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Liquidity, Trends and the Great Recession

Pablo A. Guerron-Quintana   Ryo Jinnai*

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Abstract

We study the impact that the liquidity crunch in 2008-2009 had on the U.S. economy's growth trend. To this end, we propose a model featuring endogenous growth á la Romer and a liquidity friction á la Kiyotaki-Moore. A key finding in our study is that liquidity declined around the demise of Lehman Brothers, which lead to the severe contraction in the economy. This liquidity shock was a tail event. Improving conditions in financial markets were crucial in the subsequent recovery. Had conditions remained at their worst level in 2008, output would have been 20 percent below its actual level in 2011.

1 Introduction

A few years into the recovery from the Great Recession, it is becoming clear that real GDP is failing to recover. Namely, although the economy is growing at pre-crisis growth rates, the crisis seems to have impinged a shift upon output. Figure 1 shows real GDP and its growth rate over the past decade. Without much effort, one can see that the economy is moving along a (new) trend that lies below the one prevailing in 2007.[1] It is also apparent that if the economy continues to display the dismal post-crisis growth rates (blue dashed line), it will not revert to the old trend.[2] Hence, this tepid recovery has spurred debate on whether the shift is permanent and if so what the long-term implications are for the economy.[3] In this paper, we tackle the issue of the long-run

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[1] More formally, the shift in the GDP trend is detected by the flexible estimation of trends with regime shifts recently advanced by Eo and Kim (2012). We thank Yunjong Eo for helping with the estimation using their approach.

[2] The forecast is built assuming that the economy will be growing at the average growth rate for the period: 2009Q2 - 2013Q2.

[3] This debate has received prominent attention in economic blogs like those maintained by John Cochrane, John Taylor, and Stephen Williamson. A more provocative argument that declares the end of growth in the U.S. has been advanced in Gordon (2012).

impact of the Great Recession by means of a structural model.

An emerging consensus among economic observers is that, to some degree, the Great Recession was exacerbated by a financial shock (Brunnermeier, Eisenbach, and Sannikov (2012), Christiano, Motto, and Rostagno (Forthcoming), and Stock and Watson (2012)). More precisely, the liquidity crunch following the collapse of Lehman Brothers has often been blamed for the depth and length of the recession and the subsequent sluggish recovery. But formalizing this view in RBC-style models is difficult for a number of reasons. First of all, shocks in this class of models exhibit exclusively short-run dynamics, i.e., the economy always reverts back to its pre-shock trend. A rather mechanical fix to this problem is assuming that productivity shock follows a unit root process. Such a shock can in principle "explain" a permanent shift in the trend line, but the evidence supporting this view seems to be mixed. Indeed, various measures of productivity published by the Bureau of Economic Analysis show a brief decline around 2009 but then returning to trend by 2010 (Fernald (2012) argues a similar point). Our skepticism is reinforced by Stock and Watson (2012)'s finding that productivity (as well as monetary policy and fiscal policy) shocks had a modest impact on the Great Recession. Moreover, this productivity-dominance view leaves an elephant in the room; that is, since productivity and financial market condition are completely orthogonal, the latter plays only a supporting role in the Great Recession drama at best.

In this paper, we argue that the main cause of this tension is the maintained assumption of exogenous productivity, and demonstrate that once we relax this assumption, the data strongly favors the view that the financial market condition is a main factor, not a supporting one, that drives very interesting macroeconomic dynamics during the Great Recession. As we will see below, measures of financial tightening have been improving in reality, which hints that the financial shock was most likely temporary. Our model is consistent with this observation as well, because in our novel endogenous growth model, a short-lived financial disturbance triggers a crisis with long-lasting implications.

We construct a model economy based on Romer (1990)'s framework of expanding input varieties. In the model, investment in research and development leads to the creation of new intermediate goods. A final goods producer takes these inputs to manufacture goods that are consumed and used for investment. Knowledge spillover sustains growth in the long run. The second key element in our model is a financial friction. Here, we follow Kiyotaki and Moore (2012)'s lead in assuming that financial frictions alter the liquidity of equity in the economy. More pointedly, shocks arising in the financial sector affect the resaleability of assets. In their formulation, a drop in liquidity reduces the availability of funds to finance new projects, leading to contraction in investment. In our model, this lack of funding leads to a low level of innovative activities, to weak knowledge spillover, and hence to (other things equal) a permanent shift in the economy's trend.

With our proposed model in hand, we read the recent history of U.S. economy. Specifically, we use data on economic activity including a measure of research and development to estimate the stochastic properties of the structural shocks in the model. The results from the estimation

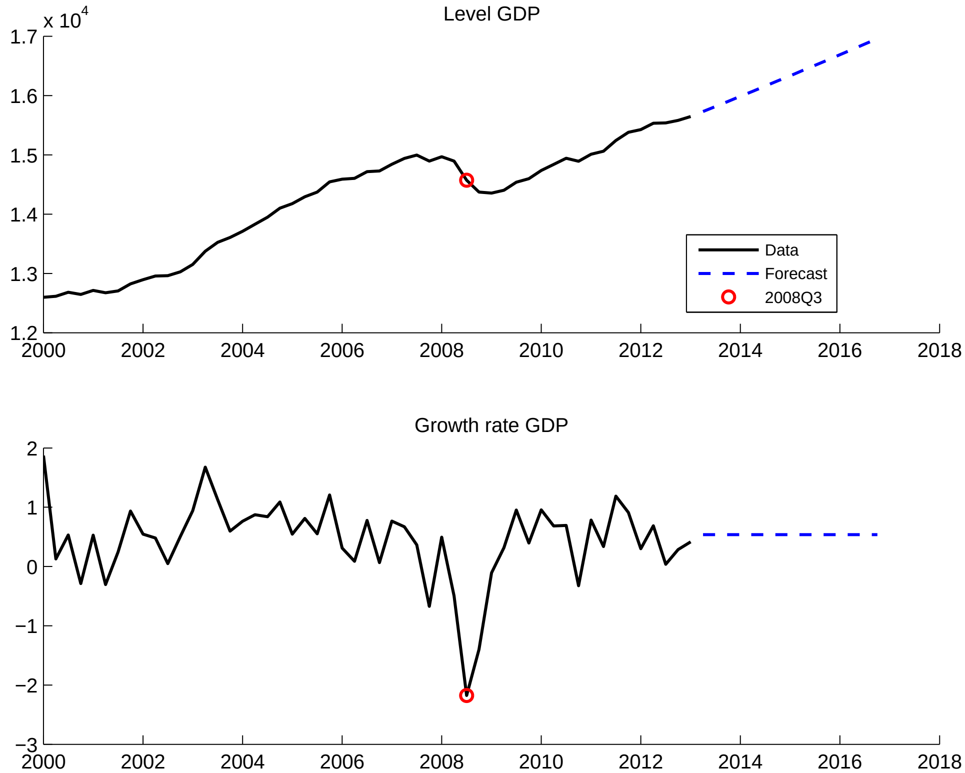

Figure 1: U.S. Real GDP

exercise provide a vivid description of the events before, during, and after the Great Recession. Chief among these findings is that our measure of liquidity reached its lowest level just after Lehman Brothers' demise in the fall of 2008. Interestingly, this measure reaches its highest value (meaning that assets are most liquid) around the same time as the peak of the credit boom estimated by Ivashina and Scharfstein (2010).[4] By relying on simulations of alternative recovery paths, we uncover that improvements in financial markets (measured by the degree of market liquidity) were critical in pushing the economy out of the recession. These results nicely square with Stock and Watson (2012)'s view that financial shocks were one of the key drivers of the recession (the other one being uncertainty shocks). It is assuring that their study and ours arrive to similar conclusions from different paths. That is, Stock and Watson rely on a dynamic factor model whereas we dissect the data using a structural macroeconometric approach.

We also read the U.S data through the lens of a standard RBC model augmented with our financial friction and exogenous non-stationary productivity. Three main messages emerge from the model. First, the estimated liquidity process points to favorable financial conditions around Lehman Brothers' demise. It is only by the second half of 2009 when liquidity became adverse. These accounts, however, are contradictory to the micro-evidence we show in the next section as

[4] They report that syndicated loans to corporations reached its highest value in billions of USD in the second quarter of 2007. This is also true if one looks at the total number of loans.

well as anecdotal reports of the crisis. Second, these adverse conditions have a little impact on the economy. But this implication is not in accordance with the emerging concensus that the financial shock played a vital role during the Great Recession. Finally, the productivity shock is critical. This is hardly a surprising message from the RBC model because it is (by construction) the only shock that can create the permanent shift in the trend from 2009 and beyond. But what seems to be counterfactual is the concentration of large negative producitivity shocks; in fact, we identify, by means of simulations, an unusually large negative productivity shock in 2008.Q3 as the single dominant factor that explains the Great Recession. In reality, none of the off-the-shelf candidate productivity shocks (commoditity prices, natural disasters, the weather, etc.) displays a discontinuity in that quarter to the extent that it permanently changes the trend that much. In contrast, our endogenous growth model attributes the permanent shift in the trend to severe but temporary credit crunch, which is supported by many empirical studies.

We also provide fresh evidence on the severity of the crisis but from the perspective of an endogenous growth model in which shocks may cause drifts in the economy's trend. Indeed, we find that the size of the financial shock around the Lehman episode was a tail event. Namely, based on the history of liquidity shocks, the Lehman shock was an event of probability less than 0.1 percent.

We would like to stress that, although the focal point of our discussion is the role of liquidity during and after the crisis, it does not mean that other aspects of the financial crisis (such as mortgage defaults and idiosyncratic risk at firm level) were unimportant. On the contrary, the crisis was a multidimensional problem of which liquidity was one of the key elements. In this respect, we view our work as complementary to those studies already in the literature. By the same token, our use of Romer (1990)'s endogenous growth model does not mean that our results crucially depend on this model's unique structure. We use this model primarily because it is parsimonious. Results similar to those discussed below should follow from other versions of, and more elaborated versions of, endogenous growth models too.

Our paper relates to several branches in macroeconomics. The first one comes from the literature on endogenous growth with seminal contributions by Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1997). Our analysis of the recent financial crisis brings us close to the literature on financial frictions in dynamic stochastic setups such as Bernanke, Gertler, and Gilchrist (1998), Jermann and Quadrini (2012), and Kiyotaki and Moore (1997), and more recently Ajello (2012), Del Negro, Eggertsson, Ferrero, and Kiyotaki (2011), and Kiyotaki and Moore (2012). The empirical treatment used in our paper relates to the extensive literature on the estimation of dynamic stochastic general equilibrium models (Fernández-Villaverde, Guerrón-Quintana, and Rubio-Ramírez (2010) and Guerron-Quintana (2010)). Finally, we borrow ideas from the unified treatment of business cycles and long-term dynamics in Comin and Gertler (2006), Comin, Gertler, and Santacreu (2008), and Queralto (2013).

The rest of the paper is organized as follows. The next section provides some financial indicators before, during, and after the 2008/09 recession. Section 3 outlines the model, derives equilibrium conditions, and discusses their implications. Our empirical strategy as well as the main results from our model are discussed in Section 4. Some discussion of our model-implied measure of liquidity is in Section 5. The last section provides some concluding remarks.

2 Some evidence on the liquidity crunch

Brunnermeier and Pedersen (2009) use margins for S&P 500 futures as a measure of liquidity. The higher the margin is the larger the amount of money an investor must maintain in a future contract. According to these authors, margins tend to increase during periods of liquidity crises. Indeed, they show that margins moved up in previous periods of illiquidity like in 1987 (Black Monday) or 1998 (Asian and LTCM crises). Figure 2 shows these margins for the last decade.[5] As one can see, the most recent crisis lead to a spike in the margins. At the peak in 2009, financiers required investors to keep 12 percent of the value of a future contract as capital requirement. Note, however, that this measure of liquidity indicates that financial conditions started to improve by 2011 and they seem to be back to more normal levels by the end of 2012. As we will see later on, our estimated measure of liquidity displays remarkably similar dynamics with the worst of the crisis happening in later 2008 and early 2009.

Figure 3 displays results from the survey of loan senior officer on lending practices published by the Federal Reserve Board. The survey goes back to 1990 so it is suitable for comparison between the recent crisis and those in 1990/91 and 2001.[6] The upper panel plots the net percentage of responders who answered that standards for commercial and industrial loans have tightened over the past three months in their banks. According to the survey, about 80 percent of loan officers reported tighter lending standards in the aftermath of Lehman's collapse. Small and large firms seem to have been affected equally by more stringent financing conditions. None of the previous two recessions saw a similar spike in this measure of tougher lending standards. The second panel in Figure 3 displays the time path of the net percentage of responders reporting an increasing gap between loan rates and the bank's cost of funding. The spike in spreads in 2008 shows that businesses (commercial and industrial; large and small) faced adverse financing conditions during the last recession.

Further evidence on harsh funding conditions comes from credit and corporate bond spreads. The upper panel of Figure 4 presents the corporate bond spreads (relative to 10-Year Treasury

[5] The margins are computed as the dollar margin divided by the product of the underlying S&P 500 index and the size of the contract ($250 in this case). Data for margins are taken from Chicago Mercantile Exchange's website (http://www.cmegroup.com/clearing/risk-management/historical-margins.html). We thank Ronel Elul for helping with computation.

[6] The survey asks senior loan officers about "changes in the standards and terms on bank loans to businesses and households over the past three months." The most recent survey (July 2013) included responses from officers at 73 domestic banks and 22 U.S. branches and agencies of foreign banking institutions.

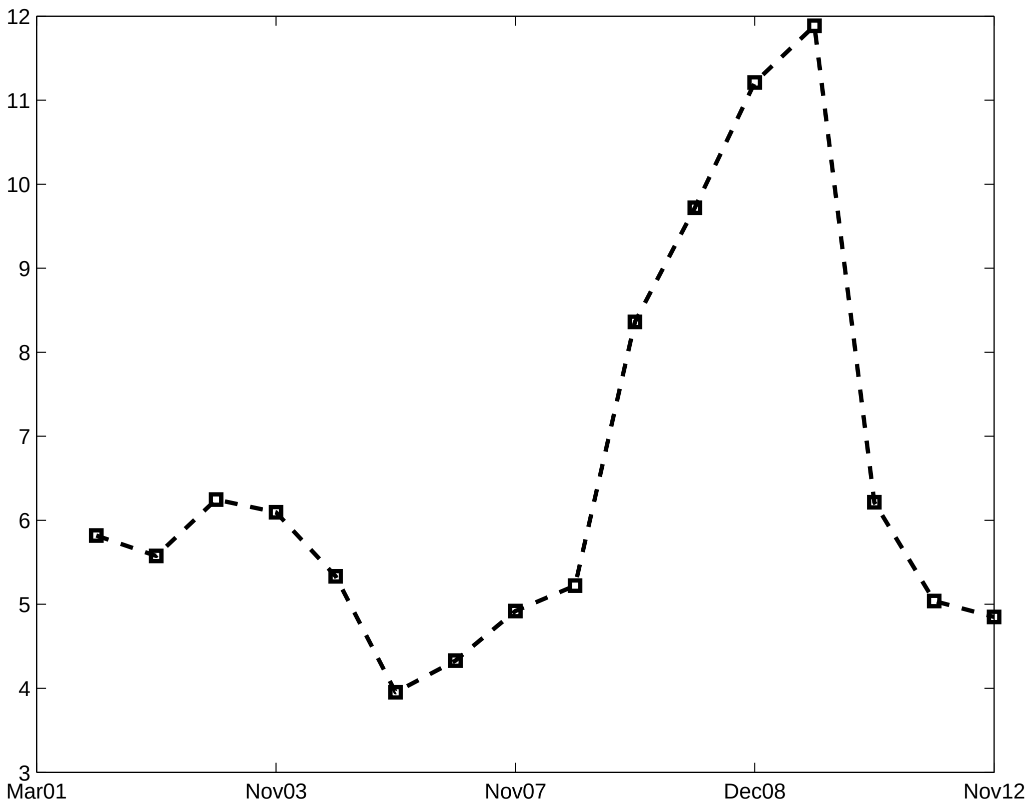

Figure 2: Margins for S&P 500 futures

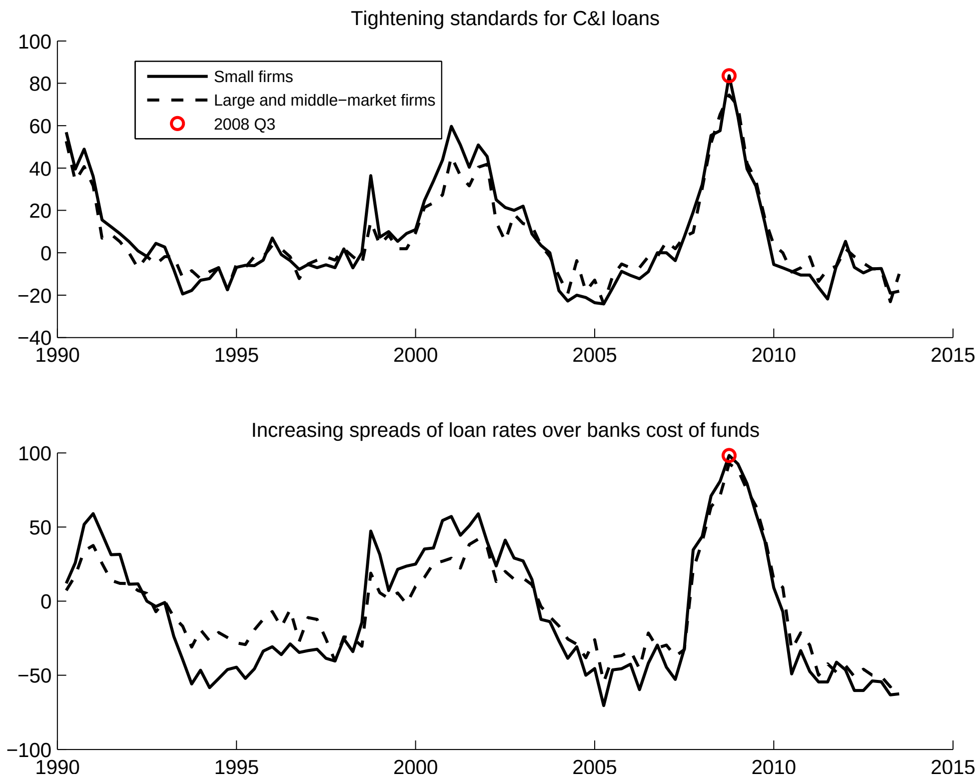

Figure 3: Senior Officer Opinion Survey on Bank Lending Practices

bond yields at constant maturity). The bottom panel displays credit spreads computed as the difference between yields on 3-month (financial or asset-backed) commercial paper and 3-month constant maturity T-Bill. These two measures point to higher spreads during the crisis. Investors moved away from commercial paper and into (the more liquid) Treasuries.

