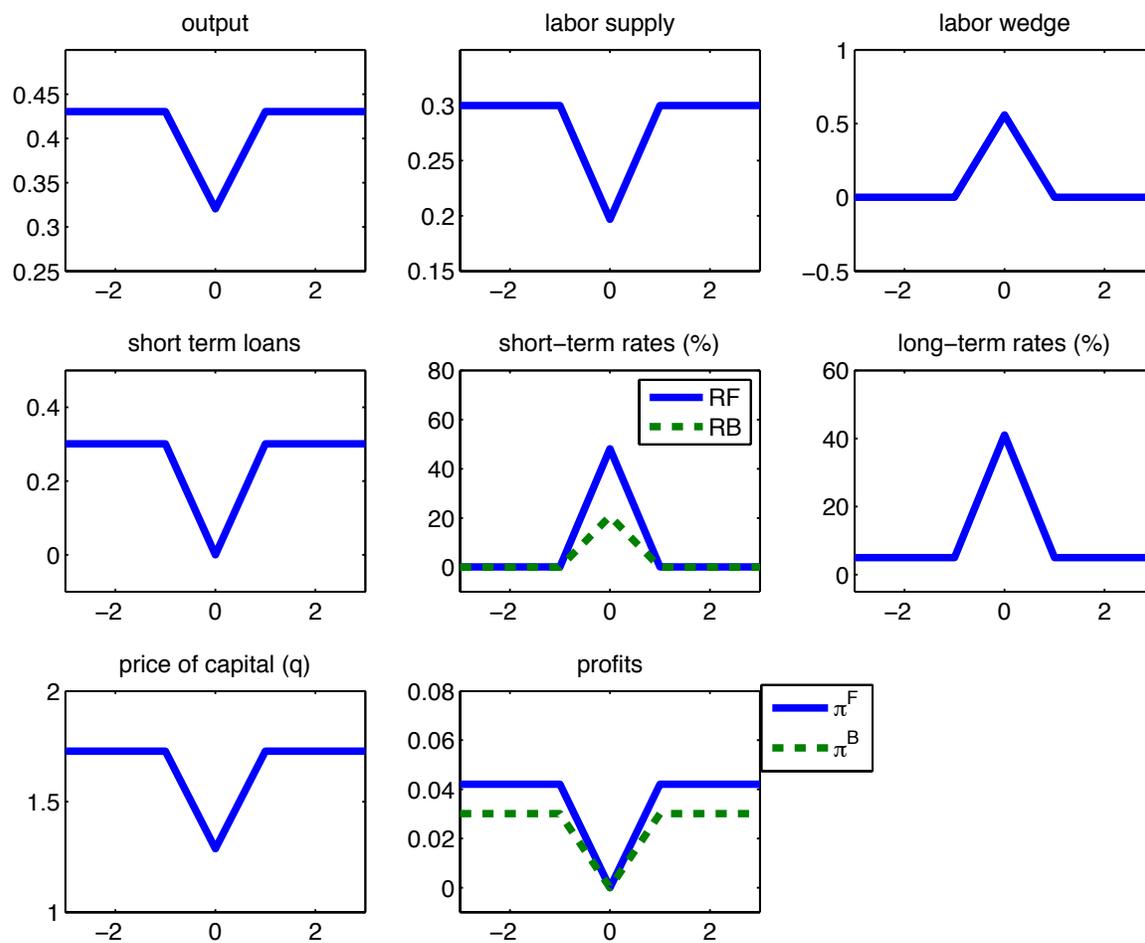


Figure 1: Liquidity crisis in the sunspot shock economy. The horizontal axis in each panel is the time period. A crisis occurs in period 0.

