JSPS Grants-in-Aid for Scientific Research (S)

Understanding Persistent Deflation in Japan

Working Paper Series

No. 023

October 2013

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The Consumer Price Index: Recent Developments

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October 24, 2013

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Abstract

The 2004 International Labour Office *Consumer Price Index Manual: Theory and Practice* summarized the state of the art for constructing Consumer Price Indexes (CPIs) at that time. In the intervening decade, there have been some significant new developments which are reviewed in this paper. The *CPI Manual* recommended the use of chained superlative indexes for a month to month CPI. However, subsequent experience with the use of monthly scanner data has shown that a significant chain drift problem can occur. The paper explains the nature of the problem and reviews possible solutions to overcome the problem. The paper also describes the recently developed Time Dummy Product method for constructing elementary index numbers (indexes at lower levels of aggregation where only price information is available).

Key Words

Consumer Price Indexes, superlative indexes, chain drift, scanner data, Time Product Dummy method, GEKS method for making international comparisons, Rolling Year indexes, elementary indexes.

Journal of Economic Literature Classification Numbers

C43, E31.

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1. Introduction

A decade has passed since the *Consumer Price Index Manual: Theory and Practice*[2] was published. Thus it seems appropriate to review the advice given in the *Manual* in the light of research over the past decade. It turns out that there have been some significant developments that should be taken into account in the next revision of the *Manual*.

In section 2 below, we review the main methodological recommendations on choosing a target index that were made in the *Manual*.[3] In subsequent sections, we will list some of the problem areas and possible solutions to these problems that have been brought forward during the past decade.

In section 3, the *chain drift problem* will be defined and possible solutions discussed. Sections 4 and 5 discuss two possible solutions to the chain drift problem.

Section 6 will discuss various problems associated with the construction of *elementary indexes*. These indexes are constructed using price information only. When value or quantity information is not available to the price statistician, then it is only possible to construct an elementary index. This type of index is used at the lowest level of aggregation when expenditure information is not available.

Section 7 will briefly review recent developments on alternative approaches to *quality adjustment*.

Section 8 will discuss additional problem areas associated with the construction of Consumer Price Indexes where further research is required.

Section 9 concludes.

2. The Consumer Price Index Manual 2004

The *Manual* distinguished four main approaches to the determination of the functional form for a *target price index* that compares the prices (and associated quantities) between two periods:

- Fixed basket and averages of fixed basket approaches;
- The test or axiomatic approach;
- The stochastic approach and
- The economic approach.

These four approaches will be explained briefly below.[4]

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[2] See the ILO/IMF/OECD/UNECE/Eurostat/The World Bank (2004). The *Manual* was written over the years 2000-2003. For brevity, in the future, we will refer to the *CPI Manual* as ILO (2004) or the *Manual*.
[3] This section can be skipped by readers who are familiar with the contents of the *Manual*.
[4] See the ILO (2004; 263-327).

2.1 Fixed Basket Approaches to Index Number Theory

A very simple approach to the determination of a price index over a group of commodities is the *fixed basket approach*. In this approach, a basket of commodities that is represented by the positive quantity vector $q \equiv [q_1,...,q_N]$ is given. Given the price vectors for periods 0 and 1, $p^0 \equiv [p_1^0,...,p_N^0]$ and $p^1 \equiv [p_1^1,...,p_N^1]$ respectively, we can calculate the cost of purchasing this same basket in the two periods, $p^0 \cdot q \equiv \sum_{n=1}^N p_n^0 q_n$ and $p^1 \cdot q \equiv \sum_{n=1}^N p_n^1 q_n$. Then the ratio of these costs is a very reasonable indicator of pure price change over the two periods under consideration, provided that the basket vector q is "representative". This leads to the *Lowe* (1823) *price index*, P_{Lo}, defined as follows:

$$(1)\ P_{Lo}(p^0,p^1,q) \equiv p^1 \cdot q / p^0 \cdot q\ .$$

As time passed, economists and price statisticians demanded a bit more precision with respect to the specification of the basket vector q. There are two natural choices for the reference basket: the period 0 commodity vector q^0 or the period 1 commodity vector q^1. These two choices lead to the Laspeyres (1871) price index P_L defined by (2) and the Paasche (1874) price index P_P defined by (3):

$$(2)\ P_L(p^0,p^1,q^0,q^1) \equiv p^1 \cdot q^0 / p^0 \cdot q^0 = \sum_{n=1}^N s_n^0 (p_n^1/p_n^0)\ ;$$

$$(3)\ P_P(p^0,p^1,q^0,q^1) \equiv p^1 \cdot q^1 / p^0 \cdot q^1 = [\sum_{n=1}^N s_n^1 (p_n^1/p_n^0)^{-1}]^{-1}$$

where the period t expenditure share on commodity n, s_n^t, is defined as $p_n^t q_n^t / p^t \cdot q^t$ for $n = 1,\ldots,N$ and $t = 0,1$. Thus the Laspeyres price index P_L can be written as a base period expenditure share weighted average of the N price ratios (or price relatives), p_n^1/p_n^0.[5] The last equation in (3) shows that the Paasche price index P_P can be written as a period 1 (or current period) expenditure share weighted *harmonic* average of the N price ratios.[6]

The problem with these index number formulae is that they are equally plausible but in general, they will give different answers. This suggests that if we require a single estimate for the price change between the two periods, then we should take some sort of evenly weighted average of the two indexes as our final estimate of price change between periods 0 and 1. Examples of such symmetric averages are the arithmetic mean, which leads to the Drobisch (1871) Sidgwick (1883; 68) Bowley (1901; 227)[7] index, $(1/2)P_L + (1/2)P_P$, and the geometric mean, which leads to the *Fisher* (1922) *ideal index*, P_F, defined as

$$(4)\ P_F(p^0,p^1,q^0,q^1) \equiv [P_L(p^0,p^1,q^0,q^1)\ P_P(p^0,p^1,q^0,q^1)]^{1/2}\ .$$

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[5] This result is due to Walsh (1901; 428 and 539).

[6] This expenditure share and price ratio representation of the Paasche index is described by Walsh (1901; 428) and derived explicitly by Fisher (1911; 365).

[7] See Diewert (1992) (1993) and Balk (2008) for additional references to the early history of index number theory.

It is very desirable for a price index formula that depends on the price and quantity vectors pertaining to the two periods under consideration to satisfy the *time reversal test*.[8] We say that the index number formula $P(p^0,p^1,q^0,q^1)$ satisfies this test if

(5) $P(p^1,p^0,q^1,q^0) = 1/P(p^0,p^1,q^0,q^1)$;

i.e., if we interchange the period 0 and period 1 price and quantity data and evaluate the index, then this new index $P(p^1,p^0,q^1,q^0)$ is equal to the reciprocal of the original index $P(p^0,p^1,q^0,q^1)$.

Diewert (1997; 138) showed that the Fisher ideal price index defined by (4) above is the *only* index that is a homogeneous symmetric mean of the Laspeyres and Paasche price indexes, P_L and P_P, and satisfies the time reversal test (5) above. Thus the *symmetric basket approach* to bilateral index number theory leads to the Fisher index (4) as being "best" from the perspective of this approach.[9]

2.2. Stochastic and Descriptive Statistics Approaches to Index Number Theory

The (unweighted) stochastic approach to the determination of the price index can be traced back to the work of Jevons (1865) (1884) and Edgeworth (1888) (1896) (1901) over a hundred years ago[10].

The basic idea behind the stochastic approach is that each price relative, p_n^1/p_n^0 for n = 1,2,…,N, can be regarded as an estimate of a common inflation rate α between periods 0 and 1; i.e., Jevons and Edgeworth essentially assumed that

(6) $p_n^1/p_n^0 = \alpha + \varepsilon_n$; $n = 1,2,\ldots,N$

where α is the common inflation rate and the ε_n are random variables with mean 0 and variance σ^2. The least squares estimator for α is the *Carli* (1804) *price index* P_C defined as

(7) $P_C(p^0,p^1) \equiv \sum_{n=1}^N (1/N)(p_n^1/p_n^0)$.

Unfortunately, P_C does not satisfy the time reversal test, i.e., $P_C(p^1,p^0) \neq 1/P_C(p^0,p^1)$[11].

[8] The concept of this test is due to Pierson (1896; 128). More formal statements of the test were made by Walsh (1901; 324) and Fisher (1922; 64).
[9] Bowley was an early advocate of taking a symmetric average of the Paasche and Laspeyres indexes: "If [the Paasche index] and [the Laspeyres index] lie close together there is no further difficulty; if they differ by much they may be regarded as inferior and superior limits of the index number, which may be estimated as their arithmetic mean … as a first approximation." Arthur L. Bowley (1901; 227). Fisher (1911; 418-419) (1922) considered taking the arithmetic, geometric and harmonic averages of the Paasche and Laspeyres indexes.
[10] For additional references to the early literature, see Diewert (1993; 37-38) (1995b) and Balk (2008; 32-36).
[11] In fact Fisher (1922; 66) noted that $P_C(p^0,p^1)P_C(p^1,p^0) \geq 1$ unless the period 1 price vector p^1 is proportional to the period 0 price vector p^0; i.e., Fisher showed that the Carli index has a definite upward

Now assume that the logarithm of each price relative, $\ln(p_n^1/p_n^0)$, is an independent unbiased estimate of the logarithm of the inflation rate between periods 0 and 1, β say. Thus we have:

$$(8)\ \ln(p_n^1/p_n^0) = \beta + \varepsilon_n \ ; \ n = 1,2,\ldots,N$$

where $\beta \equiv \ln\alpha$ and the ε_n are independently distributed random variables with mean 0 and variance σ^2. The least squares or maximum likelihood estimator for β is the logarithm of the geometric mean of the price relatives. Hence the corresponding estimate for the common inflation rate α is the *Jevons* (1865) *price index* P_J defined as:

$$(9)\ P_J(p^0,p^1) \equiv \prod_{n=1}^N (p_n^1/p_n^0)^{1/N}.$$

The Jevons price index P_J does satisfy the time reversal test and hence is much more satisfactory than the Carli index P_C. However, both the Jevons and Carli price indexes suffer from a fatal flaw: each price relative p_n^1/p_n^0 is regarded as being equally important and is given an equal weight in the index number formulae (7) and (9).[12] Keynes (1930; 76-81) also criticized the unweighted stochastic approach to index number theory on two other grounds: (i) price relatives are not distributed independently and (ii) there is no single inflation rate that can be applied to all parts of an economy; e.g., Keynes demonstrated empirically that wage rates, wholesale prices and final consumption prices all had different rates of inflation. In order to overcome the Keynesian criticisms of the unweighted stochastic approach to index numbers, it is necessary to:

- have a definite domain of definition for the index number and
- weight the price relatives by their economic importance.