Another tangible evidence of credit crunch is reported by Ivashina and Scharfstein (2010). They analyze syndicated loans, the main vehicle through which banks lend to large corporations. This market is a part of the "shadow banking" system because non-bank financial institutions are often involved to share a loan originated by a lead bank. Ivashina and Scharfstein (2010) report that the total volume of new syndicated loans fell by 47 percent during the peak period of the financial crisis (fourth quarter of 2008) relative to the prior quarter, and by 79 percent relative to the peak of the credit boom (second quarter of 2007). While commercial and industrial loans reported on the aggregate balance sheet of the U.S. banking sector sharply rose in the four weeks after the failure of Lehman Brothers (Chari, Christiano, and Kehoe (2008)), Ivashina and Scharfstein argue that this increase was actually consistent with the falling of syndicated lending because it was driven by an increase in drawdowns by corporate borrowers on existing credit lines, i.e., prior commitments by banks to lend to corporations at pre-specified rates and up to pre-specified limits.

The dramatic shrinkage of lending activities was not solely an efficient response to a change in

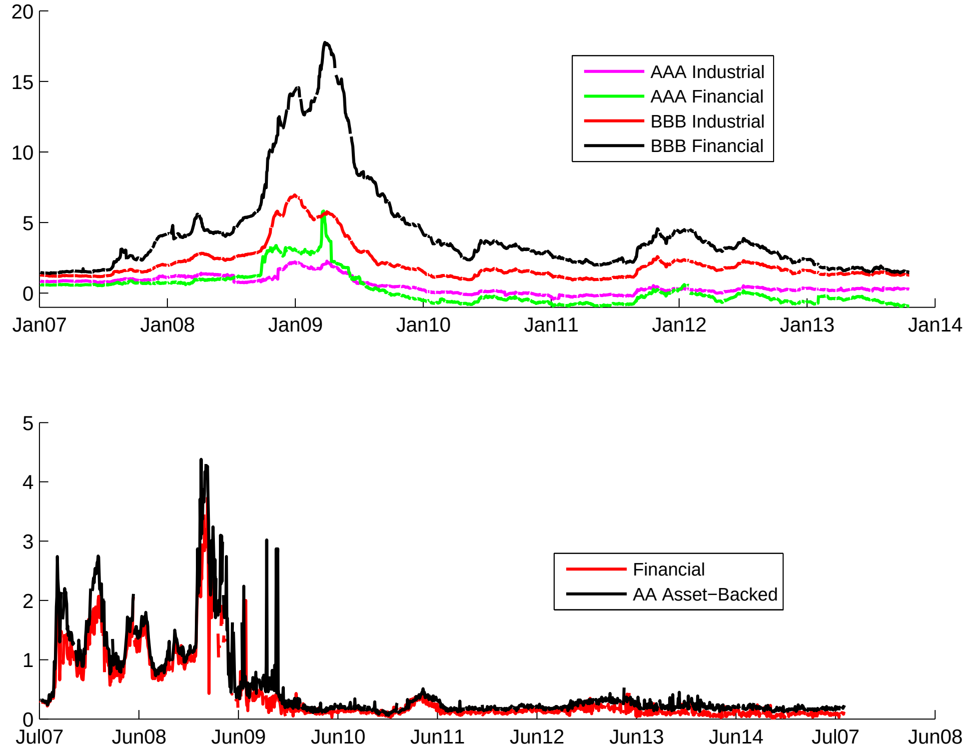

Figure 4: Credit Spreads and Corporate Bond Spreads

liquidity demands, but it had real impacts on firms' behaviors, according to Campello, Graham, and Harvey (2010). Surveying 1,050 chief financial officers (CFOs) in 39 countries in the midst of the 2008 financial crisis, they find that, after controlling other firm characteristics using a matching estimator, financially constrained firms planned to cut more investment, technology, marketing, and employment relative to financially unconstrained firms; were forced to burn a sizable portion of their cash savings and to cut more deeply their planned dividend distributions; restrict their pursuit of attractive projects; are forced to cancel valuable investments; and display a higher propensity to sell off assets. Interestingly, they also find that financially constrained firms accelerate the withdrawal of funds from their outstanding line of credit, which is consistent with Ivashina and Scharfstein (2010).

Almeida, Campello, Laranjeira, and Weisbenner (2009) support the view that the 2008 financial crisis was largely driven by an exogenous reduction in credit. These authors compare firms that needed to refinance a substantial fraction of their long-term debt over the year following August 2007 with firms that do not have a large refinancing in the period following the start of the financial crisis. After controlling other firm characteristics using a matching estimator, they find that investment of firms in the first group fell by one-third while investment in the second group showed no investment reduction. Duchin, Ozbas, and Sensoy (2010) reach a similar conclusion by comparing firms that were carrying more cash prior to the onset of the crisis with firms that were

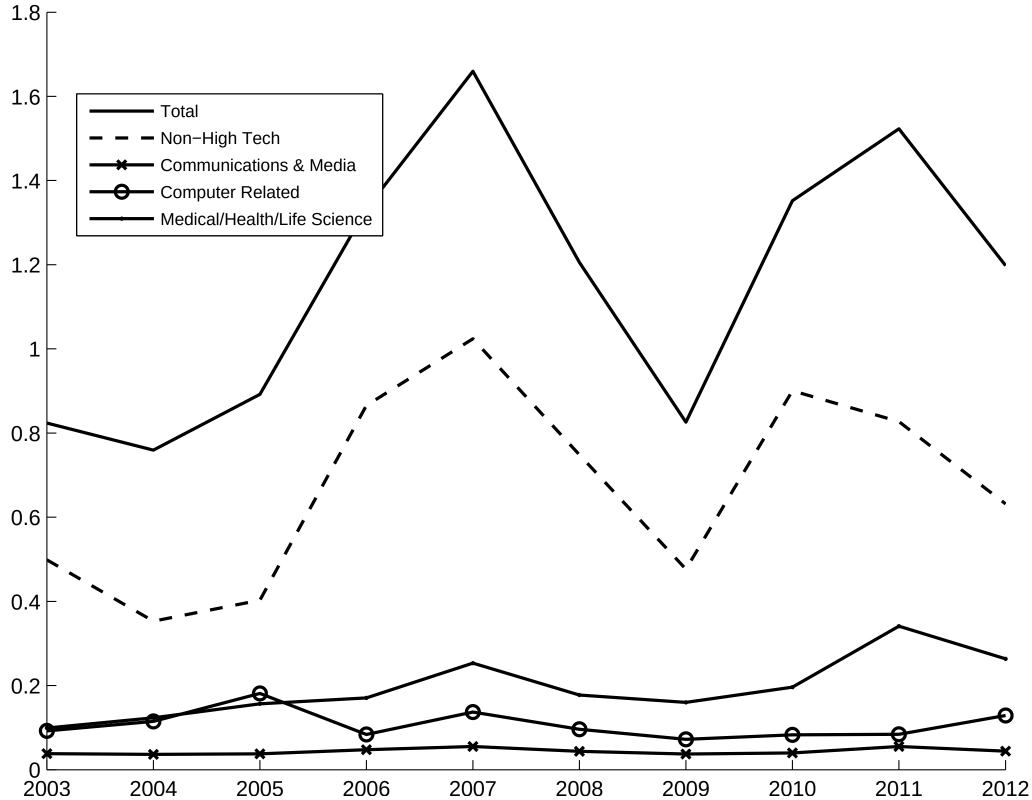

Figure 5: Private equity investment as a fraction of GDP

carrying less cash. Using a difference-in-difference approach, they find that firms with less liquid assets before the financial crisis exhibited a larger reduction in investment.

Our final evidence on the liquidity crunch comes from private equity investment data. Figure 5 plots total private equity investment as well as some of its components expressed as fractions of GDP.[7] To the extent that startups and entrepreneurs rely on private funding to finance their operations, the collapse of private equity investment (either total or its components) in 2009 indicates that otherwise profitable projects may have had a hard time securing financing during the Great Recession. In other words, the financial headwinds in 2008-2009 effectively reduced the liquidity of equity. In the next section, we develop a model that incorporates changes in liquidity and allows for these fluctuations to affect the growth path of the economy.[8]

In sum, our measures point to worsening financial conditions in the aftermath of Lehman Brothers' failure. However, as this large disturbance worked its way through the economy, the financial outlook has improved, albeit at a slow pace.

[7]A plot in levels (rather than ratios) reveals a similar contraction in 2008-2009. The data are retrieved from Thomson One Analytics.

[8]There is a plethora of anecdotal evidence on the liquidity crunch. For example, in a recent Wall Street Journal article (10/06/2013) it is reported that Berkshire Hathaway invested up to $25 billion during the crisis (when credit markets were tight) in big corporations needing funding such as Mars, Goldman Sachs, General Electric, and Dow Chemical.

3 Model

We describe our baseline model in two steps. First, we flesh out the household side where the financial friction takes place.[9] Then we switch to the endogenous growth part of the model which is primarily concentrated on the firm side of the economy.

3.1 Household

The economy is populated by a continuum of households, with measure one. Each household has a unit measure of members. At the beginning of the period, all members of a household are identical and share the household's assets. During the period, the members are separated from each other, and each member receives a shock that determines the role of the member in the period. A member will be an entrepreneur with probability $\sigma_e \in [0, 1]$ and a worker with probability $\sigma_w \in [0, 1]$. They satisfy $\sigma_e + \sigma_w = 1$. These shocks are *iid* among the members and across time.

A period is divided into five stages: household's decisions, production, innovation (R&D), consumption, and investment. In the stage of household's decision, all members of a household are together to pool their assets: k_t units of physical capital and n_t units of equities. An equity corresponds to the ownership of a firm which is a monopolistic producer of a differentiated intermediate product. Aggregate shocks to exogenous state variables are realized. The capacity utilization rate u_t is decided, which is applied to all the capital the household possesses. Because all the members of the household are identical in this stage, the household evenly divides the assets among the members. The head of the household also gives contingency plans to each member, saying if one becomes an entrepreneur, he or she spends s_t units of consumption goods to product developments (R&D), consumes c_t^e units of consumption goods, and makes necessary trades in the capital market and the equity market so that he or she returns to the household with k_{t+1}^e units of capital and n_{t+1}^e units of equities, and if one becomes a worker, he or she supplies l_t units of labor, consumes c_t^w units of consumption goods, sets aside i_t units of consumption goods for the investment stage, and makes necessary trades in the capital market and the equity market so that he or she returns to the household with k_{t+1}^w units of capital and n_{t+1}^w units of equities. After receiving these instructions, the members go to the market and will remain separated from each other until the investment stage.

At the beginning of the production stage, each member receives the shock whose realization determines whether the individual is an entrepreneur or a worker. Competitive firms produce final consumption goods from capital service, labor service, and specialized intermediate goods. Monopolistic firms produce specialized intermediate goods from final consumption goods; in other words, the production is roundabout. After production, a worker receives wage income, and an individual receives compensation for capital service and dividend income to equities. The

[9] Our implementation of Kiyotaki and Moore's financial friction is taken from Shi (2012).

government collects a uniform, lump-sum tax T_t from each member. Both a fraction $\delta(u_t)$ of capital and a fraction δ_n of products depreciate.

The third stage in the period is the R&D stage where entrepreneurs seek finance and undertake product development projects. We assume that an entrepreneur can transform any amount s_t units of consumption goods into $\vartheta_t s_t$ units of new products. The efficiency of product development ϑ_t is an endogenous variable (specified later) but individual households take it as given. Following Bilbiie, Ghironi, and Melitz (2012) and Kung and Schmid (2012), we assume that a new product starts production in the period following invention; i.e., the adoption lag is uniform and is always constant at one.[10] With this assumption, equities of new products are traded at the same price as equities of (un-depreciated) existing products that have already paid out dividends. The goods market, the capital market, and the equity market open. Individuals trade assets to finance R&Ds and to achieve the portfolio of asset holdings instructed earlier by their households. The markets close at the end of this sub-period.

In the consumption stage, a worker consumes c_t^w units of consumption goods and an entrepreneur consumes c_t^e units of consumption goods. Then, individuals return to their households. In the investment stage, the head of the household collects the resources set aside by workers and uses it as inputs for investment. The capital stock at the beginning of the next period is determined by the following equation

$$k_{t+1} = \underbrace{\left[\sigma_e k_{t+1}^e + \sigma_w k_{t+1}^w\right]}_{\text{capital before the investment stage}} + \underbrace{\left(1 - \Lambda\left(\frac{i_t}{i_{t-1}}\right)\right)\sigma_w i_t}_{\text{capital added in the investment stage}} \tag{1}$$

where $\Lambda(\cdot)$ is the investment adjustment cost function given by

$$\Lambda\left(\frac{i_t}{i_{t-1}}\right) = \frac{\bar{\Lambda}}{2}\left(g - \frac{i_t}{i_{t-1}}\right)^2$$

g is the growth rate of the economy on the non-stochastic steady state growth path.

The instructions have to satisfy a set of constraints. First, the instruction to an entrepreneur has to satisfy the intra-period budget constraint:

$$\underbrace{c_t^e + s_t + \underbrace{p_{n,t} n_{t+1}^e + p_{k,t} k_{t+1}^e}_{\text{gross asset purchases}}}_{\text{gross expenditure}} = \underbrace{\underbrace{\Pi_t n_t}_{\text{dividend}} + \underbrace{R_t(u_t k_t)}_{\text{rental}} + \underbrace{p_{n,t}(1-\delta_n) n_t + p_{k,t}(1-\delta(u_t)) k_t}_{\text{resale value}} + \underbrace{p_{n,t}\vartheta_t s_t}_{\text{IPO}} - T_t}_{\text{gross after-tax income}} \tag{2}$$

The left-hand side is the gross total expenditure, collecting bills on consumption, R&Ds, and gross asset purchases, with $p_{n,t}$ denoting the price of equity and $p_{k,t}$ denoting the price of capital,

[10] Comin and Gertler (2006) consider a more realistic adoption stage, demonstrating that endogenous diffusion can be a powerful amplification mechanism.

respectively. The right-hand side is the gross, after-tax total income, collecting dividend income, compensation for capital service, resale values of assets, and the income from the (hypothetical) initial public offerings of new products the entrepreneur has just innovated, subtracting the lump-sum tax. The constraint therefore states that the total expenditure and the total after-tax income has to be balanced within a period, in which an entrepreneur is separated from other members of the household. A similar constraint applies to a worker:

$$c_t^w + i_t + p_{n,t} n_{t+1}^w + p_{k,t} k_{t+1}^w = \Pi_t n_t + R_t \left(u_t k_t \right) + p_{n,t} \left(1 - \delta_n \right) n_t + p_{k,t} \left(1 - \delta \left(u_t \right) \right) k_t + W_t l_t - T_t \quad (3)$$

There are other, crucial constraints on trading of assets. That is, an entrepreneur can sell at most a fraction θ of new equities for products she has just innovated, but has to retain the rest of equities by herself. In addition, she can sell at most a fraction ϕ_t of both existing equities (products) and existing capital to others in the asset markets, but has to retain the rest by herself. Effectively, these constraints introduce lower bounds to equity holding and capital holding of an entrepreneur at the closing of the markets:

$$n_{t+1}^e \geq \underbrace{\left(1 - \theta \right) \vartheta_t s_t}_{\text{new equities required to retain}} + \underbrace{\left(1 - \phi_t \right) \left(1 - \delta_n \right) n_t}_{\text{existing equities required to retain}} \quad (4)$$

$$k_{t+1}^e \geq \underbrace{\left(1 - \phi_t \right) \left(1 - \delta \left(u_t \right) \right) k_t}_{\text{existing capital required to retain}} \quad (5)$$

ϕ_t is an exogenous, random variable representing shocks to asset liquidity.[11] Similar constraints apply to workers, i.e., $n_{t+1}^w \geq \left(1 - \phi_t \right) \left(1 - \delta_n \right) n_t$ and $k_{t+1}^w \geq \left(1 - \phi_t \right) \left(1 - \delta \left(u_t \right) \right) k_t$, but we drop them from consideration in the following analysis because they do not bind in the equilibrium. There are non-negativity constraints for u_t, s_t, c_t^e, l_t, i_t, c_t^w, n_{t+1}^w, and k_{t+1}^w, but we drop them too for the same reason.

We view the equity market and the capital market collectively represent the financial system, because these markets, albeit in a highly stylized manner, intermediate between investors (entrepreneurs) and capital providers (workers). In addition, as in the actual economy, our model's growth potential hinges on the efficiency of those markets to transfer funds from those who are willing to supply them to those who desperately need them to implement their great ideas. The liquidity shock is a potential clog in the fund supply conduits, and we use its fluctuation to capture variation in financial conditions we documented in the previous section.

Let q_t denote the vector of endogenous, individual state variables, i.e., $q_t = \left(n_t, k_t, i_{t-1} \right)$. The head of the household chooses instructions to its members to maximize the value function defined

[11] Brunnermeier et al. (2012) refer to this type of liquidity as *market liquidity*. Since our model does not feature irreversibilities, physical and intangible capitals are also *technologically liquid*.

as

$$v\left(q_t;\Gamma_t,\Theta_t\right)=\max\left\{\sigma_e\log\left(c_t^e\right)+\sigma_w\left[\log\left(c_t^w\right)-\psi_t\frac{l_t^{1+\zeta}}{1+\zeta}\right]+\beta_tE_t\left[v\left(q_{t+1};\Gamma_{t+1},\Theta_{t+1}\right)\right]\right\}\tag{6}$$

subject to (1), (2), (3), (4), (5), and

$$n_{t+1}=\sigma_en_{t+1}^e+\sigma_wn_{t+1}^w$$

β_t is a subjective time discount factor and ψ_t is a coefficient affecting the labor disutility schedule, both of which are common across households and are exogenous random variables. Γ_t is the vector of endogenous, aggregate state variables, i.e., $\Gamma_t=(N_t,K_t,I_{t-1})$, where N_t is the mass of products available in the economy, K_t is the capital stock in the economy, and I_{t-1} is the investment level in the previous period. Θ_t is the the vector of exogenous state variables.

