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Repos in Over-the-Counter Markets

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Repos in Over-the-Counter Markets

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Abstract

This paper presents a dynamic matching model featuring dealers and short-term investors in an over-the-counter bond market. The model illustrates that bilateral bargaining in an over-the-counter market results in an endogenous bond-liquidation cost for short-term investors. This cost makes short-term investors need repurchase agreements to buy long-term bonds. The cost also explains the existence of a margin specific to repurchase agreements held by short-term investors, if repurchase agreements must be renegotiation-proof. Without repurchase agreements, short-term investors do not buy long-term bonds. In this case, the bond yield rises unless dealers have enough capital to buy and hold bonds.

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Keywords: Repo; Over-the-counter market; Securities broker-dealer; Short-term investor; Margin.

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1 Introduction

Repurchase agreements, or repos, are one of the primary instruments in the money market. In a repo, a short-term investor buys bonds with a future contract in which the seller of the bonds, typically a bond dealer, promises to buy back the bonds at a later date. A question arises from this observation regarding why investors need such a promise when they can simply buy and resell bonds in a series of spot transactions. In this paper, I present a model to illustrate that a bond-liquidation cost due to over-the-counter (OTC) trading can explain short-term investors' need for repos in bond markets. It is not necessary to introduce uncertainty or asymmetric information to obtain this result.

In the model, investors buy long-term bonds to store cash until some fixed future dates for their cash payments. Thus, their investments are for a short term. To trade bonds, an investor must transact with a bond dealer bilaterally in an OTC bond market. This assumption is based on the fact that most bond markets are OTC markets in practice.

The model illustrates that when a short-term investor resells bonds in an OTC market, the buying dealer can negotiate down the bond price. This result is due to bilateral bargaining in an OTC market: if someone sells an asset because he needs cash soon, then the buyer of the asset can take advantage of the seller's situation to lower the asset price. This ex-post price discount in an OTC market discourages a short-term investor from buying bonds in a spot transaction ex-ante.

A repo is useful to avoid this hold-up problem. By selling bonds with a future contract to repurchase bonds, a dealer can offer the initial ask price and the repurchase price simultaneously to an investor. Thus, a dealer can adjust the two prices to ensure a sufficiently high bond return for an investor to buy bonds.

This result is robust even if a dealer cannot commit to an arbitrary repurchase price. If a repurchase agreement must be renegotiation-proof, then the pledgeable repurchase price equals the bid price in spot trade. Thus, the repurchase price cannot be free of an ex-post price discount in an OTC market as described above. In this case, a dealer can lower the initial ask price in a repo transaction, so that the investor in the transaction can earn a sufficiently high bond return despite a low repurchase price. In equilibrium, the initial ask price becomes lower than the interdealer bond price. Thus, there appears an endogenous repo margin—that is, a difference between the initial ask price and the market value of a bond. This result is consistent with the fact that short-term investors had a positive repo margin even when there were no margins in interdealer repos before the recent subprime mortgage crisis.

To further investigate the role of repos in a bond market, I compute equilibrium dynamics without repo transactions. In this case, there appears an excess return on bonds unless dealers have enough capital to buy and hold bonds. This result suggests that the disappearance of repos would have a negative effect on aggregate economic activity through a rise in long-term interest rates, such as mortgage rates.

Finally, I examine the behavior of an investor with no fixed date for a cash payment. Thus, the investor can hold bonds for a long term. In this case, the investor suffers no ex-post price discount on its bonds in an OTC market. Hence, the investor does not need a repo to buy long-term bonds. This result explains why short-term investors are the main users of repos in practice.

1.1 Related literature

This paper is related to the literature on spot trading in OTC markets intermediated by dealers. Duffie, Gârleanu and Pedersen (2005) show that search and bargaining result in bid-ask spreads in an OTC market even without inventory risk or asymmetric information. While their model abstracts from the adjustments of asset holdings by investors, Lagos and Rocheteau (2010) show that such adjustments affect trade volumes, bid-ask spreads, and trading delays in an OTC market. Further, Lagos, Rocheteau and Weill (2011) analyze dealers' liquidity provision in an OTC market during a financial crisis. Miao (2006) presents a different type of model from these papers, in which each investor endogenously chooses whether to trade with a dealer or another investor. He analyzes bid-ask spreads and liquidity in such an environment.

This paper adds to this literature by incorporating repos into a dynamic model of an OTC bond market. In this regard, this paper is also related to the paper by Monnet and Narajabad (2011). They consider an OTC asset market in which investors trade directly with each other, and show that repos and spot trades of assets co-exist if each investor has uncertainty about the future use of assets. This paper differs from their work in focusing on a repo between a dealer and an investor and analyzing the relationship between the investment horizon of an investor and the investor's need for a repo.