Theil (1967; 136-137) proposed a solution to the lack of weighting in (9). He argued as follows. Suppose we draw price relatives at random in such a way that each dollar of expenditure in the base period has an equal chance of being selected. Then the probability that we will draw the nth price relative is equal to $s_n^0 \equiv p_n^0 q_n^0/p^0\cdot q^0$, the period 0 expenditure share for commodity n. Then the overall mean (period 0 weighted) logarithmic price change is $\sum_{n=1}^N s_n^0 \ln(p_n^1/p_n^0)$. Now repeat the above mental experiment and draw price relatives at random in such a way that each dollar of expenditure in period 1 has an equal probability of being selected. This leads to the overall mean (period 1 weighted) logarithmic price change of $\sum_{n=1}^N s_n^1 \ln(p_n^1/p_n^0)$. Each of these measures of overall logarithmic price change seems equally valid so we could argue for taking a

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bias. Walsh (1901; 327) established this inequality for the case N = 2. Fisher urged users to abandon the use of the Carli index but his advice was generally ignored by statistical agencies until recently: “In fields other than index numbers it is often the best form of average to use. But we shall see that the simple arithmetic average produces one of the very worst of index numbers. And if this book has no other effect than to lead to the total abandonment of the simple arithmetic type of index number, it will have served a useful purpose.” Irving Fisher (1922; 29-30).

[12] Walsh (1901) (1921a; 82-83), Fisher (1922; 43) and Keynes (1930; 76-77) all objected to the lack of weighting in the unweighted stochastic approach to index number theory.

symmetric average of the two measures in order to obtain a final single measure of overall logarithmic price change. Theil (1967; 137) argued that a nice symmetric index number formula can be obtained if we make the probability of selection for the nth price relative equal to the arithmetic average of the period 0 and 1 expenditure shares for commodity n. Using these probabilities of selection, Theil's final measure of overall logarithmic price change is

(10) $\ln P_T(p^0,p^1,q^0,q^1) \equiv \sum_{n=1}^{N} (1/2)(s_n^0+s_n^1)\ln(p_n^1/p_n^0)$.

It is possible to give a *descriptive statistics* interpretation of the right hand side of (10). Define the nth logarithmic price ratio r_n by:

(11) $r_n \equiv \ln(p_n^1/p_n^0)$ for $n = 1,\ldots,N$.

Now define the discrete random variable, R say, as the random variable which can take on the values r_n with probabilities $\rho_n \equiv (1/2)(s_n^0+s_n^1)$ for $n = 1,\ldots,N$. Note that since each set of expenditure shares, s_n^0 and s_n^1, sums to one, the probabilities ρ_n will also sum to one. It can be seen that the expected value of the discrete random variable R is $\ln P_T(p^0,p^1,q^0,q^1)$ as defined by the right hand side of (10). Thus the logarithm of the index P_T can be interpreted as *the expected value of the distribution of the logarithmic price ratios* in the domain of definition under consideration, where the N discrete price ratios in this domain of definition are weighted according to Theil's probability weights, ρ_n.

Taking antilogs of both sides of (10), we obtain the Theil price index; P_T.[13] This index number formula has a number of good properties. In particular, P_T satisfies the time reversal test (5).[14]

Additional material on stochastic approaches to index number theory and references to the literature can be found in Selvanathan and Rao (1994), Diewert (1995b) (2004) (2005), Clements, Izan and Selvanathan (2006) and Balk (2008; 32-36)

2.3. Test Approaches to Index Number Theory

A bit of background material on price and quantity indexes and their consistency with each other is required at this point. We specify two accounting periods, t = 0,1 for which we have micro price and quantity data for N commodities pertaining to transactions by a consumer (or a well defined group of consumers). Denote the price and quantity of commodity n in period t by p_n^t and q_n^t respectively for $n = 1,2,\ldots,N$ and $t = 0,1$. Before proceeding further, we need to discuss the exact meaning of the microeconomic prices and quantities if there are *multiple* transactions for say commodity n within period t. In this case, it is natural to interpret q_n^t as the *total* amount of commodity n transacted within

[13] This index first appeared explicitly as formula 123 in Fisher (1922; 473). P_T is generally attributed to Törnqvist (1936) but this article did not have an explicit definition for P_T; it was defined explicitly in Törnqvist and Törnqvist (1937); see Balk (2008; 26).

[14] For a listing of some of the tests that P_T and P_F satisfy, see Diewert (1992; 223). In Fisher (1922), these indexes were listed as numbers 123 and 353 respectively.

period t. In order to conserve the value of transactions, it is necessary that p_n^t be defined as a *unit value* [15]; i.e., p_n^t must be equal to the value of transactions for commodity n during period t divided by the total quantity transacted, q_n^t. For t = 0,1, define *the value of transactions in period t* as:

$$(12)\ V^t \equiv \sum_{n=1}^{N} p_n^t q_n^t \equiv p^t \cdot q^t$$

where $p^t \equiv (p_1^t,\ldots, p_N^t)$ is the period t price vector, $q^t \equiv (q_1^t,\ldots, q_N^t)$ is the period t quantity vector and $p^t \cdot q^t$ denotes the inner product of these two vectors.

Using the above notation, we can now state the following *levels version of the index number problem using the test or axiomatic approach*: for t = 0,1, find scalar numbers P^t and Q^t such that

$$(13)\ V^t = P^t Q^t.$$

The number P^t is interpreted as an aggregate period t price level while the number Q^t is interpreted as an aggregate period t quantity level. The aggregate price level P^t is allowed to be a function of the period t price vector, p^t while the aggregate period t quantity level Q^t is allowed to be a function of the period t quantity vector, q^t; i.e., we have

$$(14)\ P^t = c(p^t) \text{ and } Q^t = f(q^t)\ \ ;\ \ t = 0,1.$$

However, from the viewpoint of the *test approach* to index number theory, this levels approach to finding aggregate quantities and prices comes to an abrupt halt: Eichhorn (1978; 144) showed that if the number of commodities N in the aggregate is equal to or greater than 2 and we restrict $c(p^t)$ and $f(q^t)$ to be positive if the micro prices and quantities p_n^t and q_n^t are positive, then there do not exist any functions c and f such that $c(p^t)f(q^t) = p^t \cdot q^t$ for all strictly positive p^t and q^t vectors.

In s second approach to index number theory, instead of trying to decompose the value of the aggregate into price and quantity components for a single period, we instead attempt to decompose a *value ratio* for the two periods under consideration into a *price change component* P times a *quantity change component* Q. Thus we now look for two functions of 4N variables, $P(p^0,p^1,q^0,q^1)$ and $Q(p^0,p^1,q^0,q^1)$ such that:[16]

$$(15)\ p^1 \cdot q^1/p^0 \cdot q^0 = P(p^0,p^1,q^0,q^1)Q(p^0,p^1,q^0,q^1).$$

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[15] The early index number theorists Walsh (1901; 96), Fisher (1922; 318) and Davies (1924; 96) all suggested unit values as the prices that should be inserted into an index number formula. This advice is followed in the *Manual* with the proviso that the unit value be a narrowly defined one; see the ILO (2004; 356).

[16] If N = 1, then we define $P(p_1^0,p_1^1,q_1^0,q_1^1) \equiv p_1^1/p_1^0$ and $Q(p_1^0,p_1^1,q_1^0,q_1^1) \equiv q_1^1/q_1^0$, the single price ratio and the single quantity ratio respectively. In the case of a general N, we think of $P(p_1^0,p_1^1,q_1^0,q_1^1)$ as being a weighted average of the price ratios p_1^1/p_1^0, p_2^1/p_2^0, ..., p_N^1/p_N^0. Thus we interpret $P(p_1^0,p_1^1,q_1^0,q_1^1)$ as an aggregate price ratio, P^1/P^0, where P^t is the aggregate price level for period t for t = 0,1.

If we take the test approach, then we want equation (15) to hold for all positive price and quantity vectors pertaining to the two periods under consideration, p^0,p^1,q^0,q^1. [17]

In this second test approach to index number theory, the *price index* $P(p^0,p^1,q^0,q^1)$ and the *quantity index* $Q(p^0,p^1,q^0,q^1)$ cannot be determined independently; i.e., if either one of these two functions is determined, then the remaining function is implicitly determined using equation (15). Historically, the focus has been on the determination of the price index but Fisher (1911; 388) was the first to realize that once the price index was determined, then equation (5) could be used to determine the companion quantity index.[18] This value ratio decomposition approach to index number is called *bilateral index number theory* and its focus is the determination of "reasonable" functional forms for P and Q. Fisher's 1911 and 1922 books address this functional form issue using the test approach.

Recall equation (15) above, which set the value ratio, V^1/V^0, equal to the product of the price index, $P(p^0,p^1,q^0,q^1)$, and the quantity index, $Q(p^0,p^1,q^0,q^1)$. This is called the *Product Test* and we assume that it is satisfied. This equation means that as soon as the functional form for the price index P is determined, then (15) can be used to determine the functional form for the quantity index Q. However, a further advantage of assuming that the product test holds is that we can assume that the quantity index Q satisfies a "reasonable" property and then use (15) to translate this test on the quantity index into a corresponding test on the price index P.[19]

If N = 1, so that there is only one price and quantity to be aggregated, then a natural candidate for P is p_1^1/p_1^0 , the single price ratio, and a natural candidate for Q is q_1^1/q_1^0 , the single quantity ratio.

When the number of commodities or items to be aggregated is greater than 1, then index number theorists have proposed properties or tests that the price index P should satisfy. These properties are generally multi-dimensional analogues to the one good price index formula, p_1^1/p_1^0. For a list of twenty-one tests that turn out to characterize the Fisher ideal price index, see Diewert (1992) (2012) or the ILO (2004). Thus the Fisher ideal price index receives a strong justification from the viewpoint of the test approach to index number theory.

There are two additional tests that will play a role below when we discuss the chain drift problem. These two tests are the *Circularity Test* and the *Multiperiod Identity Test* and they will be defined in section 3 below. Both the Fisher and Theil index fail these tests.

2.4 The Economic Approach to Index Number Theory

[17] When we take the economic approach in section 2.4 below, then only the price vectors p^0 and p^1 are regarded as independent variables while the quantity vectors, q^0 and q^1, are regarded as dependent variables.

[18] This approach to index number theory is due to Fisher (1911; 418) who called the implicitly determined Q, the *correlative formula*. Frisch (1930; 399) later called (15) the *product test*.

[19] This observation was first made by Fisher (1911; 400-406). Vogt (1980) also pursued this idea.

In this subsection, we will outline the theory of the cost of living index for a single consumer (or household) that was first developed by the Russian economist, A. A. Konüs (1924).[20] This theory relies on the assumption of *optimizing behavior* on the part of the consumer. Thus given a vector of commodity or input prices p^t that the consumer faces in a given time period t, it is assumed that the corresponding observed quantity vector q^t is the solution to a cost minimization problem that involves the consumer's preference or utility function f.