We will restrict our attention to the case in which $p_{n,t}\vartheta_t>1$ always holds. This is an interesting case in which revenues generated by product development exceed its costs. The liquidity constraint (4) must be binding at the optimum. Otherwise, the household can increase the utility without violating any constraints by simultaneously increasing product developments and entrepreneur's consumption from (s_t,c_t^e) to $(s_t+\Delta,c_t^e+(p_{n,t}\vartheta_t-1)\Delta)$ as long as $\Delta>0$ is sufficiently small. The liquidity constraint (5) must be binding too. Otherwise, the household can make (4) slack by letting entrepreneurs purchase products and sell capital, i.e., changing from $\left(n_{t+1}^e,k_{t+1}^e\right)$ to $\left(n_{t+1}^e+(p_{k,t}/p_{n,t})\Delta,k_{t+1}^e-\Delta\right)$, and letting workers conduct the counter trading to offset the effects to the household's portfolio of assets at the end of the period. This strategy does not violate any constraints as long as $\Delta>0$ is sufficiently small. But since the household can increase the utility if (4) is slack, this argument proves that (5) must be binding.

These binding constraints allow us to rewrite the household's problem as a maximization problem of the value function (6) by choosing u_t, s_t, c_t^e, l_t, i_t, c_t^w, n_{t+1}^w, and k_{t+1}^w subject to

$$c_t^e+p_{n,t}\left[(1-\theta)\vartheta_ts_t-\phi_t(1-\delta_n)n_t\right]+p_{k,t}\left[-\phi_t(1-\delta(u_t))k_t\right]=\Pi_tn_t+R_t(u_tk_t)+(p_{n,t}\vartheta_t-1)s_t-T_t\tag{7}$$

$$c_t^w+i_t+p_{n,t}n_{t+1}^w+p_{k,t}k_{t+1}^w=\left[\Pi_t+p_{n,t}(1-\delta_n)\right]n_t+\left[R_tu_t+p_{k,t}(1-\delta(u_t))\right]k_t+W_tl_t-T_t\tag{8}$$

$$n_{t+1}=\sigma_e\left[(1-\theta)\vartheta_ts_t+(1-\phi_t)(1-\delta_n)n_t\right]+\sigma_wn_{t+1}^w$$

$$k_{t+1}=\sigma_e\left[(1-\phi_t)(1-\delta(u_t))k_t\right]+\sigma_wk_{t+1}^w+\left(1-\Lambda\left(\frac{i_t}{i_{t-1}}\right)\right)(\sigma_wi_t)$$

We will also restrict our attention to the case in which $p_{n,t}\vartheta_t<1/\theta$ always holds, because otherwise, revenues generated by product development are too large and therefore, an entrepreneur can self-finance any amount of product development costs by selling a fraction θ of new equities. In this case, the household desires infinitely large product development, and the maximization problem cannot be properly formulated; i.e., there is no interior solution because changing the instruction to an entrepreneur from (s_t, c_t^e) to $(s_t + \Delta, c_t^e + (\theta p_{n,t}\vartheta_t - 1)\Delta)$ increases the equity holding and possibly entrepreneur's current consumption without violating any constraints, and Δ can be arbitrarily large.

First order optimality conditions are the following:

$$\beta_t E_t \left[\frac{\partial v_{t+1}}{\partial k_{t+1}}\right] \sigma_e (1-\phi_t)(-\delta'(u_t)) + \mu_t^e (p_{k,t}\phi_t(-\delta'(u_t)) + R_t) + \mu_t^w (R_t + p_{k,t}(-\delta'(u_t))) = 0 \quad (9)$$

$$\sigma_e \left(\frac{1}{c_t^e}\right) + \mu_t^e(-1) = 0 \quad (10)$$

$$\beta_t E_t \left[\frac{\partial v_{t+1}}{\partial n_{t+1}}\right] \sigma_e (1-\theta)\vartheta_t + \mu_t^e(-1+\theta p_{n,t}\vartheta_t) = 0 \quad (11)$$

$$\sigma_w \left(\frac{1}{c_t^w}\right) + \mu_t^w(-1) = 0 \quad (12)$$

$$\sigma_w \left(-\psi_t l_t^\zeta\right) + \mu_t^w(W_t) = 0 \quad (13)$$

$$\beta_t E_t \left[\frac{\partial v_{t+1}}{\partial n_{t+1}}\right] \sigma_w + \mu_t^w(-p_{n,t}) = 0 \quad (14)$$

$$\beta_t E_t \left[\frac{\partial v_{t+1}}{\partial k_{t+1}}\right] \sigma_w + \mu_t^w(-p_{k,t}) = 0 \quad (15)$$

$$\beta_t E_t \left[\frac{\partial v_{t+1}}{\partial k_{t+1}}\right] \sigma_w \left(1 - \Lambda\left(\frac{i_t}{i_{t-1}}\right) - \Lambda'\left(\frac{i_t}{i_{t-1}}\right)\frac{i_t}{i_{t-1}}\right) + \beta_t E_t \left[\frac{\partial v_{t+1}}{\partial i_t}\right] + \mu_t^w(-1) = 0 \quad (16)$$

where μ_t^e and μ_t^w are the Lagrangian multipliers associated with (7) and (8) respectively. Envelope conditions are

$$\frac{\partial v_t}{\partial n_t} = \beta_t E_t \left[\frac{\partial v_{t+1}}{\partial n_{t+1}}\right] \sigma_e (1-\phi_t)(1-\delta_n) + \mu_t^e [\Pi_t + p_{n,t}(1-\delta_n)\phi_t] + \mu_t^w [\Pi_t + p_{n,t}(1-\delta_n)] \quad (17)$$

$$\frac{\partial v_t}{\partial k_t} = \beta_t E_t \left[\frac{\partial v_{t+1}}{\partial k_{t+1}}\right] \sigma_e (1-\phi_t)(1-\delta(u_t)) + \mu_t^e [R_t u_t + p_{k,t}(1-\delta(u_t))\phi_t] + \mu_t^w [R_t u_t + p_{k,t}(1-\delta(u_t))] \quad (18)$$

$$\frac{\partial v_t}{\partial i_{t-1}} = \beta_t E_t \left[\frac{\partial v_{t+1}}{\partial k_{t+1}}\right] \sigma_w \Lambda'\left(\frac{i_t}{i_{t-1}}\right)\left(\frac{i_t}{i_{t-1}}\right)^2 \quad (19)$$

Combining (12) and (13), we obtain

$$\psi_t l_t^\zeta = W_t \left(\frac{1}{c_t^w}\right)$$

This is a standard intratemporal optimality condition, equating marginal disutility of labor (the left-hand side) to marginal utility of receiving wage income (the right-hand side). Combining (12), (14), and (15), we obtain

$$\beta_t E_t \left[\frac{\partial v_{t+1}}{\partial n_{t+1}} \right] = p_{n,t} \left(\frac{1}{c_t^w} \right)$$

$$\beta_t E_t \left[\frac{\partial v_{t+1}}{\partial k_{t+1}} \right] = p_{k,t} \left(\frac{1}{c_t^w} \right)$$

Marginal benefit of adding an addition unit of asset (the left-hand side) is equated to its opportunity cost of buying it using worker's budget (the right-hand side). These equations imply that the appropriate measure of evaluating the values of assets is worker's marginal utility of consumption, because it is not distorted by liquidity constraints. Entrepreneur's marginal utility is related to worker's as follows:

$$\left(\frac{1}{c_t^e} \right) = \left(\frac{\vartheta_t (1-\theta)}{1-\theta p_{n,t} \vartheta_t} \right) p_{n,t} \left(\frac{1}{c_t^w} \right)$$

The derivation is in the appendix. The following thought experiment demonstrates that this equation is essentially an optimality condition for product developments. An entrepreneur can increase the utility by consuming the last unit of her disposable income (the left-hand side). If, however, she devotes the same resource to product development, she can create $\vartheta_t / (1-\theta p_{n,t} \vartheta_t)$ units of new products, which is the efficiency of converting consumption goods to new products multiplied by the reciprocal of "down payment" for each unit of product development projects. Remember that a fraction of the R&D costs are financed by selling equities of new products. Among the developed products, a fraction $(1-\theta)$ are unsold in the market and therefore added to the household's asset portfolio. Lastly, since the household's subjective valuation of a product is equal to the opportunity cost of buying a product using worker's budget, the right-hand side is marginal benefit if the entrepreneur uses the last unit of her disposable income for product development. The condition says that at the optimum, the household should find these two usages of the marginal resource indifferent. The same condition can be conveniently rewritten as follows:

$$\left(\frac{1}{c_t^e} \right) = (1+\lambda_t^e) \left(\frac{1}{c_t^w} \right) \tag{20}$$

where λ_t^e is defined as

$$\lambda_t^e = \frac{p_{n,t}\vartheta_t - 1}{1-\theta p_{n,t} \vartheta_t} \tag{21}$$

λ_t^e is the variable Shi (2012) calls the liquidity services. Our assumption $1 < p_{n,t}\vartheta_t < 1/\theta$ implies that the liquidity services are always positive, and therefore, it is optimal that entrepreneurs consume less than workers. This is because freeing up a unit of resource in the entrepreneur's budget constraint is more valuable to the household than freeing up a unit of resource in the worker's budget constraint, for the entrepreneur can use the resource for product development projects whose profits are strictly positive. The liquidity services measure the relative importance to the household between incremental resource given to an entrepreneur and incremental resource given to a worker.

We derive in the appendix the following pricing equation:

$$p_{n,t} = E_t \left[\left(\beta_t \frac{c_t^w}{c_{t+1}^w} \right) \left(\Pi_{t+1} + p_{n,t+1} \left(1 - \delta_n\right) + \sigma_e \lambda_{t+1}^e \left[\Pi_{t+1} + p_{n,t+1} \left(1 - \delta_n\right) \phi_{t+1} \right] \right) \right] \tag{22}$$

As is standard, the price of a product reflects the present discounted value of future cash flow and the resale value. An interesting observation is that the pricing equation incorporates liquidity services provided by a product as well. Remember that a product delivers dividend income to its shareholders, and in addition, the equity is saleable up to a certain fraction in the equity market. Both of these features are attractive to the household because they provide liquidity to entrepreneurs. Hence, the liquidity services are incorporated into the equilibrium price. A similar pricing equation for capital is derived in the appendix:

$$p_{k,t} = E_t \left[\left(\beta_t \frac{c_t^w}{c_{t+1}^w} \right) \left(R_{t+1} u_{t+1} + p_{k,t+1} \left(1 - \delta \left(u_{t+1}\right)\right) + \sigma_e \lambda_{t+1}^e \left[R_{t+1} u_{t+1} + p_{k,t+1} \left(1 - \delta \left(u_{t+1}\right)\right) \phi_{t+1} \right] \right) \right] \tag{23}$$

The intuition is analogous to the one for the pricing equation for a product.

Finally, we derive in the appendix the first order optimality condition for investment:

$$1 = p_{k,t} \left(1 - \Lambda \left(\frac{i_t}{i_{t-1}} \right) - \Lambda' \left(\frac{i_t}{i_{t-1}} \right) \frac{i_t}{i_{t-1}} \right) + E_t \left[\left(\beta_t \frac{c_t^w}{c_{t+1}^w} \right) p_{k,t+1} \Lambda' \left(\frac{i_{t+1}}{i_t} \right) \left(\frac{i_{t+1}}{i_t} \right)^2 \right] \tag{24}$$

The left-hand side is the cost of investment measured in consumption goods. The right-hand side is the benefit of investment, which incorporates not only the value of capital created in the current period (the first term), but also investment's dynamic effects to future investment adjustment costs (the second term). The liquidity services are not involved because it is workers who invest.

3.2 Final goods sector

There is a representative firm that uses capital service KS_t, labor L_t, and a composite of intermediate goods G_t to produce the final (consumption) good according to the production technology

$$Y_t = \left(\left(KS_t\right)^{\alpha} \left(A_t L_t\right)^{1-\alpha} \right)^{1-\xi} G_t^{\xi} \tag{25}$$

where the composite G_t is defined as

$$G_t = \left[\int_0^{N_t} X_{i,t}^{\frac{1}{\nu}} di \right]^{\nu}$$

$X_{i,t}$ is intermediate good $i \in [0, N_t]$; α is the capital share; ξ is the intermediate goods share; ν is the elasticity of substitution between the intermediate goods. A_t is the exogenous, neutral productivity shock. The firm maximizes profits defined as

$$Y_t - R_t (KS_t) - W_t L_t - \int_0^{N_t} P_{i,t} X_{i,t} di$$

where $P_{i,t}$ is the price per unit of intermediate good i, which the final goods firm takes as given. Solving the cost minimization problem of purchasing intermediate goods leads to the downward-sloping demand function:

$$X_{i,t} = \left(\frac{P_{i,t}}{P_{G,t}} \right)^{\frac{\nu}{1-\nu}} G_t \tag{26}$$

where $P_{G,t}$ is the price index defined as

$$P_{G,t} = \left[\int_0^{N_t} P_{i,t}^{\frac{1}{1-\nu}} di \right]^{1-\nu}$$

The total expenditure on intermediate goods is given by

$$\int_0^{N_t} P_{i,t} X_{i,t} di = P_{G,t} G_t$$

The firm's first order optimality conditions are

$$R_t = (1-\xi)\,\alpha \frac{Y_t}{KS_t} \tag{27}$$

$$W_t = (1-\xi)(1-\alpha)\frac{Y_t}{L_t} \tag{28}$$

$$P_{G,t} = \xi \frac{Y_t}{G_t} \tag{29}$$

3.3 Intermediate goods sector

Production is roundabout, and hence, the marginal cost of producing an intermediate good is unity. The producer chooses its price $P_{i,t}$ to maximize the profits defined as

$$\Pi_{i,t} \equiv \max_{P_{i,t}} (P_{i,t} - 1) \left(\frac{P_{i,t}}{P_{G,t}} \right)^{\frac{\nu}{1-\nu}} G_t$$

Solving this problem leads to the optimal markup pricing,

$$P_{i,t} = \nu \tag{30}$$

Since optimal prices are symmetric, so are production levels and profits. Let X_t denote the symmetric intermediate production level, i.e., $X_t = X_{i,t}$ for all $i \in [0, N_t]$, and let Π_t denote the symmetric profits, i.e., $\Pi_t = \Pi_{i,t}$ for all $i \in [0, N_t]$. Profits are paid out to shareholders as dividends.

3.4 Transition rules for aggregate state variables

We assume that the technology coefficient of product developments involves both knowledge spillover á la Romer (1990) and a congestion externality effect capturing decreasing returns to scale in the innovation sector

$$\vartheta_t = \frac{\chi_t N_t}{(\sigma_e s_t)^{1-\eta} (N_t)^{\eta}} \tag{31}$$

where $\eta \in [0, 1]$ is the elasticity of new intermediate goods with respect to R&D.[12] χ_t represents an exogenous, sector-specific productivity shock in the innovation sector. N_t's transition rule is given by

$$N_{t+1} = (1 - \delta_n) N_t + \vartheta_t (\sigma_e s_t)$$

There are two important observations about this law of motion. First, the right hand side is homogeneous of degree one in N_t and s_t. The growth rate of N_t depends, except for the sector-specific productivity shock, only on the ratio of aggregate R&D spending to the mass of products available in the economy, so that innovation does not exhaust but continues to be an engine of economic growth over the long run as long as these two variables grow proportionally. Indeed, many of the endogenous variables in our model economy including s_t are non-stationary in levels, but they have common trend N_t and their ratios to this variable are stationary. Second, and more important, the growth rate of N_t is an endogenous variable. As such, temporary shocks may cause drifts in the economy's trend regardless of their origins. In the next section, we ask the data, which shock is a primary driver of the recent dramatic shift in the U.S. growth trend.

Finally, K_t's transition rule is given by

$$K_{t+1} = (1 - \delta (u_t)) K_t + \left(1 - \Lambda \left(\frac{i_t}{i_{t-1}}\right)\right) (\sigma_w i_t)$$

3.5 Government

The government spends a fraction τ_t of the value-added output $\mathcal{Y}_t$, which is defined as the gross output minus intermediate inputs

$$\mathcal{Y}_t \equiv Y_t - N_t X_t$$

We assume that the government keeps the balanced-budget:

$$\tau_t \mathcal{Y}_t = T_t$$

[12] This specification is taken from Kung and Schmid (2012).

τ_t is an exogenous, random variable.

3.6 Equilibrium

The competitive equilibrium is defined in a standard way. Market clearing conditions for production factors are

$$KS_t = u_t K_t$$

$$L_t = \sigma_w l_t$$

Goods market clearing condition is

$$Y_t = \sigma_e c_t^e + \sigma_w c_t^w + \sigma_w i_t + \sigma_e s_t + N_t X_t + T_t$$

Asset markets clearing conditions are

$$N_t = n_t$$

$$K_t = k_t$$

at the beginning of period t and

$$N_{t+1} = \sigma_e \left[(1-\theta) \vartheta_t s_t + (1-\phi_t)(1-\delta_n) N_t \right] + \sigma_w n_{t+1}^w$$

$$(1-\delta(u_t)) K_t = \sigma_e (1-\phi_t)(1-\delta(u_t)) K_t + \sigma_w k_{t+1}^w$$

at the closing of the asset markets.

Let us discuss some equilibrium relations. Using (30) and the symmetry between products, we find that the price index of intermediate goods composite is given by

$$P_{G,t} = N_t^{1-\nu} \nu \tag{32}$$

Using (25), (29), and (32), we find that the final goods production is given by

$$Y_t = \left(\frac{\xi}{\nu} \right)^{\frac{\xi}{1-\xi}} (KS_t)^{\alpha} (A_t L_t)^{1-\alpha} (N_t)^{\frac{\nu\xi - \xi}{1-\xi}} \tag{33}$$

Following Kung and Schmid (2012), we make the parameter restriction $\alpha + \frac{\nu\xi-\xi}{1-\xi} = 1$. An advantage of this assumption is that it leads to a production function that resembles the standard neoclassical one with labor augmenting technology

$$Y_t = (KS_t)^{\alpha} (Z_t L_t)^{1-\alpha}$$

where the equilibrium productivity measure is given by

$$Z_t = \left(\bar{A}\right)\left(A_t N_t\right)$$

and $\bar{A} \equiv \left(\frac{\xi}{\nu}\right)^{\frac{\xi}{(1-\xi)(1-\alpha)}} > 0$ is a constant.