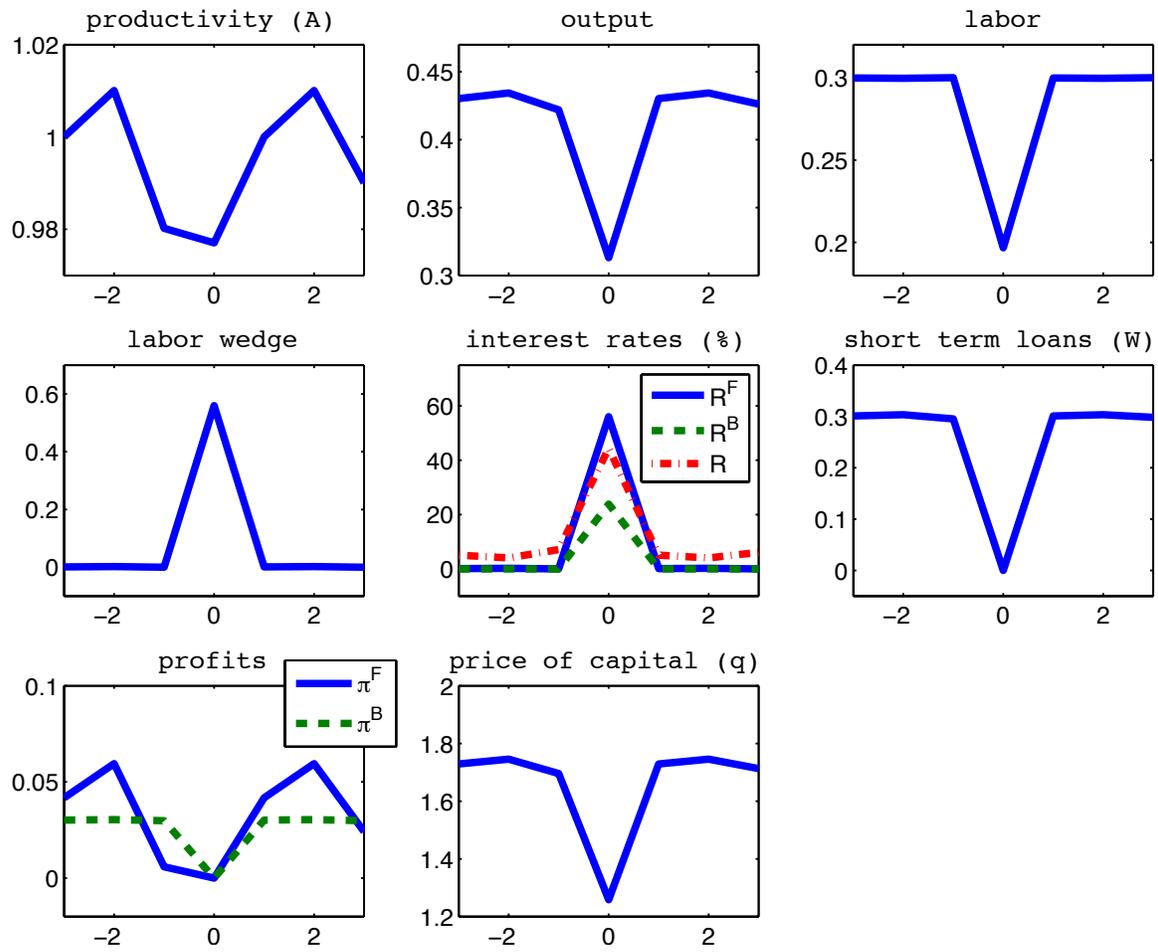


Figure 2: Liquidity crisis in the fundamental shock economy. The horizontal axis in each panel is the time period. A crisis occurs in period 0.