Other strands of literature highlight aspects of repos other than OTC transactions. Martin, Skeie and von Thadden (2010) model a repo as a liquidity insurance. Using their framework, they analyze the conditions under which dealers issuing repos suffer runs due to coordination failure among investors holding repos.

Also, there exist many works on debt financing and asset prices, which are related to repos because repos are regarded as secured loans in practice. For example, Hart and Moore (1994) and Kiyotaki and Moore (1997) show that renegotiation-proof loan contracts must be secured on collateral if borrowers have intangible human capital for production. Geanakoplos (2009) derives endogenous collateral constraints based on value-at-risk and analyzes the effects of pro-cyclical leverage in the presence of heterogeneous investor beliefs. Brunnermeier and Pedersen (2009) also model collateral constraints based on value-at-risk and analyze the linkage between market liquidity and funding liquidity in an asset market. This paper is different from these papers in focusing on the implications of bilateral bargaining in an OTC market. This paper's finding that OTC trading yields an endogenous repo margin, which is akin to a downpayment in a secured loan, contributes to the existing literature.

Finally, there is a strand of literature on special repo rates due to securities lending to short-sellers, which includes such works as Duffie (1996) and Vayanos and Weill (2008). In this paper, I focus on general repos by considering homogeneous bonds in the model.

The remainder of this paper is organized as follows. I briefly review the basic features of repos in the U.S. data in Section 2. A model of an OTC bond market is presented in Section 3. I show that OTC trading results in the short-term investors' need for repos in Section 4. I consider a renegotiation-proof repo in Section 5. The case without repos is analyzed in Section 6. I show that a long-term investor has no need for repos in Section 7. I conclude in Section 8.

2 Brief overview of the features of the U.S. repo market

In this section, I briefly review the basic features of repos in the U.S. data. Figure 1 compares the outstanding values of money market instruments. It indicates that repos issued by securities broker-dealers have maintained a non-negligible share in the money market. In particular, their figure comes close to that of bank deposits before the recent subprime mortgage crisis that started in 2007.

The main repo market in the U.S. is the so-called tri-party repo market.[1] In this market, the main holders of repos issued by securities broker-dealers are short-term investors, such as money market mutual funds (MMFs) and securities lenders, as reported by Copeland, Martin and Walker (2010).[2] Also, Table 1 from Copeland, Martin and Walker shows that most of the bonds traded in the tri-party repo market are Treasury securities, agency mortgage-backed securities, and agency debt. These securities are government-guaranteed. Thus, short-term

[1]In the tri-party repo market, a clearing bank is involved as a third party in a repurchase agreement. See the paper by Copeland, Martin and Walker (2010) for more details on this market.

[2]Securities lenders receive cash collateral from short-sellers in exchange for lending securities. They invest cash until the time when they must return the cash collateral to short-sellers.

investors need repos even to buy the least risky bonds in the economy.[3]

Repos held by short-term investors involve margins. A margin is a difference between the initial ask price that an investor pays for bonds in a repo transaction and the market price of the bonds traded. Typically, the initial ask price is lower than the market price. Krishnamurthy, Nagel and Orlov (2011) report that the margins for repos held by MMFs have been consistently around 2% of the market price of bonds if the bonds are Treasury securities. They find higher margins for other types of bonds. These findings contrast with the margins for interdealer repos, which had been zero until the recent subprime mortgage crisis, as reported by Gorton and Metrick (2012). Thus, the margins for repos held by short-term investors were positive even when those for interdealer repos were zero. Hereafter, I replicate these features of repos in a dynamic model of an OTC bond market.

3 Model of an OTC bond market

Time is discrete and its horizon is infinite. In each period, a $[0, 1]$ continuum of risk-neutral investors are born with an arbitrarily large amount e (> 0) of cash endowment each. They live for two periods and consume cash in the last period of their lives. I call investors in their first period "young" and those in their last period "old." Each investor is indexed by $i \in \mathbb{Z} \times [0, 1]$, which is a pair of the integer denoting the period when the investor is born and the real number assigned to the investor on the unit continuum of the cohort.