The economic approach assumes that "the" consumer has well defined *preferences* over different combinations of the N consumer commodities or items. The consumer's preferences over alternative possible consumption vectors q are assumed to be representable by a nonnegative, continuous, increasing, and quasiconcave utility function f, which is defined over the nonnegative orthant. It is further assumed that the consumer minimizes the cost of achieving the period t utility level $u^t \equiv f(q^t)$ for periods t = 0,1. Thus the observed period t consumption vector q^t solves the following *period t cost minimization problem*:

$$(16)\quad C(u^t,p^t) \equiv \min{}_q \{p^t\cdot q : f(q) = u^t\} = p^t\cdot q^t ; \qquad t = 0,1.$$

The period t price vector for the N commodities under consideration that the consumer faces is p^t. The Konüs (1924) family of *true cost of living indexes* $P_K(p^0,p^1,q)$ between periods 0 and 1 is defined as the ratio of the minimum costs of achieving the same utility level $u \equiv f(q)$ where q is a positive reference quantity vector:

$$(17)\quad P_K(p^0,p^1,q) \equiv C[f(q),p^1]/C[f(q),p^0].$$

We say that definition (17) defines a *family* of price indexes because there is one such index for each reference quantity vector q chosen. However, if we place an additional restriction on the utility function f, then it turns out that the Konüs price index, $P_K(p^0,p^1,q)$, will no longer depend on the reference q.

The extra assumption on f is that f be (positively) *linearly homogeneous* so that $f(\lambda q) = \lambda f(q)$ for all $\lambda > 0$ and all $q \geq 0_N$. In the economics literature, this extra assumption is known as the assumption of *homothetic preferences*. Under this assumption, the consumer's cost function, C(u,p) decomposes into uc(p) where c(p) is the consumer's *unit cost function*, $c(p) \equiv C(1,p)$, which corresponds to f. Under the assumption of cost minimizing behavior in both periods, it can be shown that the homotheticity assumption implies that equations (16) simplify to the following equations:

$$(18)\quad p^t\cdot q^t = c(p^t)f(q^t) \qquad \text{for } t = 0,1.$$

Thus under the linear homogeneity assumption on the utility function f, observed period t expenditure on the n commodities is equal to the period t unit cost $c(p^t)$ of achieving one unit of utility times the period t utility level, $f(q^t)$. Obviously, we can identify the period t

[20] For extensions to the case of many households, see Diewert (2001).

unit cost, $c(p^t)$, as the period t price level P^t and the period t level of utility, $f(q^t)$, as the period t quantity level Q^t [21].

The linear homogeneity assumption on the consumer's preference function f leads to a simplification for the family of Konüs true cost of living indexes, $P_K(p^0,p^1,q)$, defined by (17) above. Using this definition for an arbitrary reference quantity vector q and the decomposition $C(f(q),p^t) = c(p^t)f(q)$ for t = 0,1, we have:

$$(19)\quad P_K(p^0,p^1,q) \equiv C[f(q),p^1]/C[f(q),p^0] = c(p^1)f(q)/c(p^0)f(q) = c(p^1)/c(p^0).$$

Thus under the homothetic preferences assumption, the entire family of Konüs true cost of living indexes collapses to a single index, $c(p^1)/c(p^0)$, which is the ratio of the minimum costs of achieving a unit utility level when the consumer faces period 1 and 0 prices respectively.

If we use the Konüs true cost of living index defined by the right hand side of (19) as our price index concept, then the corresponding implicit quantity index can be defined as the value ratio divided by the Konüs price index:

$$(20)\ Q(p^0,p^1,q^0,q^1,q) \equiv p^1\cdot q^1/[p^0\cdot q^0\, P_K(p^0,p^1,q)] = f(q^1)/f(q^0).$$

Thus under the homothetic preferences assumption, the *implicit quantity index* that corresponds to the true cost of living price index $c(p^1)/c(p^0)$ is the *utility ratio* $f(q^1)/f(q^0)$. Since the utility function is assumed to be homogeneous of degree one, this is the natural definition for a quantity index.[22]

Recall that the Fisher price index, $P_F(p^0,p^1,q^0,q^1)$, was defined by (4). The companion *Fisher quantity index*, $Q_F(p^0,p^1,q^0,q^1)$, can be defined using the Product Test (15). Now suppose that the consumer's preferences can be represented by the homothetic utility function f defined as

$$(21)\ f(q) \equiv [q^TAq]^{1/2}$$

where $A \equiv [a_{ij}]$ is an N by N symmetric matrix that has one positive eigenvalue (that has a strictly positive eigenvector) and the remaining N−1 eigenvalues are zero or negative. Under these conditions, there will be a *region of regularity* where the function f is positive, concave and increasing and hence f can provide a valid representation of preferences over this region. Using these preferences and the assumption of cost minimizing behavior in periods 0 and 1, it can be shown that

$$(22)\ Q_F(p^0,p^1,q^0,q^1) = f(q^1)/f(q^0).$$

[21] See equations (13) and (14) above, which were used to explain the levels approach to axiomatic index number theory. In this previous approach, prices and quantities were allowed to vary independently of each other which led to Eichhorn's impossibility result. This result does not apply in the present context because in the economic approach, only prices are freely variable.
[22] Samuelson and Swamy (1974) used this homothetic approach to index number theory.

Thus under the assumption that the consumer engages in cost minimizing behavior during periods 0 and 1 and has preferences over the N commodities that correspond to the utility function f defined by (28), the Fisher ideal quantity index Q_F is *exactly* equal to the true quantity index, $f(q^1)/f(q^0)$.[23]

Let c(p) be the unit cost function that corresponds to the homogeneous quadratic utility function f defined by (21). Then it can be shown that

(23) $P_F(p^0,p^1,q^0,q^1) = c(p^1)/c(p^0)$.

Thus under the assumption that the consumer engages in cost minimizing behavior during periods 0 and 1 and has preferences over the N commodities that correspond to the utility function $f(q) = (q^TAq)^{1/2}$, the Fisher ideal price index P_F is *exactly* equal to the true price index, $c(p^1)/c(p^0)$. The significance of (22) and (23) is that we can calculate the consumer's true rate of utility growth and his or her true rate of price inflation *without having to undertake any econometric estimation*; i.e., the left hand sides of (22) and (23) can be calculated exactly using observable price and quantity data for the consumer for the two periods under consideration. Thus the present economic approach to index number theory using a *ratio approach* leads to practical solutions to the index number problem whereas the earlier *levels approach* explained in the beginning of section 2.3 did not lead to practical solutions.

A twice continuously differentiable function f(q) of N variables q can provide a *second order approximation* to another such function $f^*(q)$ around the point q^* if the level and all of the first and second order partial derivatives of the two functions coincide at q^*. It can be shown[24] that the homogeneous quadratic function f defined by (21) can provide a second order approximation to an arbitrary f^* around any point q^* in the class of twice continuously differentiable linearly homogeneous functions. Thus the homogeneous quadratic functional form defined by (21) is a *flexible functional form*.[25] Diewert (1976; 117) termed an index number formula $Q_F(p^0,p^1,q^0,q^1)$ that was *exactly* equal to the true quantity index $f(q^1)/f(q^0)$ (where f is a flexible functional form) *a superlative index number formula*.[26] Equation (22), and the fact that the homogeneous quadratic function f defined by (21) is a flexible functional form, shows that the Fisher ideal quantity index Q_F is a superlative index number formula. Since the Fisher ideal price index P_F also satisfies (23) where c(p) is the dual unit cost function that is generated by the homogeneous quadratic utility function, P_F is also a superlative index number formula.

It turns out that there are many other superlative index number formulae; i.e., there exist many quantity indexes $Q(p^0,p^1,q^0,q^1)$ that are exactly equal to $f(q^1)/f(q^0)$ and many price

[23] This result was first derived by Konüs and Byushgens (1926). For an alternative derivation and the early history of this result, see Diewert (1976; 116).
[24] See Diewert (1976; 130) and let the parameter r equal 2.
[25] Diewert (1974; 133) introduced this term to the economics literature.
[26] As we have seen earlier, Fisher (1922; 247) used the term superlative to describe the Fisher ideal price index. Thus Diewert adopted Fisher's terminology but attempted to give more precision to Fisher's definition of superlativeness.

indexes $P(p^0,p^1,q^0,q^1)$ that are exactly equal to $c(p^1)/c(p^0)$ where the aggregator function f or the unit cost function c is a flexible functional form; see Diewert (1976) or the ILO (2004).

The above results provide a reasonably strong justification for the use of the Fisher price index from the viewpoint of the economic approach. An even stronger justification[27] can be provided for the Törnqvist Theil index P_T defined by (18) as we will show below.

Suppose that the consumer's cost function, C(u,p), has the following *translog functional form*:[28]

$$(24)\quad \ln C(u,p) \equiv a_0 + \sum_{i=1}^N a_i \ln p_i + (1/2) \sum_{i=1}^N \sum_{k=1}^N a_{ik} \ln p_i \ln p_k + b_0 \ln u + \sum_{i=1}^N b_i \ln p_i \ln u + (1/2)\, b_{00} [\ln u]^2$$

where ln is the natural logarithm function and the parameters a_i, a_{ik}, and b_i satisfy the following restrictions: (i) $a_{ik} = a_{ki}$ for i,k = 1,…,N; (ii) $\sum_{i=1}^N a_i = 1$; (iii) $\sum_{i=1}^N b_i = 0$; (iv) $\sum_{k=1}^N a_{ik} = 0$ for i = 1,…,N. These restrictions ensure that C(u,p) defined by (24) is linearly homogeneous in p. It can be shown that this translog cost function can provide a second order Taylor series approximation to an arbitrary cost function.[29]

We assume that the consumer engages in cost minimizing behavior during periods 0 and 1 and has the preferences that are dual to the translog cost function defined by (24). Define the geometric average of the period 0 and 1 utility levels as $u^* \equiv [u^0u^1]^{1/2}$. Then it can be shown that the log of P_T defined by (10) is exactly equal to the log of the Konüs true cost of living index that corresponds to the reference indifference surface that is indexed by the intermediate utility level u^*; i.e., we have the following exact identity:[30]

$$(25)\quad C(u^*,p^1)/C(u^*,p^0) = P_T(p^0,p^1,q^0,q^1).$$

Since the translog cost function is a flexible functional form, the Törnqvist-Theil price index P_T is also a *superlative index*.[31] The importance of (25) as compared to the earlier exact index number results is that it is no longer necessary to assume that preferences are homothetic. However, it is necessary to choose the reference utility level on the left hand side of (25) to be the geometric mean of u^0 and u^1 in order to obtain the new exact index number result.[32]

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[27] The exact index number formula (25) is stronger than the above results because we no longer have to assume homothetic preferences.

[28] Christensen, Jorgenson and Lau (1975) and Diewert (1976) introduced this function into the economics and index number literature.

[29] It can also be shown that if $b_0 = 1$ and all of the $b_i = 0$ for i = 1,...,N and $b_{00} = 0$, then $C(u,p) = uC(1,p) \equiv uc(p)$; i.e., with these additional restrictions on the parameters of the general translog cost function, we have homothetic preferences.

[30] This result is due to Diewert (1976; 122).

[31] Diewert (1978; 888) showed that $P_T(p^0,p^1,q^0,q^1)$ approximates other superlative indexes, including the Fisher index P_F, to the second order around an equal price and quantity point.