We discuss the national income accounting. Rearranging the goods market clearing condition, we find an identity

$$\mathcal{Y}_t = \sigma_e c_t^e + \sigma_w c_t^w + \sigma_w i_t + \sigma_e s_t + T_t \tag{34}$$

The value added output is the sum of consumption, investment in capital, investment in product developments, and the government spending. Another approach to the aggregate value added output is from income. Using (27), (28), and (29), we find

$$Y_t = R_t\left(u_t K_t\right) + W_t\left(\sigma_w l_t\right) + P_{G,t} G_t$$

Since the expenditure on intermediate goods is decomposed into production costs and profits, i.e., $P_{G,t} G_t = N_t X_t + N_t \Pi_t$, we find

$$\mathcal{Y}_t = R_t\left(u_t K_t\right) + W_t\left(\sigma_w l_t\right) + N_t \Pi_t \tag{35}$$

The value added output is the sum of the compensation for capital service, the compensation for labor service, and monopoly profits.

We discuss factor shares in the value added output. Final goods firm's first order condition (29) implies that

$$\xi Y_t = P_{G,t} G_t = N_t X_t + N_t \Pi_t$$

Since intermediate goods firms always charge the constant markup, this equation implies

$$N_t X_t = \frac{\xi}{\nu} Y_t$$

$$N_t \Pi_t = \left(\frac{\nu - 1}{\nu}\right) \xi Y_t$$

and

$$\mathcal{Y}_t = \left(1 - \frac{\xi}{\nu}\right) Y_t$$

Therefore, the relation between the gross output and the value added output is linear. Factor shares of rental income, labor income, and profits in the value added output are constant too.

Finally, we discuss the budget constraints in the equilibrium. Entrepreneur's budget constraint

(7) in the equilibrium is

$$c_t^e + p_{n,t}\left[(1-\theta)\vartheta_t s_t - \phi_t (1-\delta_n) N_t\right] - p_{k,t}\phi_t (1-\delta(u_t)) K_t = \Pi_t N_t + R_t (u_t K_t) + (p_{n,t}\vartheta_t - 1) s_t - T_t \tag{36}$$

Worker's budget constraint (8) in the equilibrium is

$$\begin{aligned} & c_t^w + i_t + \frac{p_{n,t}}{\sigma_w}\left(N_{t+1} - \sigma_e\left[(1-\theta)\vartheta_t s_t + (1-\phi_t)(1-\delta_n) N_t\right]\right) \\ & + \frac{p_{k,t}}{\sigma_w}(1-\delta(u_t))(1-\sigma_e(1-\phi_t)) K_t \\ = & \left[\Pi_t + p_{n,t}(1-\delta_n)\right] N_t + \left[R_t u_t + p_{k,t}(1-\delta(u_t))\right] K_t + W_t l_t - T_t \end{aligned} \tag{37}$$

Adding (36) multiplied by σ_e and (37) multiplied by σ_w, we find

$$(\sigma_e c_t^e + \sigma_w c_t^w + \sigma_w i_t + \sigma_e s_t + T_t) - (R_t u_t K_t + W_t \sigma_w l_t + N_t \Pi_t) + p_{n,t}\left[N_{t+1} - (1-\delta_n) N_t - \vartheta_t (\sigma_e s_t)\right] = 0$$

Combining this equation with (34) and (35), we find

$$p_{n,t}\left[N_{t+1} - (1-\delta_n) N_t - \vartheta_t (\sigma_e s_t)\right] = 0$$

Since $p_{n,t}$ is always positive, we can drop the transition rule of N_t from the equilibrium conditions as long as we impose (34), (35), (36), and (37). The equilibrium conditions are summarized in the appendix.

3.7 Structural Shocks

There are six structural shocks, β_t, ϕ_t, χ_t, A_t, τ_t, and ψ_t, in our model, each of which is modeled as an AR(1) process with *iid* innovation. Hence, the generic specification of our shocks is

$$\log \frac{\varsigma_t}{\varsigma} = \rho_\varsigma \log \frac{\varsigma_{t-1}}{\varsigma} + \sigma_\varsigma \varepsilon_\varsigma,$$

where ρ_ς and σ_ς are the persistence and standard deviation of the stochastic process. The innovation or shock ε_ς is assumed to follow a normal standard distribution.

4 Results

Before discussing in detail the main results from our model, we explain how we choose the parameters.

4.1 Estimation

We take a fairly conservative approach regarding the parameterization/estimation of our model. We tie our hands by setting most structural parameters to either values used elsewhere or to match some incontrovertible ratio in the data. This means that our estimation strategy puts the emphasis on the structural shocks.

The first panel of Table 1 reports the parameters that are fixed during estimation. Following Shi (2012), we set the share of liquidity constrained agents (entrepreneurs) σ_e at 6 percent per quarter. We set product depreciation rate δ_n at 3 percent per quarter following the literature (Bilbiie, Ghironi, and Melitz (2012); Comin and Gertler (2006); and Kung and Schmid (2012)).

The second panel in turn reports the mode of the posterior distribution of the estimated parameters. We use gamma distribution priors for the elasticity of capital utilization (δ''/δ') and the adjustment cost of investment $\left(\overline{\Lambda}\right)$. The mean and standard deviations are $\{1, 0.5\}$ and $\{3, 1\}$, respectively. For the persistence of the stochastic processes, we choose a beta distribution with mean 0.5 and standard deviation 0.2. The prior for the volatility of the structural shocks is an inverse gamma distribution with parameters 6 and 1.

We estimate our model using quarterly data on output, consumption, investment, wages, labor, and data on intangible capital.[13] For the first three observables, nominal values (from NIPA) were deflated using the implicit GDP deflator. Wages correspond to nominal compensation per hour in the nonfarm business sector deflated by the implicit GDP deflator. Labor is the ratio of hours of all persons in the nonfarm business sector to civilian noninstitutional population. For the last observable, we rely on the series reported in Nakamura (2003). Without going into the details, Nakamura argues that a more accurate portray of intangible capital in the economy is given by twice the measure of software plus twice the value of R&D (both taken from NIPA) plus a measure of advertisement spending (compiled by the advertising agency McCann and Erickson). We adjust output and investment to reflect this alternative (and broader) measure of R&D (the annual series was interpolated using Fernandez (1981)'s algorithm using NIPA's R&D quarterly series as the reference entry). The sample covers 1970.Q1 - 2011.Q4. Except for labor, all variables are expressed in growth rates.

Before analyzing the Great Recession, we briefly discuss the impact of liquidity changes in our model (Figure 6). Following an adverse liquidity innovation (a decrease in ϕ_t), both equity and capital become less liquid. Entrepreneurs scale down their product development projects because they struggle to fund their businesses as cashing out assets is now not as easy as before. Weak innovative activities have detrimental impacts on future innovations through knowledge spillover. Remember that the efficiency of product development improves with the stock of products in the

[13] The use of intangible capital data reflects our view of products in the model. We define them broadly. In addition, we believe that products are able to distinguish themselves from other products not only by the formal patent system but also by informal protections surrounding trade secrets, brand images, business models, and so on. Such a consideration leads us to use an inclusive measure.

Table 1: Parameter Values

Fixed		
Parameter	Calibration	Target/Reference
β	0.92	Match mean GDP growth
ζ	1	Comin and Gertler (2006)
η	0.8	Comin and Gertler (2006)
σ_e	0.06	Shi (2012)
δ_n	0.03	Bilbiie et al. (2012) and others
ν	1.6	Comin and Gertler (2006)
δ_k	0.1	Match I/Y in data
θ	0.2	Del Negro et al. (2011) and Shi (2012)
ϕ	0.2	Del Negro et al. (2011) and Shi (2012)
τ	0.2	Match gov spending to GDP
δ'	0.19	Pinned down by equilibrium condition
α	0.36	Labor Share
ψ	0.81	Normalize labor to 1
χ	0.47	Match R&D to GDP
u	1	Normalization

Estimated		
Parameter	Mode	Prior Distribution
δ''/δ'	2.73	Gamma (1.0,0.5)
$\overline{\Lambda}$	0.02	Gamma (3.0,1.0)
ρ_β	0.22	Beta (0.5,0.2)
ρ_ϕ	0.99	Beta (0.5,0.2)
ρ_χ	0.22	Beta (0.5,0.2)
ρ_a	0.53	Beta (0.5,0.2)
ρ_τ	0.54	Beta (0.5,0.2)
ρ_ψ	0.10	Beta (0.5,0.2)
σ_β	0.05	Inverse-Gamma (0.2,0.1)
σ_ϕ	0.38	Inverse-Gamma (0.2,0.1)
σ_χ	0.04	Inverse-Gamma (0.2,0.1)
σ_a	0.03	Inverse-Gamma (0.2,0.1)
σ_τ	0.04	Inverse-Gamma (0.2,0.1)
σ_ψ	0.03	Inverse-Gamma (0.2,0.1)

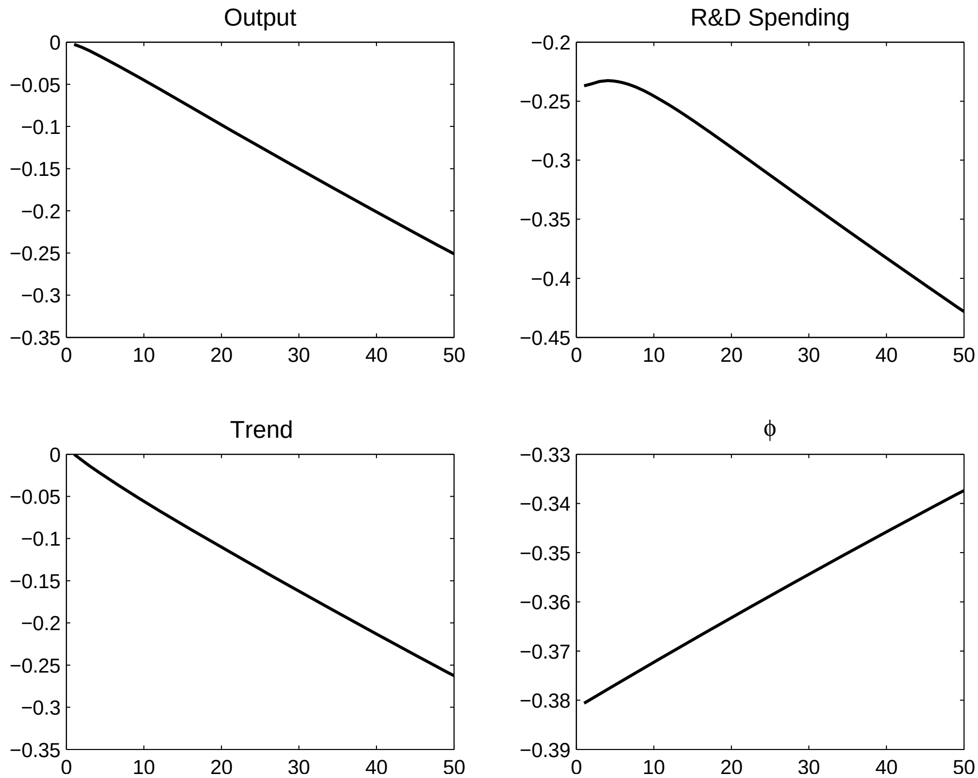

Figure 6: Impulse Responses to a Decrease in Liquidity

economy. This externality, in conjunction with slow recovery in the liquidity condition, further discourages R&D spending in subsequent periods, resulting in a prolonged weak growth in the economy.

4.2 A look at the Great Recession

Figure 7 show the smoothed paths for the stochastic processes (ς_t) around the Great Recession (the red dot indicates 2008.Q3). Two crucial observations emerge from these figures. First, the dynamics of the discount factor point to a large change in the second half of 2008, which most likely reflects the households' efforts to de-leverage (we will get back to this issue). The government spending shocks signal low demand during the recession. More important to our purposes, the liquidity condition in the asset market significantly deteriorated. Recall that a decline in ϕ_t means that entrepreneurs can resale a smaller fraction of their physical and intangible assets. Indeed, the worst liquidity shock coincided with the failure of Lehman Brothers. Our estimates suggest that the financial conditions started to improve in 2010. Low aggregate demand and adverse financial conditions translate into weakness in labor market. The adverse labor supply shock (ψ_t) further amplifies the bad situation in this market.

Figure 7 provides an interesting account of the worsening conditions in financial markets in

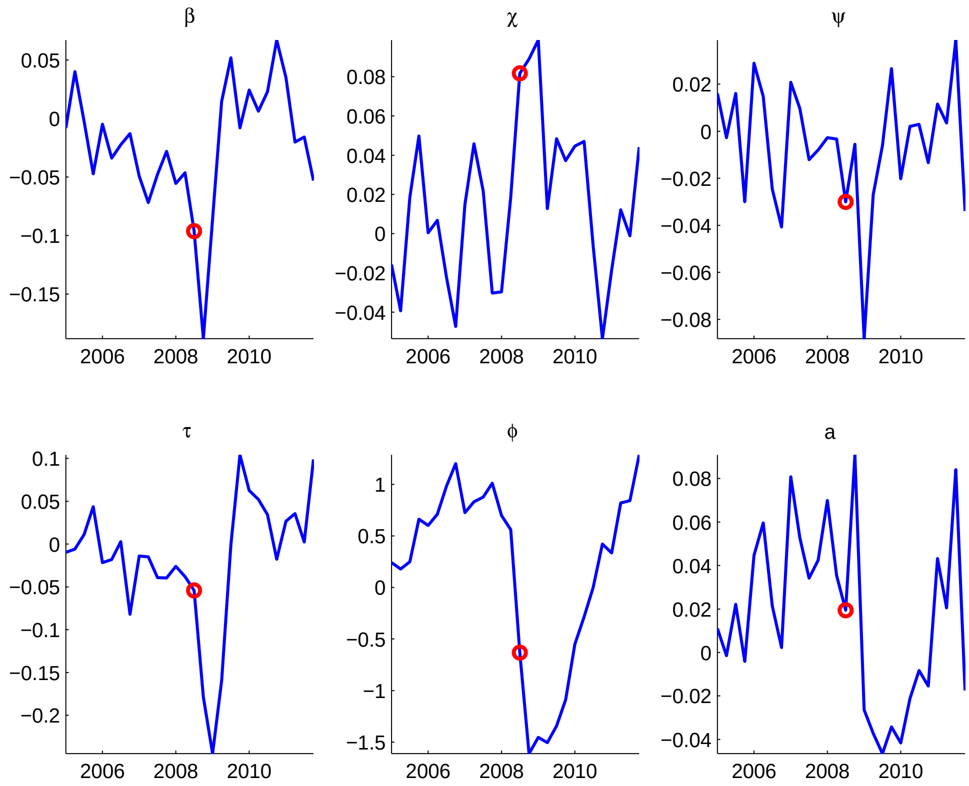

Figure 7: Smoothed Stochastic Processes

2008. In particular, the dynamics of ϕ_t indicate that tightening in credit markets started in mid-2007 (presumably due to first wave of foreclosures). Interestingly, the timing of the peak in our measure of liquidity coincides with the peak of the credit boom (Ivashina and Scharfstein (2010)) and the highest private equity investment (Figure 5). Our measure reveals that the financial crisis gained substantial momentum following the demise of Bear Stearns and in particular Lehman Brothers' collapse. Credit conditions remained tight (although improving) through 2009. It is only in mid-2010 when financial markets showed signals of more favorable financial markets. As Figure 8 shows our measure of liquidity not only agrees with the anecdotal descriptions of the crisis but also tracks very closely actual measures. Indeed, liquidity as predicted by our exercise moves in surprising coordination with the (negative) margin on S&P 500 futures reported in Section 2. This finding is quite remarkable and a favorable validation of our approach if we consider that no financial data were used to estimate the model.

The decline in the stochastic discount shock in the midst of the Great Recession is at first glance puzzling, because in a standard RBC model, a fall in the discount rate would imply a counterfactual improvement in consumption. However, a negative discount rate shock in our model economy is also associated with a negative wealth effect, which counterbalances the aforementioned intertemporal substitution effect. This is because the temporary improvement in consumption comes crucially at the expense of R&D spending in our model (unlike in the RBC model, where

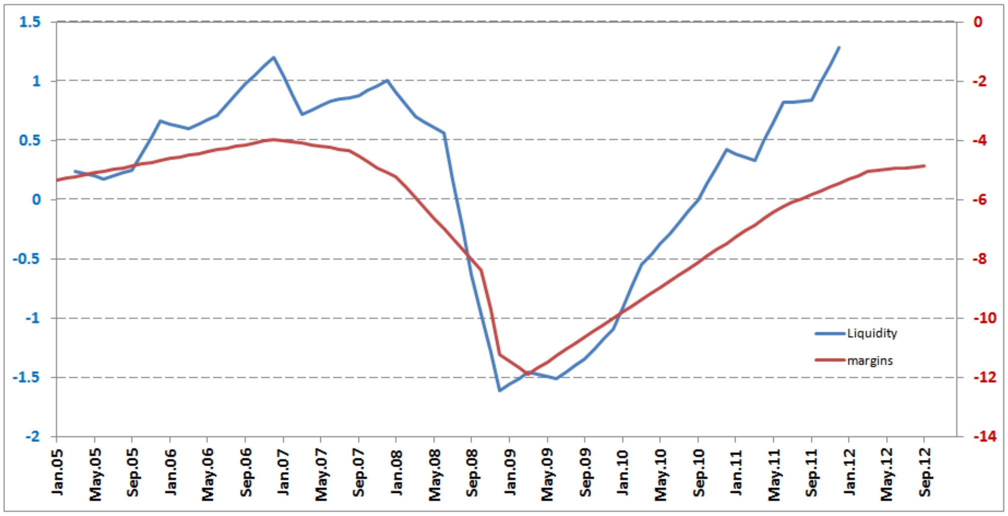

Figure 8: Estimated Liquidity and Margins on S&P 500 Futures

an increase in consumption comes solely at the expense of a contraction in investment), and the lower pace of innovation in turn implies a downward kick to the economy's trend. This mechanism is exacerbated by the household's inability to internalize knowledge spillover; i.e., cutting the R&D spending today harmfully affects the incentive to conduct R&D in the future and as a result, pushes the economy to a lower trend; individuals however do not think that this change in the trend is caused by them and hence have no private incentive to take the action that can favorably alter the trend if collectively taken.