A young investor can invest in two instruments to maximize the expected consumption of cash when old. One of the instruments is safe short-term bills that return a fixed amount $1 + r$ $(r \geq 0)$ of cash in the next period for each unit of cash invested. I call this instrument "T-bills." The other instrument is safe long-term bonds that generate a fixed amount d (> 0) of cash dividends for the holders of bonds at the beginning of every period. I call this

[3] MMFs can hold only short-term assets by regulation. Note, however, that MMFs are intermediaries for ultimate investors storing cash. While these investors hold repos through MMFs, they can put their cash on long-term bonds if they want.

instrument “bonds.” Bonds are divisible and their supply is fixed to unity. Thus, bonds are Lucas trees.

3.1 Bond market for investors

Investors can trade bonds through a $[0,1]$ continuum of dealers. Dealers are infinite-lived and risk-neutral, and maximize the expected discounted consumption of cash:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} c_{j,s}, \tag{1}$$

where β ($\in (0,1)$) is the time discount factor for a dealer, j is the index of a dealer, and $c_{j,t}$ is the consumption of cash. I assume that

$$\beta(1+r) = 1. \tag{2}$$

In each period, an investor is matched with two dealers through two types of pairwise matching. The first type of matching is for a repo transaction. In this type of matching, every investor is matched with a randomly chosen dealer when young, and meets with the same dealer again when old. Thus, a dealer can sell bonds to a young investor with a commitment that the dealer will repurchase the bonds in the next period. Nash bargaining between a young investor and a dealer determines the initial ask price and the repurchase price simultaneously as well as the quantity of bonds to trade. The bargaining powers are equal between the two parties.

The second type of matching is for a spot transaction. In this type of matching, every investor is matched with a randomly chosen dealer when young, and also with a randomly chosen dealer when old. Because there is a continuum of dealers, the probability that an investor meets with the same dealer twice through this type of matching is zero. An investor and a dealer determine the price and the quantity of bonds to trade in each match separately through Nash bargaining. The bargaining powers are equal between the two parties.

From a dealer's point of view, every dealer has four matches with investors in each period: a young and an old investor through each of the first and second types of matching. For simplicity, I assume that if a young investor buys bonds in a spot transaction, then the investor can sell the bonds only in a spot transaction when old.

3.2 Interdealer markets, settlements, and equilibrium

After meeting with investors, dealers can trade bonds in a competitive interdealer bond market.[4] Also, there exists a competitive interdealer loan market in which dealers can borrow and lend cash overnight among themselves.[5] The settlements of bond transactions take place only at the end of each period.[6] Thus, dealers can settle transactions with investors after trading bonds and cash in the interdealer markets in the same period. After the settlements, young investors can invest the residual of their cash in T-bills, T-bills return cash to old investors, and dealers and old investors can consume cash. See Figure 2 for a summary of the bond market structure.

Dealers and investors take as given the competitive interdealer bond price and interest rate. An equilibrium is such that the two prices clear the interdealer markets in each period:

$$\int a_{j,t}\, dj = 1 - \int_{i \in I_t} a_{i,t} + a'_{i,t}\, di, \tag{3}$$

$$\int b_{j,t}\, dj = 0, \tag{4}$$

[4]This assumption is based on the feature of the interdealer market for U.S. Treasury securities, in which interdealer brokers allow dealers to trade in size anonymously and distribute the best bid and ask price to dealers. See Huang, Cai and Wang (2002) and Fleming and Mizrach (2009) for more details.

[5]With this assumption, the shadow values of bonds and cash become exogenous for each dealer, given competitive interdealer prices. This feature of the model makes it tractable to solve the Nash-bargaining problem for each match between an investor and a dealer. See the next section for more details.

[6]This assumption reflects the fact that in practice, the settlement date of an asset transaction is typically set to a few days after the transaction date.

for all t, where $a_{j,t}$ denotes the bond holdings of a dealer at the end of the period; $a_{i,t}$ and $a'_{i,t}$ are the amounts of bonds purchased by a young investor in a spot and a repo transaction, respectively; I_t is the set of young investors in the period; and $b_{j,t}$ is the balance of interdealer loans for a dealer at the end of the period. There is no aggregate shock in the model. Dealers and investors have perfect foresight on aggregate variables, including competitive interdealer prices.

4 Need for repos in an OTC bond market

The model has two key features. One is bilateral bond transactions between dealers and investors. This assumption reflects the fact that most bond markets are OTC markets, as described by Harris (2003). The other is the finite time horizon for each investor's cash consumption. This assumption represents a short investment horizon of short-term investors in practice due to some needs for cash payments in the near future.

In this section, I calculate a symmetric steady-state equilibrium to show that the short investment horizon of investors makes them use repos if the bond market is an OTC market. In the calculation, I derive each dealer's value function and the competitive interdealer prices first, and then solve the Nash-bargaining problem between dealers and investors. Throughout the paper, the dealers and each cohort of investors are homogeneous in a symmetric equilibrium.