[32] For exact index number results in the context of quantity indexes and nonhomothetic preferences that are analogous to (25), see Diewert (1976; 123-124) and Diewert (2009; 241) where the first paper uses

It is somewhat mysterious how a ratio of *unobservable* cost functions of the form appearing on the left hand side of the above equation can be *exactly* estimated by an *observable* index number formula but the key to this mystery is the assumption of cost minimizing behavior and the quadratic nature of the underlying preferences. In fact, all of the exact index number results derived in this section can be derived using transformations of a quadratic identity.[33]

The important message to take home from this subsection is that the Fisher and Theil indexes, P_F, and P_T, can both be given strong justifications from the viewpoint of the economic approach to index number theory. Note that these same formulae also emerged as being "best" from the viewpoints of the basket, stochastic and test approaches to index number theory. *Thus the four major approaches to bilateral index number theory lead to the same two formulae as being best*. Which formula should then be used by a statistical agency as their target index? It turns out that for "typical" time series data, it will not matter much, since the two indexes will generally numerically approximate each other very closely.[34]

Based on the above results, the *Manual* recommended that the Fisher or Theil price index (or other superlative index) be used as a *target month to month index*, provided that monthly price and expenditure data for the class of expenditures in scope were available. Recently, grocery chains in some countries (e.g., Australia, the Netherlands and Norway) have been willing to donate their sales value and quantity information by detailed product to their national statistical agencies so it has become possible to calculate month to month superlative indexes for at least some strata of the country's Consumer Price Index. However, the issue arises: should the indexes fix a base month (for 13 months) and calculate Fisher or Theil indexes as chained indexes or as fixed base indexes? The Manual offered the following advice on this choice in the chapter on seasonal commodities:

"22.63 A reasonable method for dealing with seasonal commodities in the context of picking a target index for a month to month CPI is the following one:[35]

Malmquist (1953) quantity indexes and the second one uses Allen (1949) quantity indexes. It is also possible to generalize the result (25) to situations where the consumer changes his or her tastes going from period 0 to period 1. Again, under the assumption that the consumer has (possibly different) translog preferences in each period, it can be shown that the Törnqvist price index P_T is exactly equal to the geometric mean of two separate price indexes where the tastes for one period are used in one true cost of living index and the tastes for the other period are used in the other true cost of living index. There are some restrictions on the degree of difference in the preferences over the two periods; see Caves, Christensen and Diewert (1982; 1409-1411). On index number theory under changing preferences, see also Balk (1989).

[33] See Diewert (2002a).

[34] Diewert (1978; 888) showed that all known (at that time) superlative indexes numerically approximated each other to the second order around a point where $p^0 = p^1$ and $q^0 = q^1$. Thus if prices and quantities do not change "too much" between the two periods being compared, P_F and P_T will generate very similar indexes.

[35] For more on the economic approach and the assumptions on consumer preferences that can justify month to month maximum overlap indexes, see Diewert (1999a; 51-56).

- Determine the set of commodities that are present in the marketplace in both months of the comparison.
- For this maximum overlap set of commodities, calculate one of the three indices recommended in previous chapters; i.e., calculate the Fisher, Walsh or Törnqvist Theil index.

Thus the bilateral index number formula is applied only to the subset of commodities that are present in both periods.

22.64 The question now arises: should the comparison month and the base month be adjacent months (thus leading to chained indices) or should the base month be fixed (leading to fixed base indices)? It seems reasonable to prefer chained indices over fixed base indices for two reasons:

- The set of seasonal commodities which overlaps during two consecutive months is likely to be much larger than the set obtained by comparing the prices of any given month with a fixed base month (like January of a base year). Hence the comparisons made using chained indices will be more comprehensive and accurate than those made using a fixed base.
- In many economies, on average 2 or 3 percent of price quotes disappear each month due to the introduction of new commodities and the disappearance of older ones. This rapid sample attrition means that fixed base indices rapidly become unrepresentative and hence it seems preferable to use chained indices which can more closely follow marketplace developments." ILO (2004; 407)

Thus the *Manual* recommended chained Fisher or Törnqvist-Theil indexes as a target index concepts. As we shall see in the next section, this advice does not always work out too well.

3. The Chain Drift Problem and Possible Solutions

Suppose that we have decided on a "best" price index formula that compares the prices of period 0 with those of period 1, say $P(p^0,p^1,q^0,q^1)$. Suppose further that we have price and quantity data for 3 periods. There are at least two ways that a sequence of price levels for the three periods could be formed using the given index number formula:

- Fixed base indexes or
- Chained indexes.

The sequence of the price levels for the three periods under consideration, P^0, P^1 and P^2, using *fixed base* (or direct) *indexes* can be constructed as follows:

$$P^0 \equiv 1;\ P^1 \equiv P(p^0,p^1,q^0,q^1);\ P^2 \equiv P(p^0,p^2,q^0,q^2).$$

Thus the prices in period 2, p^2, are compared *directly* with the prices in period 0, p^0.

The sequence of the three price levels, P^0, P^1 and P^2, using *chained indexes* can be constructed as follows:

$$P^0 \equiv 1;\ P^1 \equiv P(p^0,p^1,q^0,q^1);\ P^2 \equiv P(p^0,p^1,q^0,q^1)P(p^1,p^2,q^1,q^2).$$

Thus fixed base and chained price levels coincide for the first two periods but in subsequent periods t, the fixed base indexes compare the prices in period t directly to the prices in period 0 whereas the chained indexes simply update the price level in the previous period by multiplying by the period over period *chain link index* $P(p^{t-1},p^t,q^{t-1},q^t)$.

The two methods of index construction will coincide if the bilateral price index formula $P(p^0,p^1,q^0,q^1)$ satisfies the following test:

(26) *Circularity Test*: $P(p^0,p^1,q^0,q^1)\, P(p^1,p^2,q^1,q^2) = P(p^0,p^2,q^0,q^2)$.

If there is only one commodity in the aggregate, then the price index $P(p^0,p^1,q^0,q^1)$ just becomes the single price ratio, p_1^1/p_1^0, and the circularity test becomes the equation $[p_1^1/p_1^0][p_1^2/p_1^1] = [p_1^2/p_1^0]$, which is obviously satisfied. The equation in the circularity test illustrates the difference between chained index numbers and fixed base index numbers. The left hand side of (26) uses the *chain principle* to construct the overall inflation between periods 0 and 2 whereas the right hand side uses the *fixed base principle* to construct an estimate of the overall price change between periods 0 and 2.[36]

It would be good if our preferred index number formulae, the Fisher and Törnqvist indexes (P_F and P_T), satisfied the circularity test but unfortunately, they do not satisfy (26).[37] Hence, a statistical agency compiling a CPI has to choose between the two methods of index construction. As indicated at the end of the previous section, the *Manual* favoured the use of chained superlative indexes for the reasons indicated above.

The main advantage of using chained indexes is that if prices and quantities are trending relatively smoothly, chaining will reduce the spread between the Paasche and Laspeyres indexes.[38] These two indexes each provide an asymmetric perspective on the amount of price change that has occurred between the two periods under consideration and it could be expected that a single point estimate of the aggregate price change should lie between these two estimates. Thus the use of either a chained Paasche or Laspeyres index will usually lead to a smaller difference between the two and hence to estimates that are closer to the "truth". Since annual data generally has smooth trends, the use of chained indexes is generally appropriate at this level of aggregation; see Hill (1993; 136-137).

However, the story is different at subannual levels; i.e., if the index is to be produced at monthly or quarterly frequencies. Hill (1993; 388), drawing on the earlier research of Szulc (1983) and Hill (1988; 136-137), noted that it is not appropriate to use the chain system when prices oscillate or "bounce" to use Szulc's (1983; 548) term. This phenomenon can occur in the context of regular seasonal fluctuations or in the context of

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[36] Fisher (1911; 203) introduced this fixed base and chain terminology. The concept of chaining is due to Lehr (1885) and Marshall (1887; 373).

[37] Alterman, Diewert and Feenstra (1999; 61-65) showed that if the logarithmic price ratios ln (p_n^t/p_n^{t-1}) trend linearly with time t and the expenditure shares s_i^t also trend linearly with time, then the Törnqvist index P_T will satisfy the circularity test *exactly*. They extended this exactness result to cover the case when there are monthly proportional variations in prices and the expenditure shares have constant seasonal effects in addition to linear trends. However, when sales of products at irregular intervals occur, P_T will no longer satisfy the circularity test.

[38] See Diewert (1978; 895) and Hill (1988) (1993; 387-388). Chaining under these conditions will also reduce the spread between fixed base and chained indexes using P_F or P_T as the basic bilateral formula.

sales. The extent of the *price bouncing problem* or the problem of *chain drift* can be measured if we make use of the following test due to Walsh (1901; 389), (1921b; 540):[39]

(27) *Multiperiod Identity Test*: $P(p^0,p^1,q^0,q^1)P(p^1,p^2,q^1,q^2)P(p^2,p^0,q^2,q^0) = 1.$

Thus price change is calculated over consecutive periods but an artificial final period is introduced where the prices and quantities revert back to the prices and quantities in the very first period. The test asks that the product of all of these price changes should equal unity. If prices have no definite trends but are simply bouncing up and down in a range, then the above test can be used to evaluate the amount of chain drift that occurs if chained indexes are used under these conditions. *Chain drift* occurs when an index does not return to unity when prices in the current period return to their levels in the base period; see the ILO (2004; 445). Fixed base indexes that satisfy Walsh's test will not be subject to chain drift.

The *Manual* did not take into account how severe the chain drift problem could be in practice.[40] The problem is mostly caused by periodic temporary promotional *sales* of products. An example will illustrate the problem.

Suppose that we are given the price and quantity data for 2 commodities for 4 periods. The data are listed in Table 1 below.[41]

Table 1: Price and Quantity Data for Two Products for Four Periods

Period t	p_1^t	p_2^t	q_1^t	q_2^t
1	1.0	1.0	10	100
2	0.5	1.0	5000	100
3	1.0	1.0	1	100
4	1.0	1.0	10	100

The first commodity is subject to periodic sales (in period 2), when the price drops to ½ of its normal level of 1. In period 1, we have a "normal" off sale demand for commodity 1 which is equal to 10 units. In period 2, the sale takes place and demand explodes to 5000 units.[42] In period 3, the commodity is off sale and the price is back to 1 but most

[39] This is Diewert's (1993; 40) term for the test. Walsh did not limit himself to just three periods as in T23; he considered an indefinite number of periods. If tests T3 and T22 are satisfied, then T23 will also be satisfied.
[40] Szulc (1983) (1987) demonstrated how big the chain drift problem could be with chained Laspeyres indexes but the authors of the *Manual* did not realize that chain drift could also be a problem with chained superlative indexes.
[41] This example is taken from Diewert (2012).
[42] This example is based on an actual example that used Dutch scanner data. When the price of a detergent product went on sale at approximately one half of the regular price, the volume sold shot up approximately one thousand fold; see de Haan (2008). This paper brought home the magnitude of volume fluctuations due to sales.