To assess the severity of the financial shock, we try alternative scenarios about the evolution of this shock following the collapse of Lehman Brothers. Our first counterfactual simulation (dashed red lines in Figure 9; solid blue lines corresponding to actual paths) assumes that financial markets remained frozen at their worst state in 2008, which happened in the fourth quarter (this is implemented by assuming that ϕ_t was fixed at its value in 2008.Q4). The important message from this counterfactual is that improving conditions in the financial sector played a critical role in the post-crisis recovery. Although the fictitious economy followed a path close to the actual economy in early 2009, it is clear that lingering adverse financial conditions would have led to a deeper and longer recession lasting well into the end of our sample.

We also find quite interesting (and suggestive) that labor remains contracted in our simulation when the financial friction remains at its worst state. The reason behind the labor path is as follows. Workers in the counterfactual simulation do not have to spend lots of money for purchasing assets, because assets are illiquid and as a consequence, there are not many assets sold in the market. This means that a larger amount of liquidity remains in workers' hands. They consume, rather than invest, much of this windfall of liquidity because growth prospects are dismal, which then generates a sort of "income effect" weakening labor supply condition by increasing the marginal

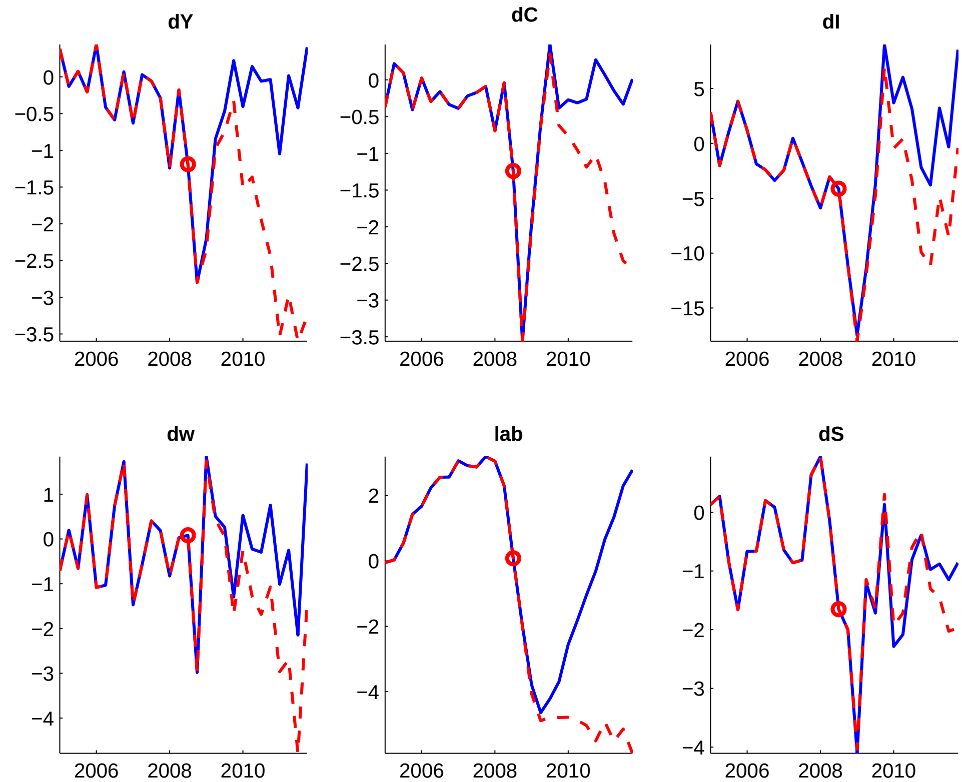

Figure 9: Counterfactual Scenario: Liquidity constraint stuck at its lowest level in 2008

rate of substitution. This cross-sectional resource misallocation, i.e., the liquidity constrained entrepreneurs cannot access to the funds they long for, but much of the resource is stuck in the workers' hands and is eaten by them, explains why labor market remains constricted in the counterfactual simulation. In short, this is a symptom of the sclerotic financial market. But our counterfactual simulation shows that dynamic consequence of the adverse liquidity condition is even severer; i.e., smaller investment in the current period reduces the stock of physical capital in the next period, which further reduces investment in the future. This vicious cycle shows no tendency of slowing down unless the liquidity condition recovers, leading to a wide gap between the actual path and the counterfactual simulation.

The dynamics of research and development are not very different between the counterfactual simulation and the actual path, at least in the time span we are concerning about. This may be puzzling at first, since other things being equal, the liquidity shock has a direct impact on entrepreneur's behavior. The reason for it is actually simple; as another counterfactual simulation we will soon show clarifies, adverse real shocks hit the research and development sector right after the Lehman shock, depressing R&D spending in the counterfactual scenario anyway. This point will be discussed in detail below.

Figure 10 in turn displays the counterfactual scenario for the variables of interest in levels (we use 2005.Q1 as the reference point to compute the series). Had financial markets remained

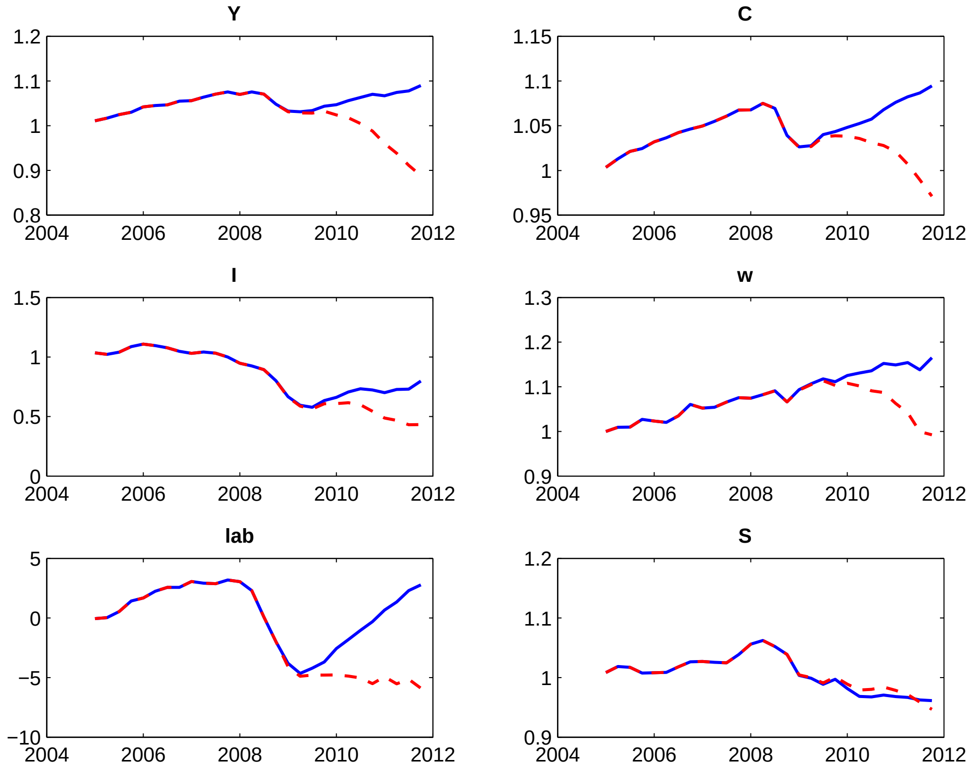

Figure 10: Counterfactual Scenario with Variables in Levels and Liquidity frozen at its 2008Q4 level

frozen, our simulations indicate that, for example, GDP would have been 20 percentage points below its actual level by the end of our sample (2011.Q4). Note that this astonishing break in the GDP trend is a consequence of the endogenous growth feature of our model. Indeed, this scenario suggests that a sclerotic financial sector could have easily lead to a collapse of the economy with a speed and severity comparable to those experienced during the Great Depression.

Another way to study the degree of financial tightening and its impact on the Great Recession is as follows. Imagine that starting in 2009.Q1, the financial shocks ($\varepsilon_{\phi,t}$) follow their estimated paths but the other shocks are replaced by random draws with replacement from their empirical distribution.[14] The resulting paths for the levels of several variables in the model are plotted in Figure 11. This exercise describes the dynamics of an average economy except in that is buffeted by the actual liquidity innovations. We think the figure speaks for itself, namely, the more favorable liquidity conditions would have eventually lead growth to positive numbers and hence bring the economy close to the trend prevalent in 2011, albeit with some delays. Indeed, this counterfactual suggests that the influence of the non-liquidity shocks were crucial for the speed of the recovery

[14] Suppose $\{\varepsilon_{\bullet,t}\}_{t=1}^{T}$ denotes the collection of estimated shocks. The simulation proceeds by randomly drawing with replacement from this collection of shocks for all disturbances except $\varepsilon_{\phi,t}$. These draws then replace the estimated ones from 2009.Q1 and beyond. The simulation is repeated many times. The figures plot the average across all these repetitions.

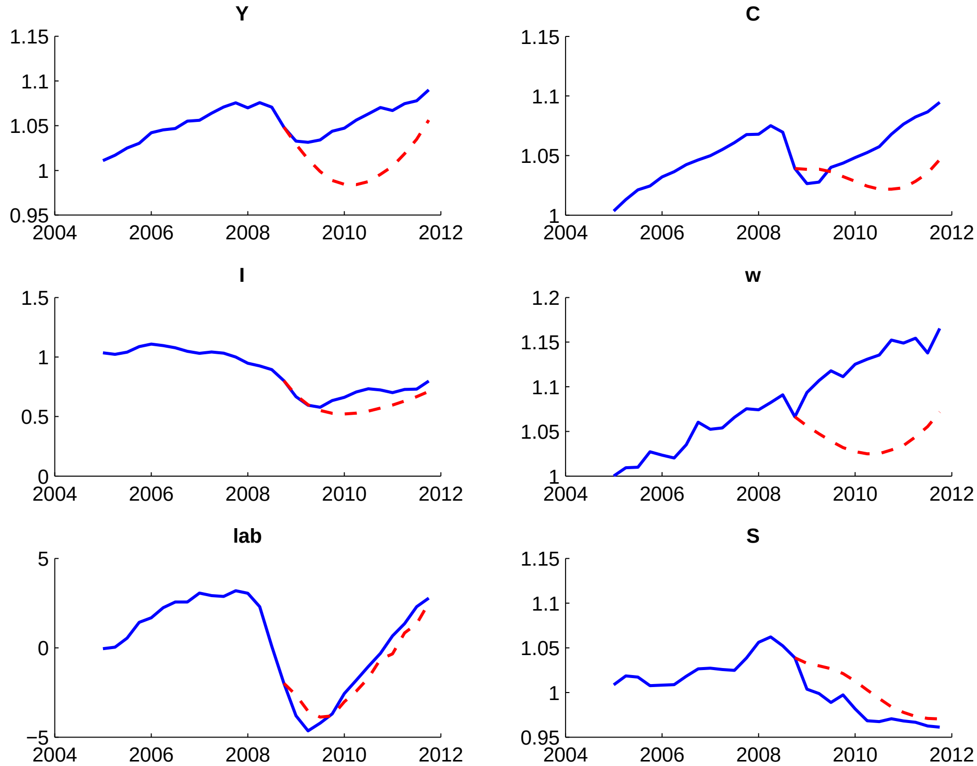

Figure 11: Counterfactual with Liquidity following Actual Path and other Shocks are Random

but not for the recovery itself. By the end of the sample, the better outlook in financial conditions was enough to get the counterfactual output trend (red dashed line) close to the actual data. Interestingly, a large part of the dynamics of the labor market is accounted for by the financial shock.

As we anticipated, research and development does not crush after the Lehman's collapse in the current counterfactual simulation. This result indicates that research and development sector was hammered twice during the Great Recession. First, it was pounded by the credit crunch following Lehman's failure. As is apparent from Figure 7, the second strike came from the shock to the efficiency of research and development (χ_t), which according to our estimation exercise happened in early 2010. The impact of this disturbance on R&D was milder and short-lived compared with the liquidity shock.

An alternative way to evaluate the role of liquidity is to back up a counterfactual path that would leave the trend of the economy growing at its steady state rate (in our model, the growth rate is endogenous but it is stationary). Figure 12 displays the results from this exercise. The liquidity paths are plotted in the upper panel while the bottom panel contains the actual and counterfactual trends. There are two striking findings. First, this simulation suggests that the Great Recession could have been averted if the financial crisis had been averted. This result further underlines the role of the liquidity shock in the Great Recession. Second, this simulation

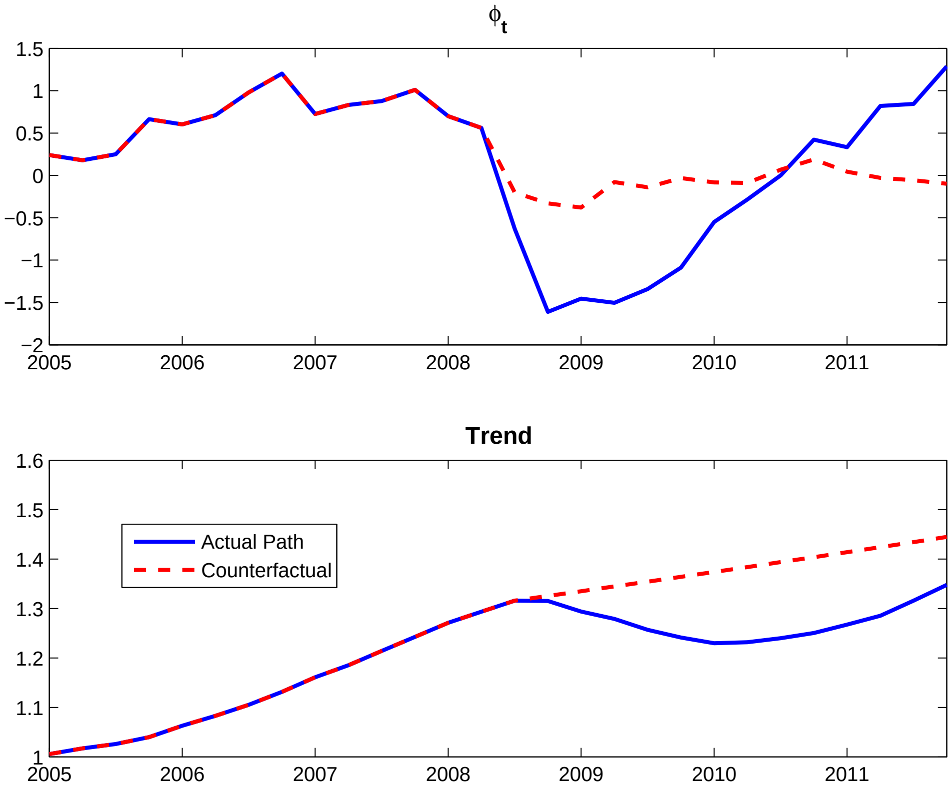

Figure 12: Liquidity and Trends

suggests that it would have sufficed that liquidity returned to its normal levels for the economy to remain around its historical trend. Notice that the counterfactual liquidity path still experiences a drop (but not a crush) that just ends the preceding liquidity boom. This means that the other structural disturbances had either a small but positive or a muted impact on the trend. This is consistent with Figure 11, in which we see that the other structural shocks expedited the recovery.

We ask now at what point in time conditions became adverse to the point that the downturn was inevitable. To this end, we simulate our model assuming that the innovations to all structural shocks, $\varepsilon_{\cdot,t}$, are randomly drawn from their distributions starting at different points in time. The idea is to assess when the innovations were sufficiently bad to pull the economy to the recession even though from that point and beyond the economy is buffeted by average shocks; in other words, by sequentially rolling back the timing of randomization, we can assess the "marginal effect" of structural shocks that hit the economy in a particular point in time, and by doing so, we can ask which vintage of shocks push the economy over the edge of a cliff. The blue dotted lines with the steepest slope in Figure 13 are the counterfactual paths corresponding to the case in which the average shocks start to hit the economy in 2007.Q4. The next blue dotted line corresponds to the case in which the random innovations start in 2008.Q3. The next two red dashed lines indicate the scenarios in which the average shocks start to hit the economy in 2008.Q4 and 2009.Q1. There are four outstanding messages from this exercise. First, macroeconomic conditions were deteriorating

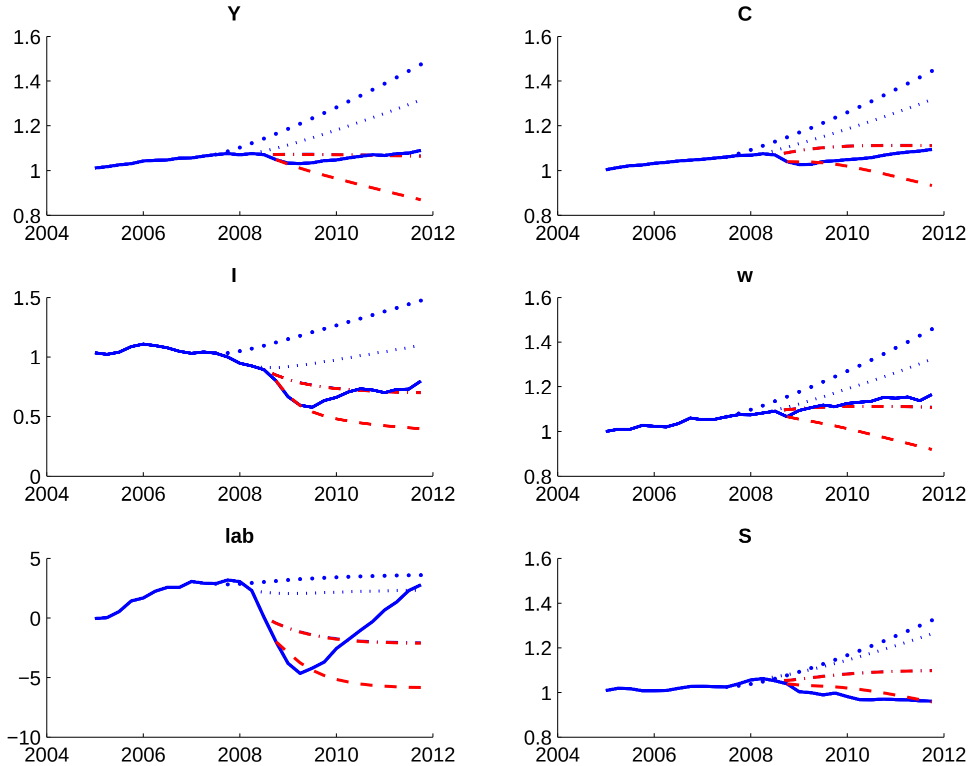

Figure 13: Trend Evolution During the Great Recession

between 2007.Q4 and 2008.Q2. Second, even as late as the second quarter of 2008, the recession could have been averted if the subsequent shocks had been replaced with random draws from their historical distributions. Third, and more important, the Lehman shock in 2008.Q3 was key in the Great Recession. Note that the economy, in spite of avoiding adverse shocks immediately after the Lehman's demise, never fully recovered in level and remained in a trajectory that coincides with the actual data at the end of 2011. Finally, the simulation suggests that conditions started to improve in 2009 and later, pushing the economy into recovery mode. This point can be seen by comparing the actual path with the counterfactual scenario in which randomization starts in 2009.Q1. In this scenario, the economy is buffeted by all the negative shocks realized in 2008 but "avoids" favorable shocks (as suggested by the smoothed estimates) realized in subsequent years. Our simulation exercise suggests that in this scenario, the economy would have fallen into a downward spiral with GDP falling by more than 20 percentage points relative to its actual level in 2011.