4.1 Dealer's value function

The flow of funds constraint for each dealer j in each period t is

$$c_{j,t} + v_t a_{j,t} + \frac{b_{j,t}}{1 + rr_t} = (d + v_t)a_{j,t-1} + b_{j,t-1} + (p_{j,t} - v_t)x_{Y,j,t} + (v_t - q_{j,t})x_{O,j,t} + (p'_{j,t} - v_t)z_{j,t} + (v_t - q'_{j,t})z_{j,t-1}, \quad (5)$$

where v_t and rr_t denote the interdealer bond price and interest rate, respectively; $p_{j,t}$ and $q_{j,t}$ are the bond prices for a young and an old investor, respectively, in spot transactions; $x_{Y,j,t}$ and $x_{O,j,t}$ are the quantities of bonds sold to a young investor and bought from an old investor, respectively, in spot transactions; $p'_{j,t}$ and $q'_{j,t}$ are the bond prices for a young and an old investor, respectively, in repo transactions; and $z_{j,t}$ is the quantity of bonds sold to a young investor in a repo transaction.

On the right-hand side of Eq. (5), the sum of $(d+v_t)a_{j,t-1}$ and $b_{j,t-1}$ is the dealer's cash holdings at the beginning of the period. The remaining terms on the right-hand side are cash flows due to bond transactions with investors. There are four terms because a dealer meets with a young and an old investor in a spot transaction in each period and the same is true in a repo transaction, as described above. These terms imply that without loss of generality, a dealer resells all of the bonds purchased from old investors in the interdealer bond market once. Then, a dealer buys back the bonds to deliver to young investors in the same market. A dealer incurs no loss from this behavior because a dealer can buy and sell bonds at an identical competitive interdealer bond price, v_t. In the last term of the right-hand side, $z_{j,t-1}$ is the amount of bonds that the dealer repurchases in period t because of a repo arranged in the previous period.

A dealer evaluates cash flows in each period by the shadow value of cash, which equals the highest marginal return from spending cash at the end of the period. A dealer has three options, as shown on the left-hand side of Eq. (5): consume cash, lend cash in the interdealer loan market, and buy bonds in the interdealer bond market. Thus, the following equation determines the shadow value of cash:

$$1+\eta_t = \max\left\{1,\ \beta(1+rr_t)(1+\eta_{t+1}),\ \frac{\beta(d+v_{t+1})(1+\eta_{t+1})}{v_t}\right\}, \tag{6}$$

where $1+\eta_t$ denotes the shadow value of a unit of cash at the end of period t for each dealer.[7] Dealers and investors take as given all of the variables on the right-hand side of Eq. (6), given the competitive interdealer bond price, v_t, and interest rate, rr_t. Hence, they regard the value of $1+\eta_t$ as exogenous.

With Eq. (6), the following value function, $V_{j,t}$, returns each dealer j's expected discounted utility at the end of each period t:

$$
\begin{aligned}
V_{j,t} = (1+\eta_t)\left[(d+v_t)a_{j,t-1} + b_{j,t-1}\right] \\
+ E_t \sum_{s=0}^{\infty} \beta^s (1+\eta_{t+s}) \left[(p_{j,t+s} - v_{t+s}) x_{Y,j,t+s} + (v_{t+s} - q_{j,t+s}) x_{O,j,t+s}\right. \\
\left. + (p'_{j,t+s} - v_{t+s}) z_{j,t+s} + (v_{t+s} - q'_{j,t+s}) z_{j,t+s-1}\right]. \quad (7)
\end{aligned}
$$

On the right-hand side, cash inflows are evaluated by the shadow value of cash, $1+\eta_{t+s}$, in each period. See Appendix A for a formal proof that Eq. (7) is the value function for each dealer's expected discounted utility in each period.

4.2 Competitive interdealer bond price and interest rate

In a steady-state equilibrium, the shadow value of cash, $1+\eta_t$, equals the marginal utility of consumption in each period:

$$1+\eta_t = 1. \quad (8)$$

Otherwise, a dealer would postpone consumption forever by investing into bonds or interdealer loans, given $1+\eta_t > 1$ for all t. In such a case, the utility-maximization problem for a dealer cannot be well-defined. I exclude such a value of η_t from a steady-state equilibrium.

[7] The variable η_t is the Lagrange multiplier for a non-negativity constraint on a dealer's consumption. See Appendix A for more details.