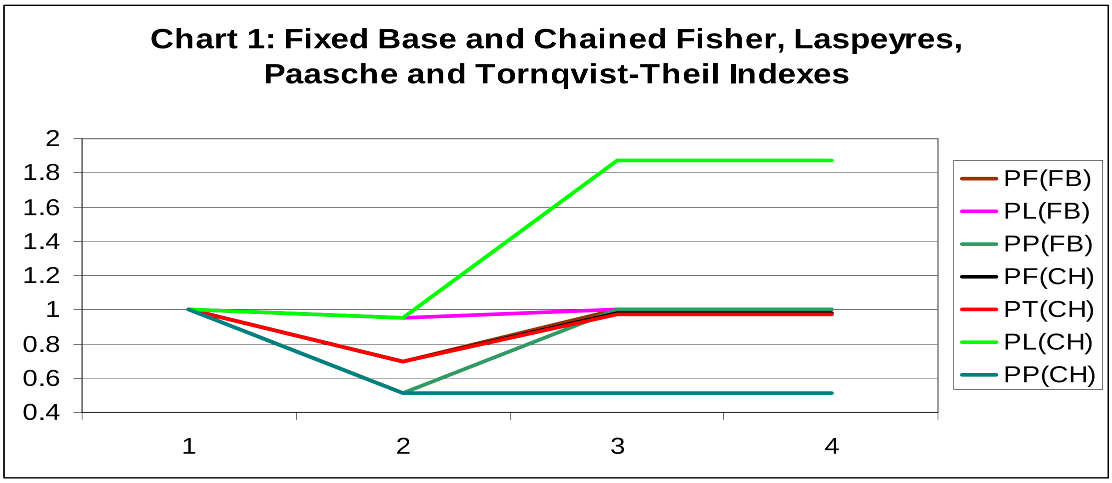
shoppers have stocked up in the previous period so demand falls to only 1 unit.[43] Finally in period 4, the commodity is off sale but we are back to the "normal" demand of 10 units. Commodity 2 is dull: its price is 1 in all periods and the quantity sold is 100 units in each period. Note that the only thing that has happened going from period 3 to 4 is that the demand for commodity one has picked up from 1 unit to the "normal" level of 10 units. Also note that, conveniently, the period 4 data are exactly equal to the period 1 data so that for Walsh's test to be satisfied, the product of the period to period chain links must equal one.

Table 2 lists the fixed base Fisher, Laspeyres and Paasche price indexes, $P_{F(FB)}$, $P_{L(FB)}$ and $P_{P(FB)}$ and as expected, they behave perfectly in period 4, returning to the period 1 level of 1. Then the chained Fisher, Törnqvist-Theil, Laspeyres and Paasche price indexes, $P_{F(CH)}$, $P_{T(CH)}$, $P_{L(FB)}$ and $P_{P(FB)}$ are listed. Obviously, the chained Laspeyres and Paasche indexes have chain link bias that is extraordinary but what is interesting is that the chained Fisher has a 2% downward bias and the chained Törnqvist has a close to 3% downward bias.

Table 2: Fixed Base and Chained Fisher, Törnqvist-Theil, Laspeyres and Paasche Indexes

Period	$P_{F(FB)}$	$P_{L(FB)}$	$P_{P(FB)}$	$P_{F(CH)}$	$P_{T(CH)}$	$P_{L(CH)}$	$P_{P(CH)}$
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	0.698	0.955	0.510	0.698	0.694	0.955	0.510
3	1.000	1.000	1.000	0.979	0.972	1.872	0.512
4	1.000	1.000	1.000	0.979	0.972	1.872	0.512

The above indexes are plotted in Chart 1 below. Because of the wide spreads between the chained Laspeyres and Paasche indexes, it is difficult to distinguish the small bias in the chained Fisher and Theil indexes. Nevertheless, these small biases are significant when they cumulate over long periods of time.



[43] Feenstra and Shapiro (2003) also looked at the chain drift problem that was caused by sales and restocking dynamics. Their suggested solution to the chain drift problem was to use fixed base indexes.

If the above data were monthly, and they repeated themselves 3 times over the year, the overall chain link bias would build up to the 6 to 8% range, which is significant.

What explains the results in the above table? The problem is this: when commodity one comes off sale and goes back to its regular price in period 3, *the corresponding quantity does not return to the level it had in period 1*: the period 3 demand is only 1 unit whereas the “normal” period 1 demand for commodity 1 was 10 units. It is only in period 4, that demand for commodity one recovers to the period 1 level. However, since prices are the same in periods 3 and 4, all of the chain links show no change (even though quantities are changing) and this is what causes the difficulties. If demand for commodity one in period 3 had immediately recovered to its “normal” period 1 level of 10, then there would be no chain drift problem.

There are at least three possible solutions to the chain drift problem that is associated with the use of a superlative index in a situation where monthly scanner data is available to the statistical agency for components of the CPI:[44]

- Stick to the usual annual basket Lowe index that uses annual expenditure weights from a past year;
- Use Rolling Year GEKS to control for chain drift or
- Use the Weighted Time Dummy Product method to control for chain drift.

The last two methods will be explained below along with a discussion of their relative merits. The problem with the first method is that the Lowe index is subject to a small amount of upper level substitution bias, usually in the range of 0.15 to 0.40 percentage points per year.[45] Note that none of the four approaches to index number theory that were described in the previous section endorsed the Lowe index as a target index. The widespread use of the Lowe index is due to its practical nature and the fact that the amount of substitution bias is generally not all that large.[46]

[44] There is a possible fourth method to avoid chain drift within a year when using a superlative index and that is to simply compute a sequence of 12 year over year monthly indexes so that say January prices in the previous year would be compared with January prices in the current year and so on. Handbury, Watanabe and Weinstein (2013) use this methodological approach for the construction of year over year monthly superlative Japanese consumer price indexes using the Nikkei point of sale data base. This data base has monthly price and expenditure data covering the years 1988 to 2010 and contains 4.82 billion price and quantity observations. This type of index number was recommended in the ILO (2004; chapter 22) as a valid year over year index that would avoid seasonality problems. However, central banks and other users require month to month CPIs in addition to year over year monthly CPIs and so the approach of Handbury, Watanabe and Weinstein does not solve the problems associated with the construction of superlative month to month indexes.

[45] For recent retrospective studies on upper level substitution bias for national CPIs, see Armknecht and Silver (2013), Diewert, Huwiler and Kohli (2009) and Huang, Wimalaratne and Pollard (2013). For studies of lower level substitution bias for a Lowe index, see Diewert, Finkel and Artsev (2009) and Diewert (2013).

[46] Recent Canadian research has indicated that the substitution bias can be reduced substantially by more frequent updating of the annual basket; see Huang, Wimalaratne and Pollard (2013).

4. The Rolling Year GEKS Approach to Index Number Theory

We turn now to an explanation of the Rolling Year GEKS method. The GEKS method for making international index number comparisons between countries is due to Gini (1931; 12). It was derived in a different fashion by Eltetö and Köves (1964) and Szulc (1964) and thus the method is known as either the GEKS or EKS method for making multilateral comparisons. Of course, it can also be adapted to making comparisons between multiple time periods. Basically, the GEKS method in the time series context works as follows. Suppose we have price and quantity information for a component of the CPI on a monthly basis for a sequence of 13 consecutive months. Now pick one month (say month k) in this augmented year as the base month and construct Fisher price indexes for all 13 months relative to this base month. Denote the resulting sequence of Fisher indexes as $P_F(1/k)$, $P_F(2/k)$, ..., $P_F(13/k)$.[47] The final set of GEKS indexes for the 13 months is simply geometric mean of all 13 of the specific month indexes; i.e., the final set of *GEKS indexes for the months in the augmented year* is any normalization of the following sequence of indexes:[48]

(28) $[\prod_{k=1}^{13} P_F(1/k)]^{1/13}, [\prod_{k=1}^{13} P_F(2/k)]^{1/13}, ... , [\prod_{k=1}^{13} P_F(13/k)]^{1/13}$.

The above GEKS indexes have a number of important properties:[49]

- They satisfy Walsh's multiperiod identity test so that if any two months in the augmented year have exactly the same price and quantity vectors, then the above index values will coincide for those two months; i.e., *the above indexes are free from chain drift*.
- The above indexes do not asymmetrically single out any single month to play the role of a base period; all possible base months contribute to the overall index values.[50]
- The above indexes make use of all possible bilateral matches of the price data between any two months in the augmented year.
- Strongly seasonal commodities make a contribution to the overall index values.

[47] Using scanner data, it is not trivial to construct these Fisher indexes. The problem is that for each pair of months, it is necessary to determine the list of products that sold in both months so that the relevant Fisher index between those two months can be constructed; see Nakamura and Steinsson (2008) and Nakamura, Nakamura and Nakamura (2011) on these difficulties.

[48] Balk (1981; 74) derived the GEKS parities using this type of argument rather than the usual least squares derivation of the GEKS parities; see Balk (1996) and Diewert (1999b) for these alternative derivations.

[49] The basic idea of adapting a multilateral method to the time series context is due to Balk (1981) who set up a framework that is very similar to the one explained here (which follows Ivancic, Diewert and Fox (2011) more closely). Balk (1981) used an index number formula due to Vartia (1976) in place of maximum overlap bilateral Fisher indexes as his basic building blocks and he considered augmented years of varying length instead of a 13 month augmented year but the basic idea of adapting multilateral methods to the time series context is certainly due to him.

[50] Thus the above GEKS procedure seems to be an improvement over the suggestion of Feenstra and Shapiro (2003) who chose only a single base month.

The last property explains why the augmented year should include at least 13 consecutive months, so that *strongly seasonal commodities* [51] can make a contribution to the overall index.

The major problem with the GEKS indexes defined by (28) is that the indexes change as the data for a new month becomes available. A headline CPI cannot be revised from month to month due to the fact that many contracts are indexed to a country's headline consumer price index. A solution to this problem was proposed by Ivancic, Diewert and Fox (2011). Their method added the price and quantity data for the most recent month to the augmented year and dropped the oldest month from the old augmented year in order to obtain a new augmented year. The GEKS indexes for the new augmented year are calculated in the usual way and the ratio of the index value for the last month in the new augmented year to the index value for the previous month in the new augmented year is used as an *update factor* for the value of the index for the last month in the previous augmented year. The resulting indexes are called *Rolling Year GEKS indexes*.

Numerical experiments with Australian and Dutch scanner data from grocery chains show that the Rolling Year GEKS indexes work well when up to date price and quantity data are made available to the statistical agency; see Ivancic, Diewert and Fox (2011), de Haan and van der Grient (2011), Johansen and Nygaard (2011), van der Grient and de Haan (2011) and Krsinich (2011). In particular, adding and dropping a month of data and recomputing the GEKS indexes does not seem to change past index values very much.[52] Basically, the method seems to control chain drift quite well.[53] More research on the method needs to be done[54] but it looks quite promising.

We turn to an alternative method that could be used to control the chain drift problem.

5. The Weighted Time Product Dummy Approach to Index Number Theory

The *Rolling Year Weighted Time Product Dummy* (RYWTPD) method for constructing indexes that are largely free of chain drift had its origins in the international comparisons literature, just as GEKS also had its origins in that literature. The *Country Product*

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[51] A strongly seasonal commodity is one that is not present in the marketplace for all months of the year.