4.3 RBC model with non-stationary productivity shock

So far we have looked at the recent U.S. macroeconomic history through the lens of our baseline model. Our finding is that the liquidity condition has consistently played important roles before, during, and after the Great Recession. In this section, we take a look at these episodes from a different perspective, i.e., through the lens of a standard RBC model. Specifically, we use a model that has productivity shock following a unit-root process and let it replace our endogenous growth mechanism. Such a model has a potential to account for the permanent downward shift in the U.S. GDP trend line that we observe during the Great Recession if a sequence of negative productivity shocks pushed productivity down to the new trend. Although this "explanation" is arguably mechanical and, in addition, the evidence seems to go against this hypothesis, it is still illustrative to analyze the data from the perspective of such an exogenous growth model since in terms of modeling, it more closely follows the real business cycle research tradition.

We make our model as simple and clean as possible. We abandon research and development sector since the most basic RBC model is a one-sector model (e.g., Cooley and Prescott (1995)). For the similar reason, we abandon monopolistically competitive intermediate goods sector too. We, however, keep some important elements to facilitate comparison between our baseline model and our RBC-type alternative. Chief among them is the liquidity shock affecting investment decision. Specifically, we follow Del Negro, Eggertsson, Ferrero, and Kiyotaki (2011), Kiyotaki and Moore (2012), and Shi (2012), and assume that investors are liquidity constrained in a similar manner as are entrepreneurs in our baseline model. The model and optimal conditions are outlined in the appendix. Since there is no research and development sector in this version, there is one less shock (the one associated with the efficiency of R&D), which leads us to estimate the model with all observables but R&D spending. As with the baseline framework, the stochastic processes and the parameters related to adjustment costs in investment and capital utilization are the only objects that we estimate.

Figure 14 displays the estimated smoothed processes under the new specification. At first sight, the estimated liquidity looks similar to the one obtained in our benchmark exercise. On closer look, however, we notice that the measure predicts that the worst part of the crisis happened in second half of 2009. In contrast with our previous results and the anecdotal evidence, liquidity was relatively benign before and after the Lehman shock, according to this model. The depth of the liquidity crunch is substantially milder in the exogenous growth version (-0.25 percent below steady state versus -1.5 percent in the benchmark). This is hardly surprising since we are pushing productivity to be the key driver of the recession. Indeed, productivity bottomed in the fourth quarter of 2008. Interestingly, the time path for the discount factor points to households trying to de-leverage during the crisis (positive innovations mean strong desire to save and consume less). This is in contrast with our benchmark model because the discount factor disturbance does not inflict permanent effects on the economy and hence does not cause a strong wealth effect.

Figure 15 compares the counterfactual paths when liquidity is frozen at its 2008.Q4 in the exogenous growth model (red dotted line). The smaller role of the liquidity shock in this model is apparent from the fact the counterfactual paths are almost indistinguishable from the actual paths (solid blue lines). As one expects, a freeze in liquidity does not drive the economy in a tailspin since liquidity leaves the economy's trend unscathed. For comparison purposes, we also

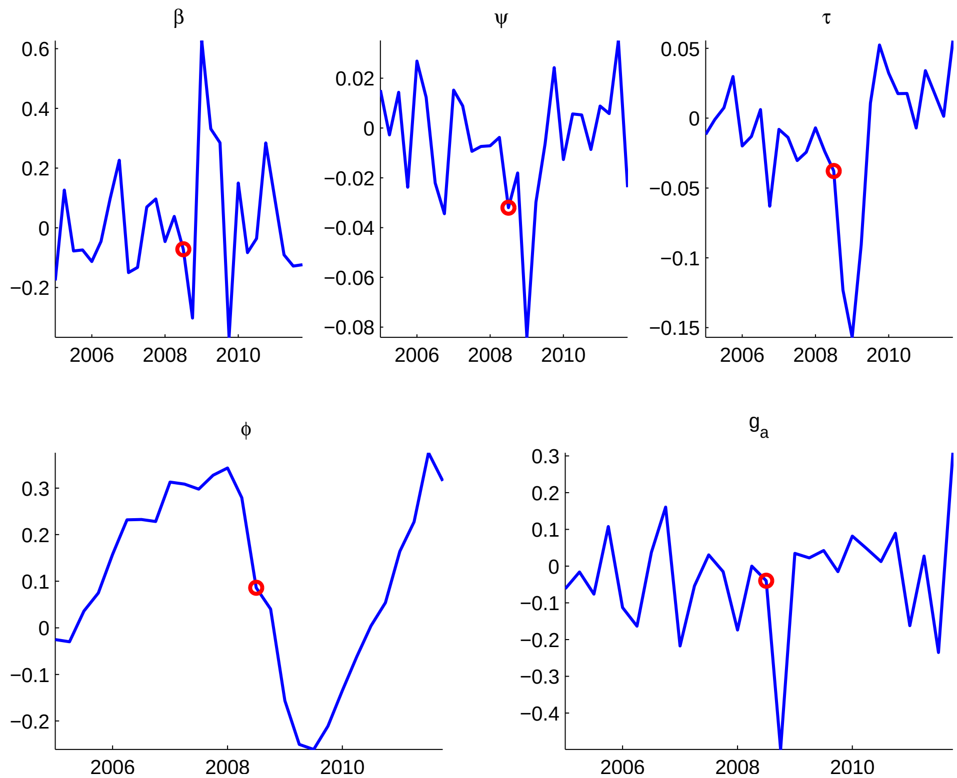

Figure 14: Smoothed Stochastic Processes in Exogenous Growth Model

include the counterfactual from our baseline model (red dashed line), which indicates that in this model, contrary to the RBC model, the liquidity shock is crucially important to account for the Great Recession and the subsequent recovery.[15]

It is the productivity shock whose role the RBC model emphasizes the most. To further appreciate this point, the following exercise even identifies an extraordinarily large productivity drop in 2008.Q4 as the single most important cause of the Great Recession in this model. That is, we plot two counterfactual scenarios (Figure 16) in which productivity shocks are randomized starting from 2008.Q4 and 2009.Q1 respectively, and found that the recession could have been largely averted in the first scenario but not in the second. Note also that the smoothed stochastic processes in the RBC model indicate that the recovery in growth rate back to the pre-crisis level is mainly due to a reversal in productivity growth rather than financial conditions. Putting it in the other way, the shift in the U.S. trend was due to a combination of a large negative productivity drop and the lackluster post-crisis productivity, according to this model. To further underline the fact that the RBC model gives prominence to the productivity shock, we freeze this shock at its 2008.Q4 level. As we see in Figure 17, the economy tailspins if the productivity growth rate is

[15] There are two investment paths because, as explained in the main text, data on R&D spending is lumped together with investment in the model with exogenous growth to make the GDP series comparable across the two exercises.

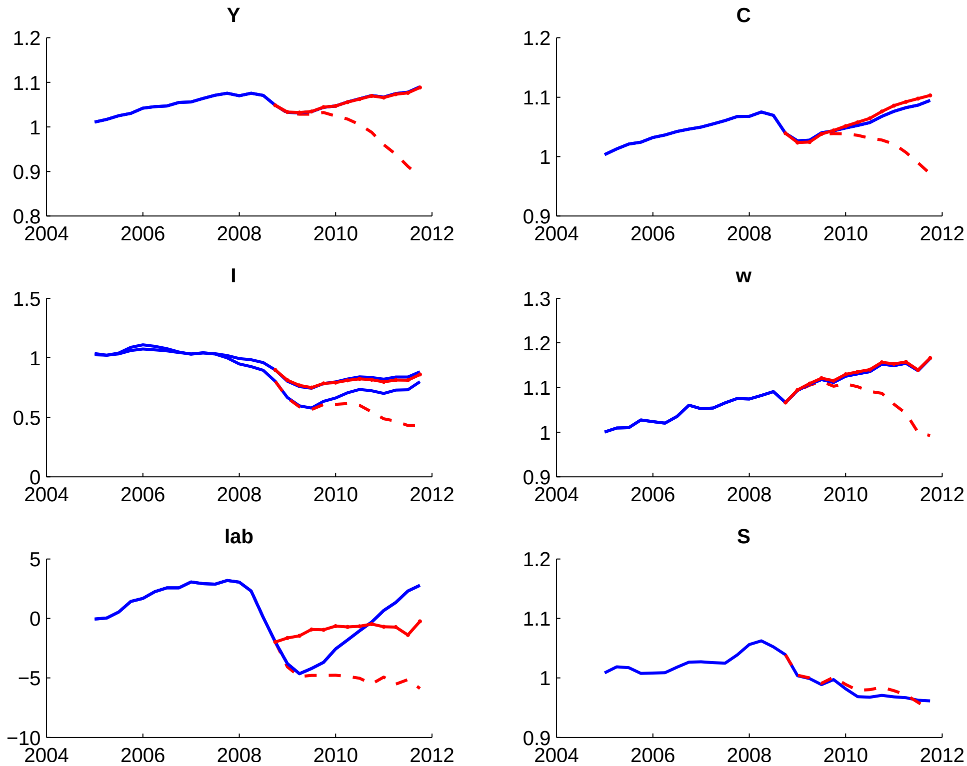

Figure 15: Liquidity Conditions frozen in the RBC Model (red dotted line) and in Endogenous Growth Model (red dashed line).

frozen (red dashed lines). It is interesting that the predicted trajectory is in fact similar to what our benchmark model predicts after a liquidity freeze. In the RBC model, however, freezing the liquidity shock does not have much impact on the simulation (red dotted line).

5 A history of liquidity shocks

In our last exercise, we put the recent crisis in perspective. Figure 18 shows the smoothed paths for the liquidity process (ϕ_t) and the innovations buffeting it. Several interesting points emerge from these figures. Our estimated paths imply that adverse liquidity conditions have been a common denominator over the past recessions. The 1975 and 1980/82 contractions involved deep but short-lived drops in our measure of liquidity. Although the sizes of the innovations buffeting these crises and the Great Recession are comparable, it is clear that the economy quickly reverted to a state with better financial conditions in these episodes. In contrast, the post-Great Recession recovery has seen a milder improvement.

We view the drop in 1975 as the model's attempt to capture the sharp increase in oil prices. Note that intermediate goods in our production function (equation (25)) enter in a similar way as in models with an oil sector. Note also that an expansion of product varieties reduces the price of

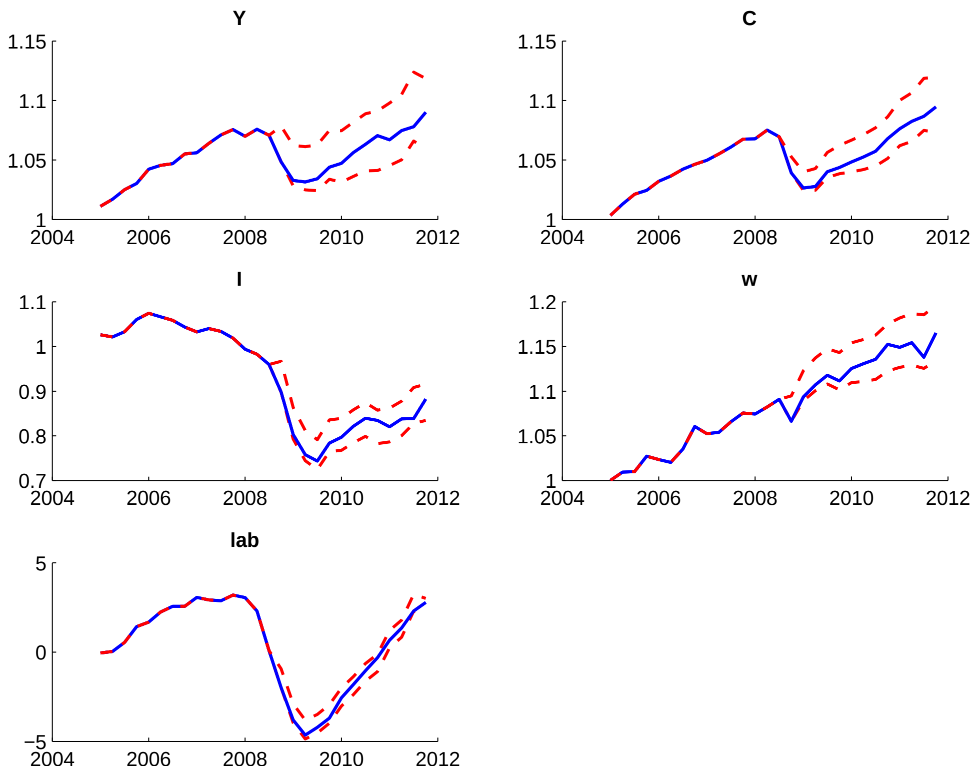

Figure 16: Counterfactual Paths in Exogenous Growth Model with Random Productivity Shocks

intermediate goods composite relative to output, and vice versa (equation (32)). Hence, the model reads the adverse liquidity shock as a worsening of the production process to create intermediate goods. The decline of liquidity in 1980/82 is such that, most likely, the model's interpretation of all the financial changes that arose at that time. Interestingly, our measure does pick up the Black Monday crash in 1987.

Consistent with the evidence reported in Brunnermeier and Pedersen (2009), liquidity fell during the first war in Iraq in the early 1990s. The estimates also reveal that the second half of the 1990s was a period of benign financial (or liquidity) conditions. Indeed, liquidity was above its historical average between 1994 and 2001. In our model, this implies ideal conditions for development of new products and hence strong growth in the economy. In contrast, liquidity was very volatile during the 1970s and 1980s, which most likely resulted in the short periods of sustained growth that the economy experienced in those decades. A similar picture emerges during the last 10 years.

How likely was the Great Recession? Using the history of estimated liquidity shocks up to 2007.Q2, we find that a 95-percent probability set covers the region $(-0.97, 0.70)$. Although the shocks between 2007.Q3 and 2008.Q2 lied in that set, the innovation immediately after the failure of Lehman Brothers (-1.20) was a tail event. To see the sheer size of this shock, Figure 19 plots the density of liquidity shocks (2007.Q2 and before), the shock in 2008.Q4 and the sequence

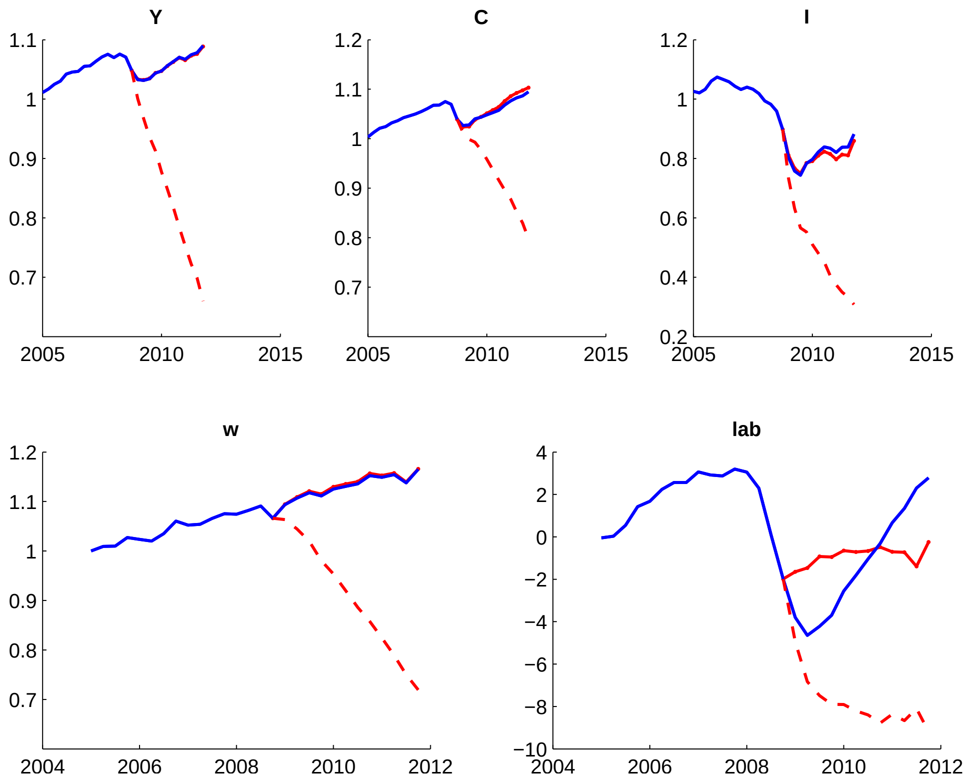

Figure 17: Productivity Growth frozen (red dashed line) and Liquidity Conditions frozen (red dotted line) in their 2008.Q4 Levels.