Given Eq. (8), the competitive interdealer bond price, v_t, satisfies

$$v_t = \beta(d + v_{t+1}), \tag{9}$$

in a steady-state equilibrium. At this price, the marginal costs of buying and selling bonds for a dealer become equal to the marginal benefit of each transaction.[8] Thus, the interdealer bond market clears without any excess demand or supply.

For a similar reason, the competitive interdealer interest rate, rr_t, satisfies

$$1 = \beta(1 + rr_{t+1}), \tag{10}$$

in a steady-state equilibrium. Given Eq. (2), Eq. (10) implies that

$$rr_t = r, \tag{11}$$

in each period.

4.3 Bond-liquidation cost in a spot transaction between an old investor and a dealer

The cash consumption of an investor equals the sum of gross returns on bonds and T-bills that the investor earns when old:

$$c_{i,t} = (d + q_{j,t})a_{i,t-1} + (d + q'_{j',t})a'_{i,t-1} + (1 + r)(e - p_{j'',t-1}a_{i,t-1} - p'_{j',t-1}a'_{i,t-1}), \tag{12}$$

where i is the index of an old investor in period t; $c_{i,t}$ is the consumption of cash; $a_{i,t-1}$ and $a'_{i,t-1}$ are the amounts of bonds that the investor buys in a spot and a repo transaction, respectively, when young; and j, j', and j'' are the indices of the dealers matched with the

[8] In Eq. (9), the left-hand side is the cost of funding to buy a bond in the interdealer bond market. The right-hand side is the present discounted value of the gross return on a bond.

investor.

If $a_{i,t-1} > 0$, then the investor must sell the bonds to a randomly chosen dealer in a spot transaction when old. The Nash-bargaining problem in this case is

$$\max_{q_{j,t}} \, (q_{j,t}a_{i,t-1} - 0)^{0.5}[(v_t - q_{j,t})a_{i,t-1}]^{0.5}, \tag{13}$$

where j is the index of a dealer matched with the old investor in a spot transaction. The left parentheses and the right square brackets contain the trade surpluses for the old investor and the dealer, respectively. In the left parentheses, zero appears as the value of the outside option for the old investor, that is, holding bonds. This value is zero because the old investor cannot postpone the timing of consumption to later periods. The right square brackets contain the difference between the values of the dealer's value function in Eq. (7) when the dealer buys the old investor's bonds (i.e., $x_{O,j,t} = a_{i,t-1}$) and when the dealer does not (i.e., $x_{O,j,t} = 0$).[9] Note that the old investor is willing to sell all of $a_{i,t-1}$, because the investor can consume cash only in the current period.

The solution for the Nash-bargaining problem is

$$q_{j,t} = 0.5v_t. \tag{14}$$

Thus, a dealer can make a profit from a spot transaction with an old investor. Intuitively speaking, a dealer can negotiate down the price of an old investor's bonds because an old investor must obtain cash to consume by the end of the current period. This price discount is a bond-liquidation cost from an old investor's perspective.

[9]The difference is the net cash flow for the dealer from buying bonds from the old investor and then reselling the bonds in the interdealer bond market. The dealer can realize this cash flow regardless of the results of the other transactions within the period, given the competitive interdealer bond price, v_t.

4.4 *No spot transaction between a young investor and a dealer due to an ex-post bond-liquidation cost*

How much bonds does a young investor buy in a spot transaction, given an ex-post bond-liquidation cost described above? The Nash-bargaining problem for a bond purchase by a young investor in a spot transaction is

$$\max_{p_{j,t},\, a_{i,t}\geq 0} \{[d + E_t q_{j',t+1} - (1+r)p_{j,t}]a_{i,t}\}^{0.5}[(p_{j,t} - v_t)a_{i,t}]^{0.5}, \tag{15}$$

where i is the index of a young investor in period t, j is the index of a dealer matched with the young investor in the current transaction, and j' is the index of a dealer matched with the investor in a spot transaction when the investor becomes old in the next period. The non-negativity constraint on $a_{i,t}$ implies that an investor cannot short-sell bonds to a dealer. I omit the budget constraint for the young investor here, given the assumption of an arbitrarily large cash endowment for each young investor, e.[10]

The left curly brackets and the right parentheses in Eq. (15) contain the trade surpluses for the young investor and the dealer, respectively, in the current transaction. In the left curly brackets, $d + E_t q_{j',t+1}$ is the gross return on a bond for the young investor. The opportunity cost of buying a bond, $(1+r)p_{j,t}$, is subtracted from this return because the young investor can invest into T-bills alternatively. In the right square brackets is the difference between the values of the dealer's value function in Eq. (7) when the dealer sells an amount $a_{i,t}$ of bonds in a spot transaction (i.e., $x_{Y,j,t} = a_{i,t}$) and when the dealer does not (i.e., $x_{Y,j,t} = 0$).[11]

The Nash-bargaining problem implies that a young investor buys a positive amount of bonds from a dealer (i.e., $a_{i,t} > 0$) only if $(d + E_t q_{j',t+1})/(1+r) \geq v_t$. Eq. (9), however,

[10] The budget constraint is $p_{j,t}a_{i,t} + p'_{j',t}a'_{i,t} \leq e$, where j' denotes the index of a dealer matched with the investor in a repo transaction in the period. The solution for Eq. (15) satisfies this constraint.