[52] Balk (1981; 77) also observed the same phenomenon as he computed his GEKS indexes using successively larger data sets. Diewert (2013) also found that Rolling Year GEKS estimates were quite close to their GEKS counterparts for his small data set on Israeli seasonal commodities.

[53] The Australian Bureau of Statistics plans to use RYGEKS for some components of its Consumer Price Index. Statistics Netherlands also computed RYGEKS indexes for some components of its CPI on an experimental basis with good results but they did not implement the method officially; see de Haan and van der Grient (2011). The problem is that the method is difficult to explain to users.

[54] An issue that requires further research is the effects of having different window lengths on the estimates.

Dummy (CPD) method[55] is due to Summers (1973) and a version of it (adapted to the time series context) will now be explained.[56]

Suppose that we are attempting to make a comparison of prices over T consecutive months over a reasonably homogeneous group of say N items. Suppose initially no expenditure weights are available for the collected prices and that exactly K outlets are sampled for each of the N items in each time period. Thus there are TNK price quotes collected across all of the time periods.

Let p_{tnk} denote the price of item n in outlet k in time period t for t = 1,…,T; n = 1,…,N; k = 1,…,K. The basic statistical model that is assumed is the following one:

$$(29)\quad p_{tnk} = a_t b_n u_{tnk}\,; \qquad t = 1,\ldots,T;\ n = 1,\ldots,N;\ k = 1,\ldots,K$$

where the a_t and b_n are unknown parameters to be estimated and the u_{tnk} are independently distributed error terms with means 1 and constant variances. The parameter a_t is to be interpreted as the *average level of prices* (over all items in this group of items) in time period t and the parameter b_n is to be interpreted as *multiplicative units of measurement factor* that is specific to product n.[57] If the error terms are all unity, then it can be seen that the N item prices move in a proportional manner over time and thus weighting is not important since all reasonable price index formula will generate the a_t as the overall price levels up to a factor of proportionality. Thus the a_t are the period t price levels that we want to determine while the b_n are product effects. The basic hypothesis is that the price of product n in outlet k in time period t is equal to a price level a_t times an item commodity adjustment factor b_n times a random error that fluctuates around 1. Taking logarithms of both sides of (29) leads to the following model:

$$(30)\quad y_{tnk} = \alpha_t + \beta_n + \varepsilon_{tnk}\,; \qquad t = 1,\ldots,T;\ n = 1,\ldots,N;\ k = 1,\ldots,K$$

where $y_{tnk} \equiv \ln p_{tnk}$, $\alpha_t \equiv \ln a_t$, $\beta_n \equiv \ln b_n$ and $\varepsilon_{tnk} \equiv \ln u_{tnk}$.

The model defined by (30) is a linear regression model where the independent variables are dummy variables. The least squares estimators for the α_c and β_n can be obtained by solving the following least squares minimization problem:

$$(31)\quad \min_{\alpha_t,\beta_n} \{\textstyle\sum_{t=1}^{T} \sum_{n=1}^{N} \sum_{k=1}^{K} [y_{tnk} - \alpha_t - \beta_n]^2\}.$$

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[55] This method can be viewed as a simple type of hedonic regression model or alternatively, as a descriptive statistics method that summarizes price movements into simple indexes along the lines pioneered by Theil (1967; 136-138).

[56] This material on the TPD method is adapted from Rao (2004) (2005) and Diewert (2004) (2005). The term TPD is due to de Haan and Krsinich (2012) (2013). The adaptation of the CPD method to the time series context is due to Aizcorbe, Corrado and Doms (2003).

[57] The model assumes that the quality of the outlet k is the same for each product n. If this is not the case, then each product in each outlet should be considered a separate commodity and the k index disappears from the model.

We also require a normalization on the α_t and β_n such as $\alpha_1 = 0$.[58] Solve (31) for the least squares solution parameters α_t^* and β_n^* and let $a_t \equiv \exp[\alpha_t^*]$ for t = 2,3,...,T and $b_n \equiv \exp[\beta_n^*]$ for n = 1,...,N. It turns out that the price level for period t, a_t, is the following expression:

$$(32)\ a_t = \prod_{n=1}^{N} \prod_{k=1}^{K} p_{tnk}^{1/NK} / \prod_{n=1}^{N} \prod_{k=1}^{K} p_{1nk}^{1/NK}\ ; \qquad t = 1,...,T.$$

Thus the TPD price level for period t (using the balanced sample of prices) is equal to the geometric mean of all of the period t prices divided by the geometric mean of all of the period 1 prices. However, the solution is much more complicated when some outlet prices are missing from some period or when the number of outlets varies from period to period. We will deal with these more complicated situations below when we introduce weighting.

Now introduce weighting into the picture. Thus for product n in time period t, we assume that there are K(t,n) outlets that have transactions in product n[59] and that the *unit value price* for the kth such transaction is p_{tnk} and the associated *quantity transacted* is q_{tnk} for k = 1,2,…,K(t,n). Again, $y_{tnk} \equiv \ln p_{tnk}$ is the logarithm of the price p_{tnk}. For each time period t, we use the prices and quantities p_{tnk} and q_{tnk} in order to form the following *period t expenditure shares* across all products n and all outlets k:

$$(33)\ s_{tnk} \equiv p_{tnk}q_{tnk} / \sum_{i=1}^{N} \sum_{j=1}^{K(t,n)} p_{tij}q_{tij}\ ; \qquad t = 1,...,T\ ;\ n = 1,\ldots,N\ ;\ k = 1,\ldots,K(t,n).$$

For each time period t, these expenditure shares sum up to 1:

$$(34)\ \sum_{n=1}^{N} \sum_{k=1}^{K(t,n)} s_{tnk} = 1\ ; \qquad t = 1,\ldots,T.$$

The *Weighted Time Product Dummy* (WTPD) counterpart to the unweighted least squares minimization problem (31) above is:

$$(35)\ \min_{\alpha_t,\beta_n} \{\sum_{t=1}^{T} \sum_{n=1}^{N} \sum_{k=1}^{K(t,n)} s_{tnk}\,[y_{tnk} - \alpha_t - \beta_n]^2\}.$$

Again, the parameters α_t and β_n cannot be uniquely identified so we will choose to set the price level in period 1, $a_1 \equiv \exp[\alpha_1]$, equal to 1, which implies the following normalization on the parameters appearing in (35):

$$(36)\ \alpha_1 = 0.$$

In order to obtain a classical regression model that has a solution consistent with the least squares minimization problem (35) subject to the constraint (36), we need to multiply

[58] This normalization implies that $a_1 = 1$; i.e., the aggregate price level is set equal to unity in the first period. Thus the price levels for subsequent periods a_t become price indexes (relative to the level of prices in period 1).

[59] We allow K(t,n) to be zero; i.e., it can be the case that for some time periods t, there are no price quotes collected for product n.

each y_{cnk} by the square root of the associated expenditure share s_{cnk} defined by (33); i.e., the counterparts to our linear regression equations (30) are now the following equations:

$$(37)\quad s_{tnk}^{1/2}\, y_{tnk} = s_{tnk}^{1/2}\, \alpha_t + s_{tnk}^{1/2}\, \beta_n + \varepsilon_{tnk}\ ; \qquad t = 1,\ldots,T;\ n = 1,\ldots,N;\ k = 1,\ldots,K(c,n)$$

where $y_{tnk} \equiv \ln p_{tnk}$ and the α_t for t = 2,…,T and β_n for n = 1,….,N are parameters to be estimated (α_1 is set equal to 0) and the ε_{tnk} are assumed to be independently distributed error terms with means 0 and variances σ^2. If for any t and n, K(t,n) = 0 so that there are no item n prices collected in time period t, then the corresponding equations in (37) are dropped.

In order to rigorously justify the linear regression model (37) from an econometric point of view, we need to assume that the variance of y_{tnk} is proportional to σ^2/s_{tnk} for t = 1,…,T; n = 1,…,N; k = 1,…,K(t,n).[60] This means that the smaller is the expenditure share s_{tnk}, the bigger will be the variance of y_{tnk}. This assumption is not likely to be precisely justified from a statistical point of view but solving the weighted least squares problem (37) leads to very reasonable estimates for the period t price levels, $a_t \equiv \exp[\alpha_t^*]$ for t = 2,3,...,T where the α_t^* are the least squares estimates of the α_t for the linear regression model defined by (37). These estimates are reasonable from the viewpoint of classical index number theory, where weighting by economic importance is regarded as being extremely important. It is worth quoting Irving Fisher on the importance of weighting:

"It has already been observed that the purpose of any index number is to strike a 'fair average' of the price movements—or movements of other groups of magnitudes. At first a *simple* average seemed fair, just because it treated all terms alike. And, in the absence of any knowledge of the relative importance of the various commodities included in the average, the simple average *is* fair. But it was early recognized that there are enormous differences in importance. Everyone knows that pork is more important than coffee and wheat than quinine. Thus the quest for fairness led to the introduction of weighting." Irving Fisher (1922; 43).

"But on what principle shall we weight the terms? Arthur Young's guess and other guesses at weighting represent, consciously or unconsciously, the idea that relative *money values* of the various commodities should determine their weights. A value is, of course, the product of a price per unit, multiplied by the number of units taken. Such values afford the only common measure for comparing the streams of commodities produced, exchanged, or consumed, and afford almost the only basis of weighting which has ever been seriously proposed." Irving Fisher (1922; 45).

Thus it can be argued that solving (35) leads to index numbers that are reasonable from a descriptive statistics point of view; i.e., the resulting price levels are a reasonable way of summarizing overall price trends in the data, where the relative economic importance of each unit value price is taken into account in the model.[61]

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[60] An alternative way for justifying the weighted model (37) is to argue that each logarithmic price ln p_{tnk} should be repeated according to its economic importance; i.e., if consumers are spending e_{tnk} dollars on commodity n during time period t, then ln p_{tnk} should appear e_{tnk} times in the regression instead of only once. In order to standardize these weights across time periods, we change the e_{tnk} weight to s_{tnk}. This type of argument was used by Diewert (2005) (2006).

[61] There is another way of proceeding and that is to solve the weighted least squares problem but instead of assuming the stochastic specification given below (37), assume that $y_{cnk} = \alpha_c + \beta_n + \varepsilon_{cnk}$ where the ε_{cnk} are independently distributed and have mean zero and variance σ^2. We still solve (35) for the weighted least

It can be verified that if the expenditure and price data are exactly the same for any two periods, then the WTDP method will generate price levels for the two periods that are also identical. Thus the WTPD estimates satisfy an identity test and hence are free of chain drift over the T periods in the sample.[62]

The WTPD price level estimates suffer from the same problem that the GEKS estimates suffer from: the addition of one more period to the sample will change all of the estimates. Thus Ivancic, Diewert and Fox (2009) proposed a Rolling Year approach to the Weighted Time Product Dummy (RYTPD) estimation procedure; i.e., set T = 13 and as a new month's data is added, delete the data for the oldest month in the sample, obtain new WTPD estimates and use the month over month movement in the estimated price levels for the last two months to update the previous estimates.[63]

When we move from WTPD estimates to RYWTPD estimates, the Multiperiod Identity Test is no longer satisfied by the price level estimates and so the rolling year variant of the method is subject to possible chain drift. However, as was the case with the move from GEKS to RYGEKS, empirically very little difference is found between the rolling year indexes and their fixed sample counterparts.[64] Thus both the RYGEKS and RYWTPD methods seem to be largely free of chain drift.