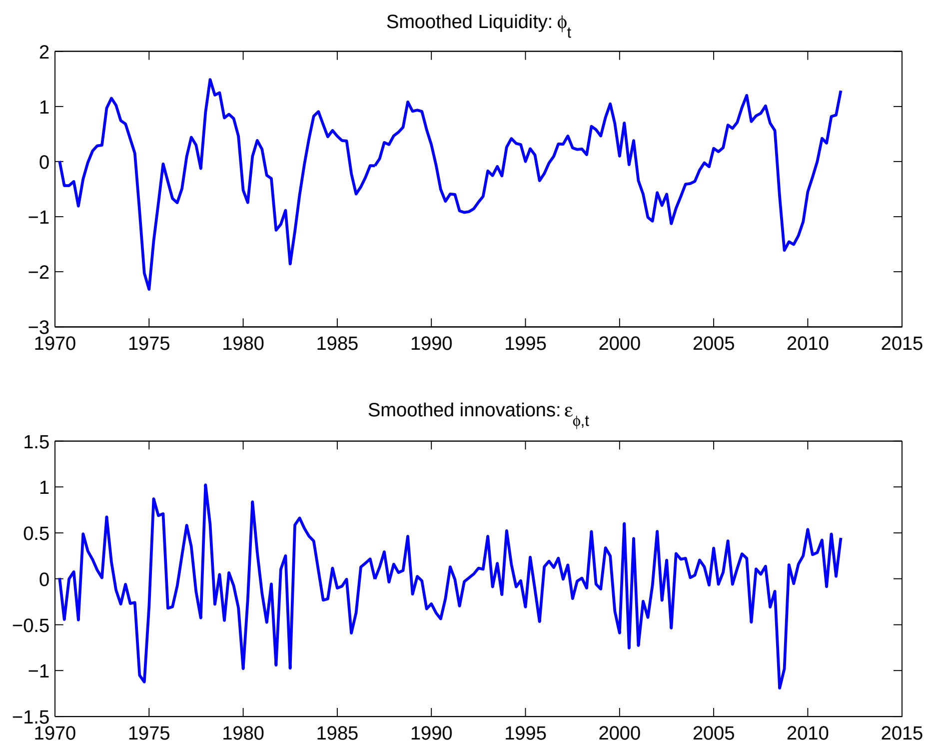

Figure 18: Time paths for liquidity and its innovations

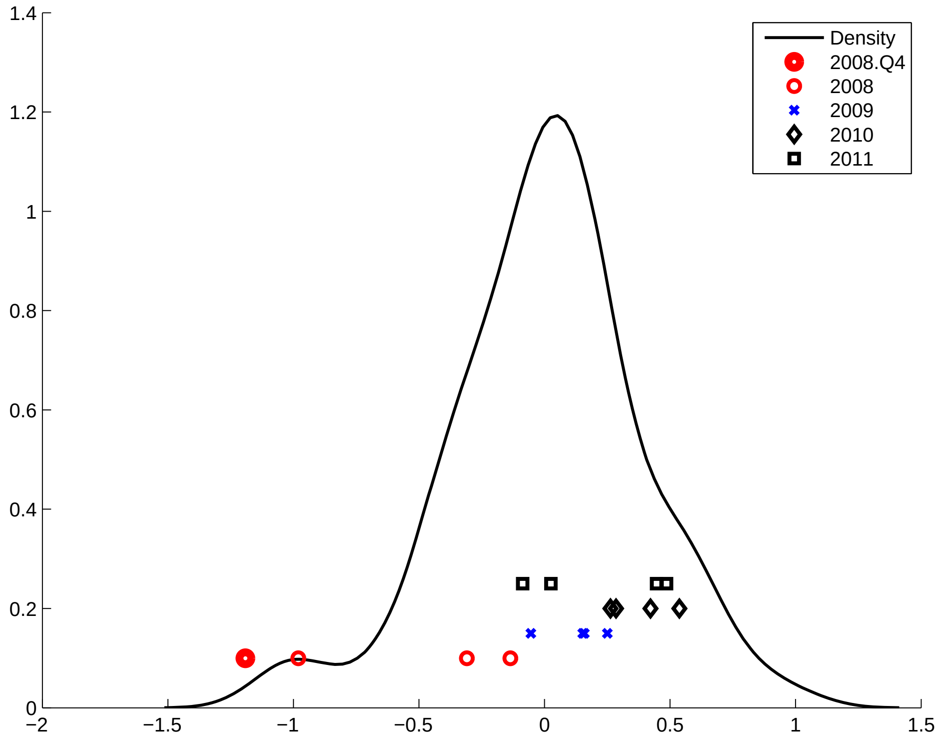

Figure 19: Density of smoothed liquidity shocks $\varepsilon_{\phi,t}$

of shocks between 2008 and 2011. The density also indicates that the distribution is skewed to the left (skewness = -0.29) and has a kurtosis of 3.7. Together these findings suggest that the structural shocks are far from normal distributed (see Fernandez-Villaverde et al. (2013) for a formal treatment of the role of asymmetric distributions in business cycles).

6 Conclusion

Adverse financial conditions during the recent recession seem to have played a critical role. Our model shows that financial shocks affecting the resaleability of equity is an example of the cross-winds the economy faced in 2008-09. But illiquidity was far from being the solely financial malice. Default risk is another instance that most likely exacerbated the crisis. As a consequence, our model captures just one aspect of the Great Recession and hence our results may well be a lower bound of the true impact that financial frictions had during the crisis.

7 Appendix

7.1 Solving the household's problem

Equation (20) is derived as follows. Combining (11) and (14), we find

$$\frac{\mu_t^e/\sigma_e}{\mu_t^w/\sigma_w} = \frac{p_{n,t}\vartheta_t\,(1-\theta)}{1-\theta p_{n,t}\vartheta_t} \tag{38}$$

Combining (10), (12), and (38), we obtain equation (20).

Equation (22) is derived as follows. Combining (14) and (17), we find

$$\frac{\partial v_t}{\partial n_t} = \left(\frac{\mu_t^w}{\sigma_w}\right)\sigma_e\,(1-\phi_t)\,(1-\delta_n)\,p_{n,t} + \mu_t^e\left[\Pi_t + p_{n,t}\,(1-\delta_n)\,\phi_t\right] + \mu_t^w\,(\Pi_t + p_{n,t}\,(1-\delta_n))$$

Substituting it into (14), we find

$$p_{n,t} = E_t\left[\begin{array}{c}\left(\beta_t\frac{\mu_{t+1}^w}{\mu_t^w}\right)\\ \left(\begin{array}{c}\sigma_e\left(1-\phi_{t+1}\right)(1-\delta_n)\,p_{n,t+1}\\ +\sigma_w\frac{\mu_{t+1}^e}{\mu_{t+1}^w}\left[\Pi_{t+1}+p_{n,t+1}\,(1-\delta_n)\,\phi_{t+1}\right]+\sigma_w\,(\Pi_{t+1}+p_{n,t+1}\,(1-\delta_n))\end{array}\right)\end{array}\right]$$

Since $\sigma_w = 1-\sigma_e$, we can rewrite the previous equation as

$$p_{n,t} = E_t\left[\begin{array}{c}\left(\beta_t\frac{\mu_{t+1}^w}{\mu_t^w}\right)\\ \left(\Pi_{t+1}+p_{n,t+1}\,(1-\delta_n)+\sigma_e\left(-1+\frac{\sigma_w}{\sigma_e}\frac{\mu_{t+1}^e}{\mu_{t+1}^w}\right)\left[\Pi_{t+1}+p_{n,t+1}\,(1-\delta_n)\,\phi_{t+1}\right]\right)\end{array}\right]$$

Using (21) and (38), we obtain equation (22).

Equation (23) is derived as follows. Combining (15) and (18), we find

$$\frac{\partial v_t}{\partial k_t} = \left(\frac{\mu_t^w}{\sigma_w}\right)\sigma_e\,(1-\phi_t)\,(1-\delta\,(u_t))\,p_{k,t} + \mu_t^e\left[R_t u_t + p_{k,t}\,(1-\delta\,(u_t))\,\phi_t\right] + \mu_t^w\,(R_t u_t + p_{k,t}\,(1-\delta\,(u_t)))$$

Substituting it into (15), we find

$$p_{k,t} = E_t\left[\begin{array}{c}\left(\beta_t\frac{\mu_{t+1}^w}{\mu_t^w}\right)\\ \left(\begin{array}{c}\sigma_e\left(1-\phi_{t+1}\right)(1-\delta\,(u_{t+1}))\,p_{k,t+1}\\ +\sigma_w\frac{\mu_{t+1}^e}{\mu_{t+1}^w}\left[R_{t+1}u_{t+1}+p_{k,t+1}\,(1-\delta\,(u_{t+1}))\,\phi_{t+1}\right]+\sigma_w\,(R_{t+1}u_{t+1}+p_{k,t+1}\,(1-\delta\,(u_{t+1})))\end{array}\right)\end{array}\right]$$

Since $\sigma_w = 1-\sigma_e$, we can rewrite the previous equation as

$$p_{k,t} = E_t\left[\begin{array}{c}\left(\beta_t\frac{\mu_{t+1}^w}{\mu_t^w}\right)\\ \left(R_{t+1}u_{t+1}+p_{k,t+1}\,(1-\delta\,(u_{t+1}))+\sigma_e\left(-1+\frac{\sigma_w}{\sigma_e}\frac{\mu_{t+1}^e}{\mu_{t+1}^w}\right)\left[R_{t+1}u_{t+1}+p_{k,t+1}\,(1-\delta\,(u_{t+1}))\,\phi_{t+1}\right]\right)\end{array}\right]$$

Using (21) and (38), we obtain equation (23).

Equation (24) is derived as follows. Combining (9) and (15), we find

$$(1-\phi_t)\left(-\delta'(u_t)\right)p_{k,t}+\frac{\sigma_w}{\sigma_e}\frac{\mu_t^e}{\mu_t^w}\left(R_t+p_{k,t}\phi_t\left(-\delta'(u_t)\right)\right)+\frac{\sigma_w}{\sigma_e}\left(R_t+p_{k,t}\left(-\delta'(u_t)\right)\right)=0$$

Using (38), we find

$$R_t+p_{k,t}\left(-\delta'(u_t)\right)+\sigma_e\lambda_t^e\left(R_t+p_{k,t}\phi_t\left(-\delta'(u_t)\right)\right)=0$$

Combining (15) and (19),

$$\frac{\partial v_t}{\partial i_{t-1}}=\mu_t^w p_{k,t}\Lambda'\left(\frac{i_t}{i_{t-1}}\right)\left(\frac{i_t}{i_{t-1}}\right)^2 \tag{39}$$

Substituting (15) and (39) into (16), we obtain equation (24).

7.2 Model summary

The following eighteen equations jointly determine the equilibrium dynamics of eighteen endogenous variables, Y_t, Z_t, K_t, c_t^e, c_t^w, λ_t^e, s_t, M_t, $p_{n,t}$, Π_t, N_t, i_t, u_t, $p_{k,t}$, W_t, R_t, l_t, and ϑ_t:

$$Y_t=(u_tK_t)^{\alpha}\left(Z_t\sigma_w l_t\right)^{1-\alpha}$$

$$Z_t=\left(\bar{A}\right)\left(A_tN_t\right)$$

$$\psi_t l_t^{\zeta}=W_t\left(\frac{1}{c_t^w}\right)$$

$$W_t=(1-\xi)(1-\alpha)\frac{Y_t}{\sigma_w l_t}$$

$$\left(\frac{1}{c_t^e}\right)=\left(1+\lambda_t^e\right)\left(\frac{1}{c_t^w}\right)$$

$$\lambda_t^e=\frac{p_{n,t}\vartheta_t-1}{1-\theta p_{n,t}\vartheta_t}$$

$$p_{n,t}=E_t\left[M_{t+1}\left(\Pi_{t+1}+p_{n,t+1}(1-\delta_n)+\sigma_e\lambda_{t+1}^e\left[\Pi_{t+1}+p_{n,t+1}(1-\delta_n)\phi_{t+1}\right]\right)\right]$$

$$M_{t+1}=\beta_t\left(\frac{c_t^w}{c_{t+1}^w}\right)$$

$$\Pi_t=\left(\frac{\nu-1}{\nu}\right)\xi\frac{Y_t}{N_t}$$

$$R_t=(1-\xi)\alpha\frac{Y_t}{u_tK_t}$$

$$p_{k,t}=E_t\left[M_{t+1}\left(R_{t+1}u_{t+1}+p_{k,t+1}\left(1-\delta(u_{t+1})\right)+\sigma_e\lambda_{t+1}^e\left[R_{t+1}u_{t+1}+p_{k,t+1}\left(1-\delta(u_{t+1})\right)\phi_{t+1}\right]\right)\right]$$

$$R_t + p_{k,t}\left(-\delta'\left(u_t\right)\right) + \sigma_e \lambda_t^e \left(R_t + p_{k,t}\phi_t\left(-\delta'\left(u_t\right)\right)\right) = 0$$

$$1 = p_{k,t}\left(1 - \Lambda\left(\frac{i_t}{i_{t-1}}\right) - \Lambda'\left(\frac{i_t}{i_{t-1}}\right)\frac{i_t}{i_{t-1}}\right) + E_t\left[M_{t+1}p_{k,t+1}\Lambda'\left(\frac{i_{t+1}}{i_t}\right)\left(\frac{i_{t+1}}{i_t}\right)^2\right]$$

$$\begin{aligned} & c_t^e + p_{n,t}\left[\left(1-\theta\right)\vartheta_t s_t - \phi_t\left(1-\delta_n\right)N_t\right] - p_{k,t}\phi_t\left(1-\delta\left(u_t\right)\right)K_t \\ = \quad & \Pi_t N_t + R_t\left(u_t K_t\right) + \left(p_{n,t}\vartheta_t - 1\right)s_t - \tau_t\left(1 - \frac{\xi}{\nu}\right)Y_t \end{aligned}$$

$$\begin{aligned} & c_t^w + i_t + \frac{p_{n,t}}{\sigma_w}\left(N_{t+1} - \sigma_e\left[\left(1-\theta\right)\vartheta_t s_t + \left(1-\phi_t\right)\left(1-\delta_n\right)N_t\right]\right) \\ & + \frac{p_{k,t}}{\sigma_w}\left(1-\delta\left(u_t\right)\right)\left(1-\sigma_e\left(1-\phi_t\right)\right)K_t \\ = \quad & \left[\Pi_t + p_{n,t}\left(1-\delta_n\right)\right]N_t + \left[R_t u_t + p_{k,t}\left(1-\delta\left(u_t\right)\right)\right]K_t + W_t l_t - \tau_t\left(1-\frac{\xi}{\nu}\right)Y_t \end{aligned}$$

$$\vartheta_t = \chi_t\left(\frac{\sigma_e s_t}{N_t}\right)^{\eta-1}$$

$$\left(1-\tau_t\right)\left(1-\frac{\xi}{\nu}\right)Y_t = \sigma_e c_t^e + \sigma_w c_t^w + \sigma_w i_t + \sigma_e s_t$$

$$K_{t+1} = \left(1-\delta\left(u_t\right)\right)K_t + \left(1-\Lambda\left(\frac{i_t}{i_{t-1}}\right)\right)\left(\sigma_w i_t\right)$$

Variables having a trend are detrended by the common stochastic trend N_t. The resulting system involves only stationary variables, being summarized by the following equations:

$$\hat{Y}_t = \left(u_t\hat{K}_t\right)^{\alpha}\left(\left(\bar{A}\right)\left(A_t\right)\left[\sigma_w l_t\right]\right)^{1-\alpha}$$

$$\psi_t l_t^{\zeta} = \frac{\hat{W}_t}{\hat{c}_t^w}$$

$$\hat{W}_t = \left(1-\xi\right)\left(1-\alpha\right)\frac{\hat{Y}_t}{\sigma_w l_t}$$

$$\frac{1}{\hat{c}_t^e} = \left(1+\lambda_t^e\right)\frac{1}{\hat{c}_t^w}$$

$$\lambda_t^e = \frac{p_{n,t}\vartheta_t - 1}{1-\theta p_{n,t}\vartheta_t}$$

$$p_{n,t} = E_t\left[M_{t+1}\left(\Pi_{t+1} + p_{n,t+1}\left(1-\delta_n\right) + \sigma_e\lambda_{t+1}^e\left[\Pi_{t+1} + p_{n,t+1}\left(1-\delta_n\right)\phi_{t+1}\right]\right)\right]$$

$$M_{t+1} = \beta_t\frac{1}{g_{t+1}}\frac{\hat{c}_t^w}{\hat{c}_{t+1}^w}$$

$$\Pi_t = \left(\frac{\nu - 1}{\nu}\right) \xi \hat{Y}_t$$

$$R_t = (1 - \xi)\, \alpha \frac{\hat{Y}_t}{u_t \hat{K}_t}$$

$$p_{k,t} = E_t \left[M_{t+1} \left(R_{t+1} u_{t+1} + p_{k,t+1} \left(1 - \delta\left(u_{t+1}\right)\right) + \sigma_e \lambda^e_{t+1} \left[R_{t+1} u_{t+1} + p_{k,t+1} \left(1 - \delta\left(u_{t+1}\right)\right) \phi_{t+1} \right] \right) \right]$$

$$R_t + p_{k,t} \left(-\delta'\left(u_t\right)\right) + \sigma_e \lambda^e_t \left(R_t + p_{k,t} \phi_t \left(-\delta'\left(u_t\right)\right)\right) = 0$$

$$1 = p_{k,t} \left(1 - \Lambda\left(g_t \frac{\hat{\imath}_t}{\hat{\imath}_{t-1}}\right) - \Lambda'\left(g_t \frac{\hat{\imath}_t}{\hat{\imath}_{t-1}}\right)\left(g_t \frac{\hat{\imath}_t}{\hat{\imath}_{t-1}}\right)\right) + E_t \left[M_{t+1} p_{k,t+1} \Lambda'\left(g_{t+1} \frac{\hat{\imath}_{t+1}}{\hat{\imath}_t}\right)\left(g_{t+1} \frac{\hat{\imath}_{t+1}}{\hat{\imath}_t}\right)^2 \right]$$

$$\begin{aligned} & \hat{c}^e_t + p_{n,t} \left[(1 - \theta)\, \vartheta_t \hat{s}_t - \phi_t \left(1 - \delta_n\right) \right] - p_{k,t} \phi_t \left(1 - \delta\left(u_t\right)\right) \hat{K}_t \\ = \quad & \Pi_t + R_t \left(u_t \hat{K}_t\right) + \left(p_{n,t} \vartheta_t - 1\right) \hat{s}_t - \tau_t \left(1 - \frac{\xi}{\nu}\right) \hat{Y}_t \end{aligned}$$

$$\begin{aligned} & \hat{c}^w_t + \hat{\imath}_t + \frac{p_{n,t}}{\sigma_w} \left(g_{t+1} - \sigma_e \left[(1 - \theta)\, \vartheta_t \hat{s}_t + \left(1 - \phi_t\right)\left(1 - \delta_n\right) \right]\right) \\ & + \frac{p_{k,t}}{\sigma_w} \left(1 - \delta\left(u_t\right)\right) \left(1 - \sigma_e \left(1 - \phi_t\right)\right) \hat{K}_t \\ = \quad & \Pi_t + p_{n,t} \left(1 - \delta_n\right) + \left[R_t u_t + p_{k,t} \left(1 - \delta\left(u_t\right)\right) \right] \hat{K}_t + \hat{W}_t l_t - \tau_t \left(1 - \frac{\xi}{\nu}\right) \hat{Y}_t \end{aligned}$$

$$\vartheta_t = \chi_t \left(\sigma_e \hat{s}_t\right)^{\eta - 1}$$

$$\left(1 - \tau_t\right) \left(1 - \frac{\xi}{\nu}\right) \hat{Y}_t = \sigma_e \hat{c}^e_t + \sigma_w \hat{c}^w_t + \sigma_w \hat{\imath}_t + \sigma_e \hat{s}_t$$

$$g_{t+1} \hat{K}_{t+1} = \left(1 - \delta\left(u_t\right)\right) \hat{K}_t + \left(1 - \Lambda\left(g_t \frac{\hat{\imath}_t}{\hat{\imath}_{t-1}}\right)\right) \left(\sigma_w \hat{\imath}_t\right)$$

Hat variables denote the original variables divided by N_t, i.e., $\hat{Y}_t = Y_t / N_t$, and so on, and $g_{t+1} = N_{t+1} / N_t$.