[11] The difference equals the net cash flow for the dealer from buying bonds in the interdealer market and reselling them to the young investor. The dealer can realize this cash flow regardless of the results of the other transactions within the period, given the competitive interdealer bond price, v_t.

implies that

$$v_t > \frac{d + E_t q_{j',t+1}}{1+r}, \tag{16}$$

in a steady-state equilibrium, given Eqs. (2) and (14). Thus, a young investor buys no bonds in a spot transaction, that is, $a_{i,t} = 0$. This result indicates that an ex-post bond-liquidation cost implied by Eq. (14) discourages a young investor from buying bonds in a spot transaction.

4.5 Use of a repo by a young investor

In contrast, a young investor can buy bonds in a repo transaction. This result is due to the feature of a repo transaction that an investor is matched with a dealer when young, and meets with the same dealer again when old. Thus, a dealer can sell bonds to a young investor with an offer of a repurchase price at which the dealer will buy back the bonds in the next period.

The Nash-bargaining problem for this transaction is

$$\max_{p'_{j,t},\, q'_{j,t+1},\, a'_{i,t} \geq 0} \{[d + q'_{j,t+1} - (1+r)p'_{j,t}]a'_{i,t}\}^{0.5}\{[p'_{j,t} - v_t + \beta(v_{t+1} - q'_{j,t+1})]a'_{i,t}\}^{0.5}, \tag{17}$$

where i and j are the indices of a young investor and a dealer, respectively, matched in a repo transaction. As in Eq. (15), I omit the budget constraint for a young investor here given an arbitrarily large cash endowment for each young investor. In the left curly brackets is the net return on bonds for the young investor, as in a spot transaction specified by Eq. (15). The right curly brackets contain the difference between the values of the dealer's value function in Eq. (7) when the dealer sells an amount $a'_{i,t}$ of bonds in a repo transaction (i.e., $z_{j,t} = a'_{i,t}$) and when the dealer does not (i.e., $z_{j,t} = 0$), given Eq. (8). The difference consists of the cash flows for the dealer from a bond sale in the current period and from a bond repurchase in the next period.[12]

Given Eq. (9) in a steady-state equilibrium, the Nash-bargaining problem yields

$$(1+r)p'_{j,t} - q'_{j,t+1} = d. \tag{18}$$

Thus, the dealer adjusts both the initial ask price for the young investor's bond purchase, $p'_{j,t}$, and the repurchase price, $q'_{j,t+1}$, so that the young investor can earn the T-bill rate, r, on a repo. In this case, the young investor can buy bonds because the investor will not suffer a low bond return due to a bond-liquidation cost in a spot transaction.

4.6 Terms of a repo in a symmetric steady-state equilibrium

Eqs. (17) and (18) imply that any value of $a'_{i,t}$ can be a solution for the Nash-bargaining problem. This result holds because the rates of return on bonds and interdealer loans for a dealer equal the rate of return on a repo for an investor, as implied by Eqs. (2), (9), (11) and (18). Thus, there is no arbitrage opportunity for a dealer in a repo transaction. Further, given that the rate of return on a repo equals that on T-bills, $1+r$, there is no trade surplus for a young investor either. Hence, a dealer and a young investor are indifferent to the amount of bonds to trade, $a'_{i,t}$, when trading in a repo transaction.

In a symmetric equilibrium, $a'_{i,t}$ for each i cannot be more than one, because otherwise the total amount of bonds held by a cohort of young investors, which is a unit continuum, would exceed the total supply of bonds, which equals one. Thus,

$$a'_I \in [0,1], \tag{19}$$

[12] In each transaction, the dealer uses the interdealer bond market to hold no bonds after the transaction. The dealer incurs no loss from this action because the dealer can buy and sell bonds at an identical competitive price in the market when necessary.

where a'_I denotes the value of $a'_{i,t}$ for all i and t in a symmetric steady-state equilibrium.[13] If $a'_I < 1$, then dealers hold the remaining bonds (i.e., $a_{j,t} = 1 - a'_I$ for all j and t), so that the interdealer bond market clears.