How do the RYGEKS estimates compare with the corresponding RYWTPD estimates when using the same data set? Empirical experience is limited but in the studies that have compared the two methods, there is a general tendency for the RYWTPD estimates to be slightly less than their RYGEKS counterparts.[65]

A possible explanation for the differences in the indexes generated by the two methods may be due to the *democratic weighting* that is inherent in the GEKS method. Thus the GEKS estimates are formed by averaging a series of 13 separate sets of index numbers where the data of each month in the augmented rolling year are used as the base price and quantities in each of the bilateral indexes. If the data for one month is sparse so that the value of transactions in that month is unusually low and perhaps not "typical", then these atypical indexes are averaged with all of the other 12 sets of indexes and given an equal

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squares α_t^* and β_n^* but the resulting parameter estimates are no longer minimum variance unbiased for the new stochastic specification. However, the resulting estimates are still unbiased under the new stochastic specification and they are representative from the viewpoint of index number theory. Hill and Timmer (2006) take this point of view. Note also that Diewert (2005) derived an explicit index number formula for a_2 using the weighted least squares model defined by (37) for the two period case; i.e., the case where T = 2. Diewert also showed that the resulting index number formula approximated the Törnqvist-Theil index to the second order around an equal price and quantity point.

[62] De Haan and Krsinich (2012) noted this property of the method.

[63] Ivancic, Diewert and Fox (2009) is essentially the same as Ivancic, Diewert and Fox (2011) except the former paper had an extra section in it which compared the RYWTPD method to the RYGEKS method using Australian scanner data.

[64] See Ivancic, Diewert and Fox (2009) and de Haan and Krsinich (2012) (2013).

[65] See Ivancic, Diewert and Fox (2009) and de Haan and Krsinich (2012) (2013). The latter authors used the Törnqvist-Theil index formula as their basic bilateral formula in their RYGEKS estimates instead of the Fisher index but it is unlikely that this formula difference would affect the results.

weight in the averaging process.[66] On the other hand, the WTPD method would automatically give a much lower weight to the possibly atypical prices in the low volume month. The WTPD method works on a principle that tries to fit heterogeneous price movements over the sample into a simpler framework where all price movements are approximated by proportional movements in prices over time, taking into account the economic importance of the prices.

A simple (extreme) example may help to illustrate possible problems with the GEKS methodology. Suppose we have price and expenditure share data for 3 products for 3 periods but each product is present in only 2 of the 3 periods. Suppose the first product is present in periods 1 and 2 with prices p_1^1, p_1^2, the second product is present in periods 2 and 3 with prices p_2^2, p_2^3 and the third product is present in periods 1 and 3 with prices p_3^1, p_3^3. The period 1 expenditure shares for products 1 and 3 are s_1^1 and s_3^1, the period 2 expenditure shares for products 1 and 2 are s_1^2 and s_2^2 and the period 3 expenditure shares for products 2 and 3 are s_2^3 and s_3^3. The expenditure shares for each period sum to one. Because of the missing data, we can only calculate 3 matched product bilateral indexes across the 3 periods. The Fisher index for period 2 relative to period 1, $P_F(2/1)$, turns out to equal the price ratio p_1^2/p_1^1; the Fisher index for period 3 relative to period 2, $P_F(3/2)$, turns out to equal the price ratio p_2^3/p_2^2 and the Fisher index for period 3 relative to period 1, $P_F(3/1)$, turns out to equal the price ratio p_3^3/p_3^1. We can compute three separate set of price levels using different combinations of the available bilateral indexes.

The first set of parities uses the index $P_F(2/1)$ to determine the period 2 price level relative to the period 1 level and the period 3 price level relative to the period 1 level is determined as $P_F(3/1)$. The resulting price levels are the following ones:

(38) $P^1 = 1$; $P^2 = (p_1^2/p_1^1)$; $P^3 = (p_3^3/p_3^1)$.

The second set of parities uses the index $P_F(2/1)$ to determine the period 2 price level relative to the period 1 level and the period 3 price level relative to the period 1 level is determined as the product $P_F(2/1)$ times $P_F(3/2)$.[67] The resulting price levels are the following ones:

(39) $P^1 = 1$; $P^2 = (p_1^2/p_1^1)$; $P^3 = (p_1^2/p_1^1)(p_2^3/p_2^2)$.

The third set of parities uses the index $P_F(3/1)$ to determine the period 3 price level relative to the period 1 level and the period 2 price level relative to the period 1 level is determined as the product $P_F(3/1)$ times $P_F(2/3)$. The resulting price levels are the following ones:

(40) $P^1 = 1$; $P^2 = (p_3^3/p_3^1)(p_2^2/p_2^3)$; $P^3 = (p_3^3/p_3^1)$.

[66] This situation occurs frequently in the context of making international comparisons of prices using the GEKS method.

[67] Because there is only one commodity whose price is compared in each bilateral Fisher index, these bilateral Fisher (and Laspeyres and Paasche) indexes all collapse down to simple price ratios.

The price levels defined by (38) are a normalization of the Fisher parities generated by using period 1 as the base period, while the price levels defined by (39) and (40) are normalizations of the Fisher parities that use periods 2 and 3 as the base periods respectively.

Taking the geometric mean of the above price levels leads to the following GEKS price levels:

(41) $P^1 = 1$; $P^2 = [(p_1^2/p_1^1)^2(p_3^3/p_3^1)(p_2^2/p_2^3)]^{1/3}$; $P^3 = [(p_1^2/p_1^1)(p_2^3/p_2^2)(p_3^3/p_3^1)^2]^{1/3}$.

Note that the GEKS price levels do not depend on the expenditure shares. However, the Weighted Time Product Dummy price levels for this example *will* depend on the expenditure shares. The exact formula for these price levels is too complicated to be exhibited here but we know that the WTPD price levels will be weighted according to the size of the expenditure shares in each period.[68] In particular, suppose the commodity 3 expenditure shares, s_3^1 and s_3^3, are tiny. Then the WTPD price levels will be close to the price levels defined by (39) (which do not involve the prices p_3^1 and p_3^3). On the other hand, suppose the commodity 2 expenditure shares, s_2^2, s_2^3, are close to zero. Then the WTPD price levels will be close to the price levels defined by (38) (which do not involve the prices p_2^2 and p_2^3). Finally, suppose the commodity 1 expenditure shares, s_1^1, s_1^2, are close to zero. Then the WTPD price levels will be close to the price levels defined by (40) (which do not involve the prices p_1^1 and p_1^2). In each of the three cases just considered, the WTPD price levels are very reasonable; the unimportant commodity is given a low weighting in the overall index but this is not the case for the GEKS price levels: the GEKS price levels remain the same under all three scenarios! Thus if price movements are far from proportional over time, so that the price levels defined by (38)-(40) are very different, then the GEKS indexes may be rather far removed from their WTPD counterparts, which will be much more reasonable in each of the three cases considered above. These possible problems with the GEKS indexes carry over to Rolling Year GEKS indexes.

More research is required on pinning down the differences between the GEKS estimates and their WTPD counterparts but at this stage, we tentatively conclude that in the case where the period to period data is sparse and there is a lack of product matching for each pair of periods under consideration, the WTPD estimates may be preferable to the corresponding GEKS estimates (and the RYWTPD estimates may be preferable to the corresponding RYGEKS estimates).

6. Elementary Indexes: New Developments

The ILO *Manual* basically recommended the Jevons formula for elementary indexes.[69] This advice was based on the axiomatic approach to elementary indexes; see Diewert (1995a). In the case of complete data on a sample of products with no sample attrition,

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[68] In this example, K(t,n) is always equal to 1; i.e., we have only one outlet for each product.

[69] Elementary indexes are constructed using item prices only (due to the unavailability of quantity information).

we end up with the formula (32) in the previous section. But real life does not generate complete samples with no attrition; products disappear and then reappear[70] and some products disappear permanently. If a product disappearance is thought to be temporary, price statisticians typically impute the missing prices. But there are many imputation methods[71] and this creates a certain amount of uncertainty about the accuracy of the index at any give time period.

There is another problem associated with elementary indexes that the *Manual* did not deal with and that is the fact that many statistical agencies do not chain their elementary indexes every month: they choose a reference month and then calculate item prices relative to the item price in the base month for 13 months.[72] The problem with this strategy is that the procedure depends asymmetrically on the choice of the base month. Some items will not be available in the base month and so how are we to treat these items which appear in subsequent months?

Thus chaining elementary price quotes is problematic (due to the necessity of imputing prices for temporarily disappearing items and for strongly seasonal items) and using a fixed base methodology for elementary indexes is also problematic (due to the fact that some products may not be available in the base month and more generally, due to the asymmetry of choosing one month out of 12 months as the base month).

A solution to these problems was suggested by de Haan and Krsinich (2012) and Diewert (2012): use the Time Product Dummy methodology in order to construct elementary indexes. These TPD indexes are generated by solving the least squared minimization problem (35) above, except the expenditure shares s_{tnk} are all set equal to one. The resulting elementary indexes, $a_1 = 1$, $a_2 = \exp[\alpha_2]$, ..., $a_N = \exp[\alpha_N]$ have a large number of good axiomatic properties.[73] This new approach to the construction of elementary indexes seems promising. De Haan and Krsinich (2012) and Diewert (2012) suggested that the TPD methodology could be generalized into a Rolling Year Time Product Dummy (RYTPD) method where a moving sample of 13 consecutive months of item price data is used to generate TPD price levels and then the movement in the index for the last two months is used to update the previous index.

The RYTPD method for constructing elementary indexes seems to be very promising. It is relatively easy to implement, there are no imputations required for the method and it treats the price data for each period in a symmetric manner.

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[70] This can happen with strongly seasonal products or the temporary disappearance may be due to the fact that retailers sometimes rotate the brand items that they sell in order to generate price discounts from manufacturers.

[71] See Feenstra and Diewert (2001) for a review of alternative imputation methods and references to the literature.

[72] Thus the Retail Prices Index in the UK uses January as its base month whereas the Harmonized Index of Consumer Prices used as the official Eurostat measure of European household inflation uses December as its base month for a sequence of 13 months. In many other countries, month to month chained elementary indexes are used.

[73] See de Haan and Krsinich (2012) (2013).