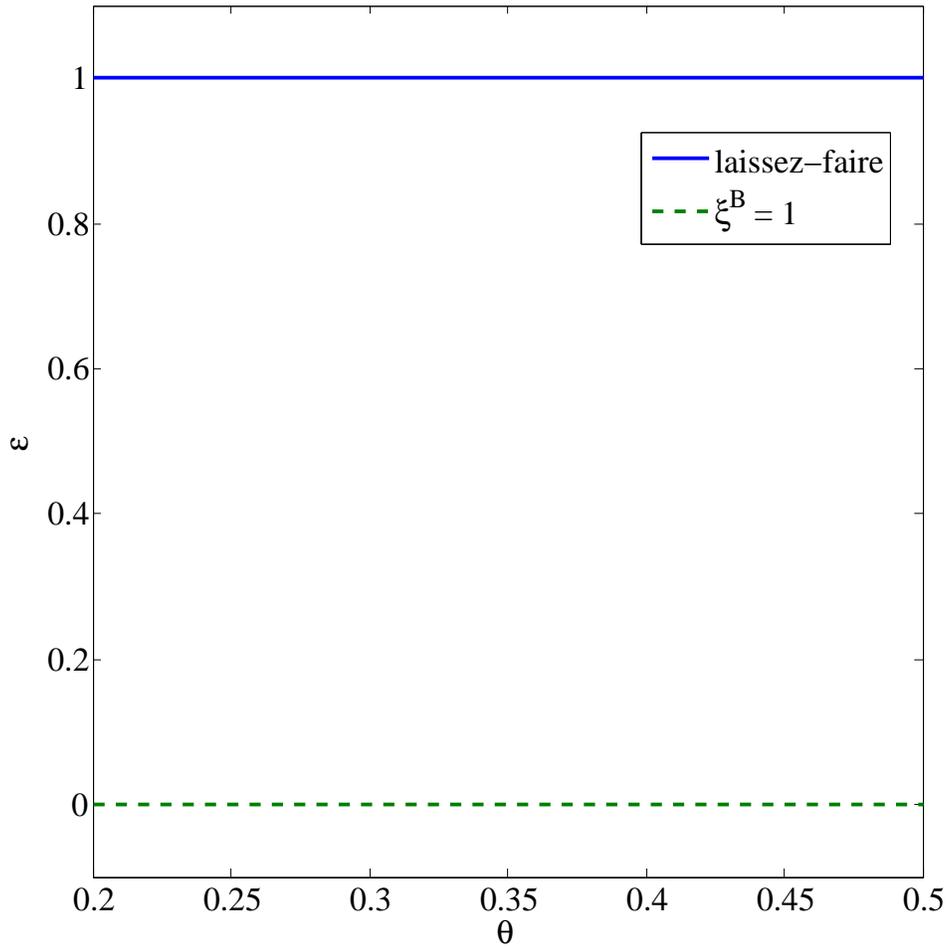


Figure 3: The upper bound of the probability of a crisis ($\bar{\varepsilon}$) in the class of sunspot equilibria for different values of θ that expresses the firms' ability to commit to make the factor payments. The solid line denotes the case without government intervention. The dashed line corresponds to the case where the government guarantees bank deposits so that $\xi^B = 1$.

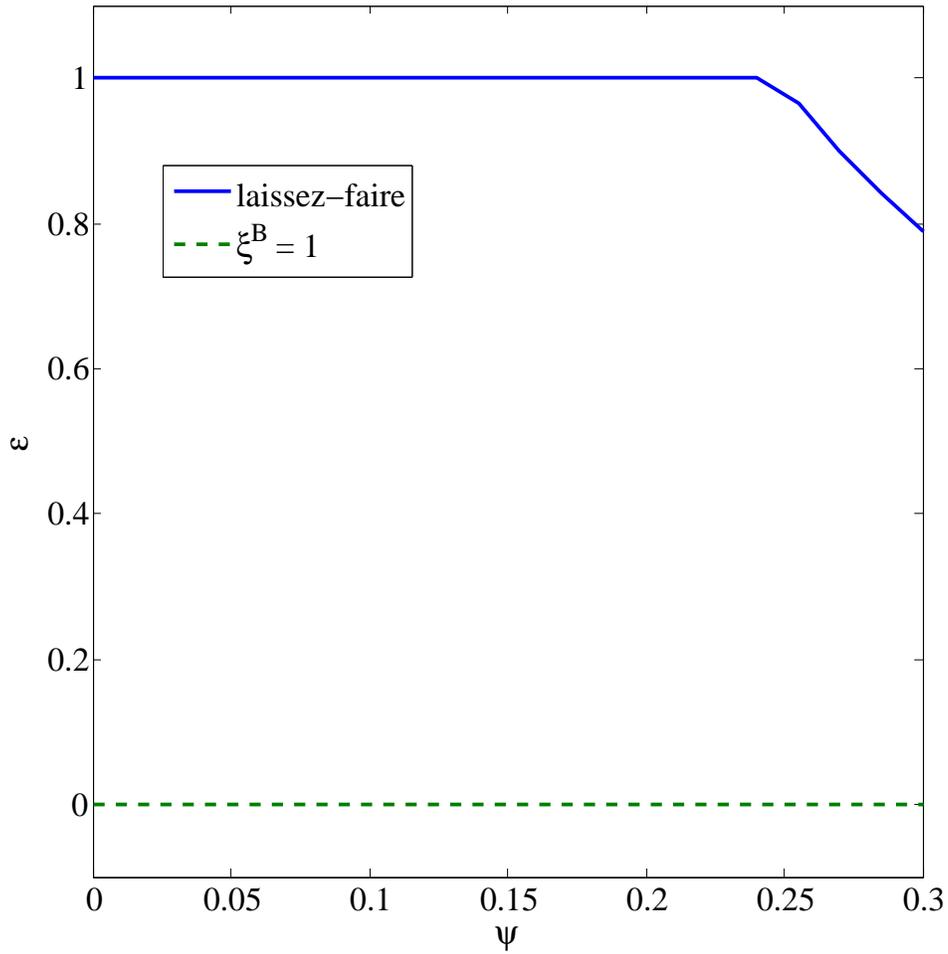


Figure 4: The upper bound of the probability of a crisis ($\bar{\varepsilon}$) in the class of sunspot equilibria for different values of the degree of banks' moral hazard (ψ). The solid line denotes the case without government intervention. The dashed line corresponds to the case where the government guarantees bank deposits so that $\xi^B = 1$.