Given Eq. (5), the flow of funds for a dealer in a symmetric steady-state equilibrium is:

$$c_D = rr \cdot b + d(1 - z) + (p' - q')z, \tag{20}$$

where rr, c_D, b, z, p', and q' denote the steady-state values of rr_t, $c_{j,t}$, $b_{j,t}$, $z_{j,t}$, $p'_{j,t}$, and $q'_{j,t}$ for all j, respectively. In Eq. (20), the market clearing condition for bonds, $a_{j,t} = 1 - a'_I$, and the equality between the amounts of bonds sold by a dealer and bought by a young investor, $z = a'_I$, are substituted into $a_{j,t}$.

In a symmetric steady-state equilibrium, the balance of interdealer loans for each dealer, b, must be zero to clear the interdealer loan market. Further, Eq. (19) implies $z \in [0, 1]$ given $z = a'_I$. Thus, the values of p', q', and z must be non-negative values satisfying Eq. (18) and $c_D \geq 0$, given Eq. (20), $b = 0$, and $z \in [0, 1]$. For all $z \in [0, 1]$, these conditions are satisfied if

$$p' \in \left[\frac{d}{1 + r}, \frac{d}{r}\right], \tag{21}$$

$$q' = (1 + r)p' - d. \tag{22}$$

5 Endogenous repo margin in a renegotiation-proof repo

The values of p' and q' become unique if I add one more condition to the equilibrium: a repo must be renegotiation-proof. In this case, an old investor and a dealer can renegotiate the repurchase price, q', when the dealer repurchases the investor's bonds in a repo transac-

[13]The value of a'_I must be non-negative, as implied by the non-negativity constraint on $a'_{i,t}$ in Eq. (17). This constraint implies that an investor cannot short-sell bonds to a dealer, as described above.

tion. Expecting the possibility of a renegotiation, a young investor and a dealer in a repo transaction choose such a value of q' in advance that a renegotiation will be unnecessary.

The Nash-bargaining problem for a renegotiation of a repo is the same as that for a spot transaction between an old investor and a dealer, because subsequent events after the two transactions, and hence the outside options of the trading parties, are all the same. Thus, the renegotiation-proof value of q' is identical to the one in Eq. (14):

$$q' = 0.5v, \tag{23}$$

where v denotes the steady-state value of the interdealer bond price, v_t.

Given Eqs. (2) and (23), Eqs. (9) and (18) imply that

$$p' = \frac{d + 0.5v}{1 + r} < v = \frac{d + v}{1 + r}. \tag{24}$$

In this case, there appears a margin in a repo transaction, that is, a difference between the market value of a bond, v, and the initial ask price paid by a young investor, p'. The size of the margin is such that a dealer's loss from a negative cash flow due to the margin is compensated by an ex-post profit from a bond repurchase in the next period:

$$v - p' = \beta(v - q'), \tag{25}$$

which can be derived from Eqs. (2), (23), and (24). Because $v - p' < v - q'$, each dealer can finance a repo margin in each period by a profit from a bond repurchase in the same period, as implied by Eq. (20).[14]

This result illustrates that a bond-liquidation cost due to bilateral bargaining in an OTC market causes a repo margin even without the bond price risk. This result is consistent with

[14]Eq. (20) implies that $c_D \geq 0$ if $p' > q'$, given $b = 0$ in a symmetric steady-state equilibrium.

the fact that the size of a repo margin depends on the type of trading party. For example, Gorton and Metrick (2012) report that the margins for interdealer repos on various high-rated securities had been zero before the subprime mortgage crisis. In contrast, Krishnamurthy, Nagel and Orlov (2011) find that regardless of the type of security, the margins for repos held by MMFs have been positive before and after the crisis. Given that MMFs are short-term investors, the model can explain the observed difference in repo margins as a consequence of a bond-liquidation cost for short-term investors in an OTC market.

6 What happens if repos are unavailable?

In the model presented above, a young investor is indifferent between a repo and T-bills because the two instruments have the same rate of return. This feature of the model, however, does not mean that a repo is redundant in the presence of T-bills. In this section, I eliminate the matching for repo transactions from the model, so that investors and dealers are only randomly matched for spot transactions in each period. In this case, a lack of repos raises the bond yield unless dealers have enough capital to buy and hold bonds.