More research into the method is required. It would also be useful to look at variants of the method that allowed for longer windows; e.g., instead of using 13 consecutive months of data, perhaps more stable estimates may be obtained using 25 consecutive months of data in each RYTPD regression.[74]

7. New Approaches to Quality Adjustment

A problem with the RYGEKS and RYWTPD methods described above is that these methods do not deal adequately with the introduction of new products. Thus if a new product enters the marketplace during the last period in the Rolling Year, it will have no effect on the index for the current period and all previous periods. De Haan and Krsinich (2012) (2013) invented a method that deals with this problem. The basic building block in their method is a time dummy hedonic regression model that uses the data for two periods. The dependent variable in the model is the logarithm of the item price and a time dummy and various characteristics of the product enter the regression as independent variables. The time dummy coefficient and the characteristic "prices" are the result of a weighted least squares minimization problem. If an item appears in both periods under consideration, the weights in the weighted regression are the (arithmetic) average of the expenditure shares for the item in the two periods; if the item appears in only one of the two periods, one half of the expenditure share on the item for that period is used as the weight. The resulting bilateral price index turns out to equal the usual Törnqvist index if all items are present in both periods but for unmatched items, an imputed price for the missing price enters the index number formula and this imputed price is obtained as a predicted price using hedonic regression. Thus in the general case when there are unmatched items in the two periods under consideration, we obtain a generalization of the usual Törnqvist index that makes use of imputed prices from the hedonic regression and hence de Haan and Krsinich (2012) call the resulting bilateral index number formula the *Imputation Törnqvist index*.[75] De Haan and Krsinich (2012) (2013) proposed the following variation of the Rolling Year GEKS method: instead of using bilateral Fisher indexes as the basic building blocks, the Fisher indexes are replaced by bilateral Imputation Törnqvist indexes. They call the resulting indexes ITRYGEKS indexes.[76]

Which of the three methods discussed above is "best"? Methods 1 and 2 (RYGEKS and RYWTPD) have the disadvantage that unmatched items in any bilateral index used as building blocks in these methods have no impact on the resulting indexes. But these methods have the advantage that no information on product characteristics is required in order to implement these indexes. Method 3 (ITRYGEKS) has the advantage that it is likely to have the least amount of bias due to the introduction of new models and the disappearance of old models but of course, it has the disadvantage that product characteristics information is required in order to implement the method. The bottom line

[74] If the time period is longer than an (augmented) year, then the term Rolling *Window* TPD method is more appropriate.
[75] This index is derived in de Haan and Krsinich (2012)(2013) and draws on earlier contributions by Diewert (2003) and de Haan (2003) (2004).
[76] It might be more appropriate to call these indexes ITRYCCD indexes since multilateral Caves, Christensen and Diewert (1982) indexes are used in place of multilateral GEKS indexes in the case where all items are matched.

is that ITRYGEKS may be the best method that can deal with chain drift and quality change in the context of using scanner data.[77]

De Haan and Krsinich (2012) (2013) and Krsinich (2013) showed that for electronic products in New Zealand, the RYWTPD indexes were closer to their "gold standard" ITRYGEKS indexes than their RYGEKS counterparts. This is a somewhat surprising result since it is known that in the two period case where all products are present in both periods, RYGEKS and RYWTPD approximate each other closely.[78] However, the results presented by Krsinich indicate that this close correspondence does not necessarily hold in more realistic environments when not all products are present in all periods. The implication of the results presented by de Haan and Krsinich is that when information on product characteristics is not available, the RYWTPD method may be preferred to the RYGEKS method. This is an important result but more research on this is required.[79]

The results derived by de Haan and Krsinich required the availability of scanner data on sales and the prices of various electronic products. Suppose the statistical agency does not have access to data on sales and prices. What is the "best" approach to quality adjustment when only price data is collected? This is an open question.

The work by de Haan and Krsinich is perhaps the most important work on the theory and practice of quality adjustment that has appeared since the publication of the *Manual*. Of lesser importance is the research by Diewert, Heravi and Silver (2009) and de Haan (2010) that examines more closely the differences between the time dummy approach and the hedonic imputation approach to hedonic regressions.

8. Long Time Problems with the CPI that Still Need to be Addressed

There are several long standing problems associated with the construction of a CPI that have troubled national statisticians over the years. My list of vexing problems is the following list:

- Should the CPI be compiled on a domestic, national or household inflation basis?
- What is the appropriate treatment of Owner Occupied Housing (OOH) in the CPI?
- How exactly should financial services be treated in the CPI?
- How can strongly seasonal commodities make a contribution to the month to month CPI?
- Is it possible to construct real time CPIs using current month information on prices and older information on household expenditure shares that will approximate a superlative CPI that is constructed later when additional data on expenditures shares become available?

8.1. National versus Domestic Versus Household Inflation

[77] However, it should be possible to adapt the WTPD method to deal with quality change.
[78] See Diewert (2005; 564).
[79] This result reinforces the earlier misgivings about the democratic nature of the GEKS indexes in the context of sparse data.

It seems to me that the national CPI should be calculated on a domestic basis; i.e., we look at the expenditures of households that primarily reside in the nation. Many economists use the domestic CPI to deflate nominal consumer expenditures to look at the welfare of residents in the country. Thus from the viewpoint of welfare economics, nations should provide a national CPI. However, when we look at the production accounts of a country (and the associated Multifactor Productivity or Total Factor Productivity accounts of a nation), we require a "domestic" CPI which is usually labelled as the Domestic Consumption Deflator. Thus we require *both* the national and domestic consumption deflators to fill in the deflator cells in the System of National Accounts.[80]

The Harmonized Index of Consumer Prices that was introduced by Eurostat to achieve comparability across countries belonging to the European Union was introduced as a household inflation index that central banks could use to gauge relative inflation rates across member countries. This is a valid household inflation index but it does not provide a substitute for the national and domestic consumer price indexes described in the above paragraph.[81]

8.2 The Problem of Owner Occupied Housing in the CPI

There is no consensus on how Owner Occupied Housing (OOH) should be treated in the CPI. The main approaches to the treatment of OOH are as follows:

- The *rental equivalence approach*. In this approach, the value of the services of OOH is the rent that the owned unit could accrue if it were rented.
- The *monetary expenditures approach*. In this approach, the out of pocket costs of home ownership are totalled to provide an imputed "rent".[82]
- The *acquisitions approach*. In this approach, the ownership of previously purchased housing properties is ignored; only newly constructed housing units are in scope.[83]
- The *user cost approach*. In this approach, the imputed value of housing services is set equal to the financial cost of tying up owner's capital in the house.

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[80] The main difference between the two indexes is the expenditures of nationals abroad (this is in scope for the national concept) and the expenditures of tourists in the home country (this is in scope for the domestic concept).

[81] The Eurostat HICP index was originally introduced as an index that would make absolutely no imputations. However, over time, it was recognized that quality change required imputations and eventually the HICP allowed imputations for quality change. Diewert (2002b) criticized the HICP from the viewpoint that it did not fit into the System of National Accounts. However, if central banks want a minimal imputation index that measures consumer inflation, then the HICP fills a very useful function.

[82] The problem with this approach is that it does not list all of the (opportunity) costs and benefits of ownership. The main missing costs are depreciation and the financial capital tied up in the equity of the housing unit and the main benefit that is missing is the expected capital gains (or losses) on the property. This is my least preferred alternative treatment of owner occupied housing.

[83] This is the variant that the HICP has chosen to implement. There are two variants of the method: include only the structure portion of the new building or include the structure and land components together.

- The *opportunity cost approach*. In this approach, the maximum of the user cost and rental equivalent price is used as the valuation of the services of the housing unit.[84]

Since no consensus on the appropriate approach to the valuation of the services of Owner Occupied Housing has been achieved in the literature, it seems reasonable to ask national statistical agencies to provide analytical series for all five approaches to the valuation of housing services. To date, these analytical series have not been forthcoming!

8.3 The Measurement of Financial Services in the CPI

Financial services are an important component of GDP and a somewhat important component of household consumption. What is amazing is that there is absolutely no agreement on how to measure these services. The main components of financial services for households are the services provided by their monetary deposits and insurance services for property and life. There are other financial services that are easier to measure such as stock trading (this is basically a margin industry and can be treated in a manner similar to wholesaling and retailing).[85]

I will not go into all of the alternative treatments of financial services that have been suggested in the literature. Suffice it to say that there is extreme heterogeneity in these treatments.[86] It would be good if academics could turn their attention to these basic measurement problems in the area of financial services in the near future.

8.4 How Can Strongly Seasonal Commodities Make a Contribution to the Month to Month CPI?

The answer to the above question is relatively straightforward in the light of the analysis that we have done above in looking at the properties of the GEKS and WTPD methods. For both of these methods, strongly seasonal commodities play a role in the overall index construction. Similarly, if we look at elementary indexes, the TPD method explained above deals adequately with strongly seasonal commodities. This is encouraging!

The practical question to be resolved in the coming years is whether the RYGEKS is better than the RYWTPD method. This is an open question.

At the elementary level, the RYTPD method seems to be a much superior option to other methods for constructing elementary indexes.

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[84] See Diewert (2011) and Diewert, Nakamura and Nakamura (2009) for a description of the alternative approaches.
[85] However, in wholesaling and retailing, the price of the service is the margin times the price of the products being purchased. In the case of stock trading, the appropriate price is not completely clear.
[86] I have been working in this area with my coauthors (Fixler and Zieschang) for several years and I can report that it is difficult for the three of us to agree on a suitable framework for modeling financial transactions. For our recent efforts, see Diewert, Fixler and Zieschang (2013a) (2013b) and Diewert (2013b).

8.5 Predicting Superlative CPIs Using Current Prices and Past Expenditure Shares

Is it possible to construct a real time CPI using current month information on prices and older information on household expenditure shares that will approximate a superlative CPI that is constructed later when additional data on expenditures shares become available? It seems unlikely that we will be able to approximate a superlative CPI perfectly using currently available data but recent research has indicated that it is possible to obtain pretty good approximations to a superlative CPI using current data; see Armknecht and Silver (2013) and Huang, Wimalaratne and Pollard (2013). This is an promising area which requires more research.

9. Conclusion

In the decade since the *Manual* appeared, there have been some significant new developments in the theory and practice of CPI construction. A new development is the fact that some supermarket firms are willing to share their price and quantity data with national statistical agencies. Hopefully, this spirit of cooperation will spread to other countries.[87]

With the advent of scanner data availability, it becomes possible to compute the type of indexes that have been recommended by index number theorists over the past century. But new problems have emerged; in particular the problem of chain drift for superlative indexes has emerged.

This review paper has indicated methods for overcoming the chain drift problem. When price and quantity information is available, the Rolling Year GEKS or the Rolling Year Weighted Time Product Dummy method is recommended. For high tech products that are undergoing rapid technological change, the methods for quality adjustment developed by de Haan and Krsinich are recommended. At the level of elementary indexes, the Rolling Year Time Product Dummy method is recommended.

However, the above methods have not been thoroughly tested and so perhaps some caution is in order. Hopefully, further research in the coming years will demonstrate whether the suggested methods are definitely preferred.

Finally, in section 8 above, some long standing problem areas with respect to CPI construction have been highlighted. It would be good if some progress could be made on resolving these problems in the coming decade.

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[87] Many retailers have the information in files; the costs of providing this information to national statistical agencies is trivial.

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