7.3 Steady state

This section briefly describes how to find the steady state. The steady state sector-specific productivity shock χ, the steady state subjective discount factor β, the steady state neutral technology level A, the steady state labor disutility shock ψ, and the capital depreciation rate δ_k are set to match the following five targets: the steady state growth rate g, the steady state R&D share $\sigma_e \hat{s} / \hat{\mathcal{Y}}$, steady state investment to output ratio, the steady state gross output $\hat{Y}$ which is normalized to be unity, and the steady state labor l which is also normalized to be unity.

$$\Pi = \frac{\Pi}{\hat{Y}} = \underbrace{\left(\frac{\nu - 1}{\nu}\right)\xi}_{\text{known}}$$

$$\sigma_e \hat{s} = \frac{\sigma_e \hat{s}}{\hat{Y}} = \underbrace{\left(1 - \frac{\xi}{\nu}\right)\left(\frac{\sigma_e \hat{s}}{\hat{\mathcal{Y}}}\right)}_{\text{known}}$$

$$\hat{s} = \underbrace{(\sigma_e \hat{s})\left(\frac{1}{\sigma_e}\right)}_{\text{known}}$$

$$\vartheta = \underbrace{\frac{g - 1 + \delta_n}{\sigma_e \hat{s}}}_{\text{known}}$$

$$p_k = 1$$

$$\hat{W} = (1 - \xi)(1 - \alpha)\frac{\hat{Y}}{\sigma_w l} = \underbrace{(1 - \xi)(1 - \alpha)\frac{1}{\sigma_w}}_{\text{known}}$$

$$Ru = (1 - \xi)\,\alpha \frac{\hat{Y}}{\hat{K}} = (1 - \xi)\,\alpha \frac{1}{\hat{K}}$$

$$\hat{K}(g - 1 + \delta_k) = \sigma_w \hat{\imath}$$

The following six equations solve six unknowns $\hat{c}^e$, $\hat{c}^w$, λ^e, p_n, $\hat{K}$, and M.

$$\hat{c}^w = (1 + \lambda^e)\,\hat{c}^e$$

$$\lambda^e = \frac{(p_n)(\vartheta) - 1}{1 - \theta(p_n)(\vartheta)}$$

$$p_n = M\left(\Pi + p_n(1 - \delta_n) + \sigma_e \lambda^e\left[\Pi + p_n(1 - \delta_n)\phi_n\right]\right)$$

$$1 = M\left((1 - \xi)\,\alpha \frac{1}{\hat{K}} + 1 - \delta_k + \sigma_e \lambda^e\left[(1 - \xi)\,\alpha \frac{1}{\hat{K}} + (1 - \delta_k)\phi_k\right]\right)$$

$$\hat{c}^e + p_n\left[(1 - \theta)(\vartheta)(\hat{s}) - \phi_n(1 - \delta_n)\right] - \phi_k(1 - \delta_k)\hat{K} = \Pi + (1 - \xi)\,\alpha + (p_n(\vartheta) - 1)(\hat{s}) - \tau\left(1 - \frac{\xi}{\nu}\right)$$

$$(1 - \tau)\left(1 - \frac{\xi}{\nu}\right) = \sigma_e \hat{c}^e + \sigma_w \hat{c}^w + \hat{K}(g - 1 + \delta_k) + \sigma_e \hat{s}$$

The subjective discount rate β is backed out:

$$\beta = \underbrace{(g)(M)}_{\text{known}}$$

Other steady state values are found by

$$R = \underbrace{(1-\xi)\,\alpha \frac{1}{u}\frac{1}{\hat{K}}}_{\text{known}}$$

$$\hat{\imath} = \underbrace{\frac{1}{\sigma_w}\,(g-1+\delta_k)\,\hat{K}}_{\text{known}}$$

$$\delta'(u)\,u = \underbrace{\left(\frac{1+\sigma_e\lambda^e}{1+\sigma_e\lambda^e\phi_k}\right)Ru}_{\text{known}}$$

The steady state a is backed out from

$$1 = \hat{Y} = \left(u\hat{K}\right)^{\alpha}\left(\bar{A}e^{a}\sigma_w\right)^{1-\alpha}$$

The following parameters are backed out:

$$\psi = \underbrace{\frac{\hat{W}}{\hat{c}^w}}_{\text{known}}$$

$$\chi = \underbrace{\frac{\vartheta}{(\sigma_e\hat{s})^{\eta-1}}}_{\text{known}}$$

7.4 RBC model with non-stationary productivity shock

We consider a large household which has a unit measure of members. A member will be an investor with probability $\sigma_i \in [0,1]$ and a worker with probability $\sigma_w \in [0,1]$. They satisfy $\sigma_i + \sigma_w = 1$. These shocks are *iid* among the members and across time.

A period is divided into four stages: household's decisions, production, investment, and consumption. In the stage of household's decision, all members of a household are together to pool their assets, i.e., k_t units of physical capital. Aggregate shocks to exogenous state variables are realized. The capacity utilization rate u_t is decided, which is applied to all the capital the household possesses. Because the members in the household are identical in this stage, the household evenly divides the assets among the members. The head of the household also gives contingency plans to each member, saying if one becomes an investor, he or she invests i_t units of consumption goods, consumes c_t^i units of consumption goods, and makes necessary trades in the capital market so that he or she returns to the household with k_{t+1}^i units of capital, and if one becomes a worker, he or she supplies l_t units of labor, consumes c_t^w units of consumption goods, and makes necessary trades in the capital market so that he or she returns to the household with k_{t+1}^w units of capital.

After receiving these instructions, the members go to the market and will remain separated from each other for the remaining of the period.

At the beginning of the production stage, each member receives the shock whose realization determines whether the individual is an investor or a worker. Competitive firms produce final consumption goods from capital service and labor service. After production, a worker receives wage income, and an individual receives compensation for capital service. The government collects a uniform, lump-sum tax T_t from each member. Then, a fraction $\delta(u_t)$ of capital depreciates.

The third stage in the period is investment stage. The goods market and the capital market are open, and investors seek finance and undertake investment of the scale instructed by the household. We assume that an investor can transform i_t units of consumption goods into the following units of capital goods

$$\left(1 - \Lambda\left(\frac{i_t}{i^*_{t-1}}\right)\right) i_t$$

where $\Lambda(\cdot)$ is the capital adjustment costs given by

$$\Lambda\left(\frac{i_t}{i^*_{t-1}}\right) = \frac{\bar{\Lambda}}{2}\left(g - \frac{i_t}{i^*_{t-1}}\right)^2$$

i^*_{t-1} is the cross-sectional average investment level of investors in the previous period, which an individual household takes as given, and g is the steady state growth rate of the technology level.[16] Individuals trade assets to finance investment and to achieve the portfolio of asset holdings instructed earlier by their households. In the consumption stage, both a worker and an investor consume.

The instructions have to satisfy a set of constraints. First, the instruction to an investor has to satisfy the intra-period budget constraint:

$$\underbrace{c^i_t + i_t + \underbrace{p_{k,t}k^i_{t+1}}_{\text{gross equity purchases}}}_{\text{gross expenditure}} = \underbrace{\underbrace{R_t(u_t k_t)}_{\text{rental}} + \underbrace{p_{k,t}\left[(1-\delta(u_t))k_t + \left(1 - \Lambda\left(\frac{i_t}{i^*_{t-1}}\right)\right) i_t\right]}_{\text{resale value + value of new capital}} - T_t}_{\text{gross after-tax income}} \quad (40)$$

The left-hand side is the gross total expenditure, collecting bills on consumption, investment, and gross equity purchases, where $p_{k,t}$ is the price of capital. The right-hand side is the gross after-tax total income, collecting compensation for capital service, resale values of assets, and the value of newly installed capital goods, subtracting the lump-sum tax. The constraint therefore states that

[16] Notice that investment adjustment cost function is formally similar to the one in the baseline model since $i^*_{t-1} = i_{t-1}$ holds in equilibrium, but is different from it because the household in the current model does not internalize effects of current investment on future investment adjustment costs. This is for analytical convenience; the liquidity constraint and dynamic investment adjustment costs are hard to be analyzed together. Del Negro, Eggertsson, Ferrero, and Kiyotaki (2011) and Shi (2012) make similar assumptions so that investment adjustment costs only depend on the current level of investment.

the total expenditure and the total income has to be balanced within a period, in which an investor is separated from other members of the household. A similar constraint applies to a worker:

$$c_t^w + p_{k,t} k_{t+1}^w = R_t (u_t k_t) + p_{k,t} (1 - \delta (u_t)) k_t + W_t l_t - T_t \tag{41}$$

There are other, crucial constraints on trading of assets. That is, an investor can sell at most a fraction θ of newly installed capital, but has to retain the rest by herself. In addition, she can sell a fraction ϕ_t of existing capital to others in the asset markets, but has to retain the rest by herself. Effectively, these constraints introduce a lower bound to the equity holdings of an investor at the end of the period:

$$k_{t+1}^i \geq \underbrace{(1-\theta)\left(1 - \Lambda\left(\frac{i_t}{i_{t-1}^*}\right)\right) i_t}_{\text{newly installed capital required to retain}} + \underbrace{(1-\phi_t)(1-\delta(u_t)) k_t}_{\text{existing capital required to retain}} \tag{42}$$

Similar constraint applies to workers, i.e., $k_{t+1}^w \geq (1-\phi_t)(1-\delta(u_t)) k_t$, but we drop it because it does not bind in the equilibrium. There are non-negativity constraints for u_t, c_t^i, l_t, i_t, c_t^w, and k_{t+1}^w, but we drop them too because they do not bind in the equilibrium either.

The head of the household chooses the instructions to its members to maximize

$$v(k_t; \Gamma_t, \Theta_t) = \max \left\{ \sigma_i \log\left(c_t^i\right) + \sigma_w \left[\log\left(c_t^w\right) - \psi_t \frac{l_t^{1+\zeta}}{1+\zeta}\right] + \beta_t E_t \left[v\left(k_{t+1}; \Gamma_{t+1}, \Theta_{t+1}\right)\right] \right\} \tag{43}$$

subject to (40), (41), (42), and

$$k_{t+1} = \sigma_i k_{t+1}^i + \sigma_w k_{t+1}^w \tag{44}$$

β_t is a subjective time discount factor and ψ_t is a coefficient affecting the labor disutility schedule, both of which are common across households and are exogenous random variables. Γ_t is the vector of endogenous, aggregate state variables, i.e., $\Gamma_t = \left(K_t, i_{t-1}^*\right)$, where K_t is the capital stock in the economy. Θ_t is the vector of exogenous state variables.

We will restrict our attention to the case in which the inequality $p_{k,t} > p_{i,t}$ always holds in the equilibrium, where $p_{i,t}$ is the shadow price of newly installed capital defined as

$$p_{i,t} \equiv \left(1 - \Lambda\left(\frac{i_t}{i_{t-1}^*}\right) - \Lambda'\left(\frac{i_t}{i_{t-1}^*}\right)\frac{i_t}{i_{t-1}^*}\right)^{-1} \tag{45}$$

The liquidity constraint (42) must be binding at the optimum. Otherwise, the household can increase the utility without violating any constraints by marginally increasing investment by $\Delta > 0$ units, which creates $\Delta / p_{i,t}$ units of new capital, selling it in the capital market, and increasing investor's consumption by $(p_{k,t}/p_{i,t} - 1)\Delta$ units.

With this binding constraint given, we can rewrite the household's problem as choosing u_t, i_t,

c_t^i, l_t, c_t^w, and k_{t+1}^w to maximize the value function (43) subject to (41) and

$$c_t^i + i_t + p_{k,t}\left[-\theta\left(1-\Lambda\left(\frac{i_t}{i_{t-1}^*}\right)\right)i_t - \phi_t\left(1-\delta\left(u_t\right)\right)k_t\right] = R_t\left(u_t k_t\right) - T_t \tag{46}$$

$$k_{t+1} = \sigma_i\left[\left(1-\theta\right)\left(1-\Lambda\left(\frac{i_t}{i_{t-1}^*}\right)\right)i_t + \left(1-\phi_t\right)\left(1-\delta\left(u_t\right)\right)k_t\right] + \sigma_w k_{t+1}^w$$

The representative final good producing firm uses capital service KS_t and labor L_t to produce the final (consumption) good according to the production technology

$$Y_t = \left(KS_t\right)^\alpha \left(A_t L_t\right)^{1-\alpha}$$

A_t is the productivity shock following a trend-stationary process:

$$A_t = e^{a_t}$$

$$\Delta a_t = \log\left(g\right) + \rho_a \Delta a_{t-1} + \varepsilon_{a,t}$$

The firm maximizes profits defined as

$$Y_t - R_t\left(KS_t\right) - W_t L_t$$

In the symmetric equilibrium, $i_{t-1}^* = i_{t-1}$ holds and the aggregate capital stock K_t evolves as

$$K_{t+1} = \left(1-\delta\left(u_t\right)\right)K_t + \left(1-\Lambda\left(\frac{i_t}{i_{t-1}}\right)\right)\left(\sigma_i i_t\right)$$

The government spends a fraction τ_t of the output Y_t. We assume that the government keeps the balanced-budget:

$$\tau_t Y_t = T_t$$

The competitive equilibrium is defined in a standard way. Market clearing conditions for production factors are

$$KS_t = u_t K_t$$

$$L_t = \sigma_w l_t$$

Goods market clearing condition is

$$Y_t = \sigma_i c_t^i + \sigma_w c_t^w + \sigma_i i_t + T_t \tag{47}$$

Asset market clearing conditions are

$$K_t = k_t$$

at the beginning of the period and

$$K_{t+1} = k_{t+1}$$

at the end of the period.

The derivations of both first order conditions and equilibrium conditions in the current model are analogous to those in the benchmark model. The following equations summarize the model economy:

$$Y_t = (u_t K_t)^{\alpha} \left(e_t^{a_t} \sigma_w l_t\right)^{1-\alpha}$$

$$\psi_t l_t^{\zeta} = W_t \frac{1}{c_t^w}$$

$$W_t = (1-\alpha) \frac{Y_t}{\sigma_w l_t}$$

$$\frac{1}{c_t^i} = (1+\lambda_t) \frac{1}{c_t^w}$$

$$\lambda_t = \frac{(p_{k,t}/p_{i,t}) - 1}{1 - \theta (p_{k,t}/p_{i,t})}$$

$$R_t = \alpha \frac{Y_t}{u_t K_t}$$

$$1 = p_{i,t} \left(1 - \Lambda\left(\frac{i_t}{i_{t-1}}\right) - \Lambda'\left(\frac{i_t}{i_{t-1}}\right)\frac{i_t}{i_{t-1}}\right)$$

$$p_{k,t} = E_t \left[\left(\beta_t \frac{c_t^w}{c_{t+1}^w}\right)\left(R_{t+1} u_{t+1} + p_{k,t+1}\left(1 - \delta\left(u_{t+1}\right)\right) + \sigma_i \lambda_{t+1}\left[R_{t+1} u_{t+1} + p_{k,t+1}\left(1 - \delta\left(u_{t+1}\right)\right)\phi_{t+1}\right]\right)\right]$$

$$R_t + p_{k,t}\left(-\delta'\left(u_t\right)\right) + \sigma_i \lambda_t \left(R_t + p_{k,t} \phi_t \left(-\delta'\left(u_t\right)\right)\right) = 0$$

$$c_t^i + i_t + p_{k,t}\left[-\theta\left(1 - \Lambda\left(\frac{i_t}{i_{t-1}}\right)\right) i_t - \phi_t \left(1 - \delta\left(u_t\right)\right) K_t\right] = R_t\left(u_t K_t\right) - \tau_t Y_t$$

$$\begin{aligned} & c_t^w + \frac{p_{k,t}}{\sigma_w}\left(K_{t+1} - \sigma_i\left[(1-\theta)\left(1 - \Lambda\left(\frac{i_t}{i_{t-1}}\right)\right) i_t + (1-\phi_t)\left(1 - \delta\left(u_t\right)\right) K_t\right]\right) \\ = \; & \left[R_t u_t + p_{k,t}\left(1 - \delta\left(u_t\right)\right)\right] K_t + W_t l_t - \tau_t Y_t \end{aligned}$$

$$(1-\tau_t) Y_t = \sigma_i c_t^i + \sigma_w c_t^w + \sigma_i i_t$$

The twelve equations jointly determine the equilibrium dynamics of twelve endogenous variables, Y_t, K_t, c_t^i, c_t^w, λ_t, i_t, u_t, $p_{k,t}$, $p_{i,t}$, W_t, R_t, and l_t.

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