Suppose that the values of the competitive interdealer bond price, v_t, and interest rate, r_t, remain the same as in Eqs. (9) and (11). Given no repo transactions, young investors do not buy bonds in this case, as shown above. Thus, dealers must have enough cash to buy and hold the bonds sold by old investors. The aggregate amount of cash held by dealers in each period t equals the sum of dividends on their bonds, $\int da_{j,t-1}dj$, given no bond transactions with young investors under Eqs. (9) and (11).[15] In this section, I assume that

$$a_{j,-1} = 0, \tag{26}$$

[15]Note that cash inflows for each dealer through selling bonds and taking loans in the interdealer markets are cash transfers among dealers. To clear the interdealer markets, the aggregate amount of cash held by dealers must be enough to buy old investors' bonds.

for all j in period -1, so that Eqs. (9) and (11) do not hold in period 0. Given Eq. (26), I calculate the perfect foresight dynamics from period 0 onward in a symmetric equilibrium.[16]

I denote by T the first period in which dealers hold all of the bonds in the economy after period 0. Thus, the economy converges to the steady state with Eqs. (9) and (11). For $t = 0, 1, ..., T-1$, dealers do not have enough cash to hold all of the bonds. For these periods, the competitive interdealer bond price, v_t, must take such a value that a young investor is willing to buy bonds in a spot transaction. The Nash-bargaining problem for a spot transaction between a young investor and a dealer, Eq. (15), implies the following condition for v_t:

$$v_t = \frac{d + E_t q_{j',t+1}}{1+r} = \frac{d + 0.5 v_{t+1}}{1+r}, \tag{27}$$

for $t = 0, 1, ..., T-1$, where the second equality is due to Eq. (14). In this case, there are no trade surpluses for a young investor and a dealer. Hence, a young investor is indifferent to the amount of bonds to buy from a dealer.[17]

Given Eq. (2), Eq. (27) implies that the present discounted value of the rate of return on bonds for a dealer, $\beta(d + v_{t+1})/v_t$, exceeds the marginal utility from consumption, that is, 1. Thus, dealers consume no cash:

$$c_{j,t} = 0, \tag{28}$$

[16] If repo transactions are available, the symmetric steady-state equilibrium shown above sustains with Eq. (26). To see this result, note that the value of $a_{i,t}$ can be any non-negative value in this equilibrium.

[17] On one hand, if the left-hand side of Eq. (27) is larger than the right-hand side, then a young investor buys no bonds in a spot transaction (i.e., $a_{i,t} = 0$). Thus, dealers must have enough cash to hold all of the bonds in the economy. If this is possible, however, $t \geq T$ must hold, which contradicts $t < T$. On the other hand, if the left-hand side is smaller than the right-hand side, then the trade surpluses for a young investor and a dealer become positive. In this case, $a_{i,t}$ becomes arbitrarily large as each young investor spends all of its cash endowment, e, on bonds. As a result, there is excess demand in the interdealer bond market as dealers try to buy bonds to deliver to young investors in the market.

for all j and $t = 0, 1, ..., T-1$. Also, for all t, the gross interdealer interest rate, $1 + rr_t$, equals the rate of return on bonds for a dealer:

$$1 + rr_t = \frac{d + v_{t+1}}{v_t}, \tag{29}$$

so that the interdealer loan market clears with a zero balance of interdealer loans for each dealer in a symmetric equilibrium:

$$b_{j,t} = 0, \tag{30}$$

for all j and t.[18]

Hence, dealers spend all of their cash on bonds until period T. Substituting Eqs. (28) and (30) and no repo transactions ($z_{j,t} = 0$ for all j and t) into Eq. (5) yields the following law of motion for a dealer's bond holdings at the end of the period, $a_{j,t}$:

$$v_t a_{j,t} = (d + v_t) a_{j,t-1} + (p_{j,t} - v_t) x_{Y,j,t} + (v_t - q_{j,t}) x_{O,j,t}, \tag{31}$$

for $t = 0, 1, ..., T-1$, where $p_{j,t}$ and $q_{j,t}$ are the spot prices of bonds that the dealer offers to a young and an old investor, respectively, and $x_{Y,j,t}$ and $x_{O,j,t}$ are the amounts of bonds that a dealer sells to a young investor and purchases from an old investor, respectively, in spot transactions.

In a symmetric equilibrium, $x_{O,j,t}$ equals the amount of bonds held by each young investor at the end of the previous period, $a_{i,t-1}$. Further, given Eq. (27), the Nash-bargaining

[18] On one hand, if the left-hand side of Eq. (29) is greater than the right-hand side, then every dealer would provide interdealer loans by shorting bonds in the interdealer bond market. On the other hand, if the left-hand side is smaller than the right-hand side, then every dealer would borrow interdealer loans to buy bonds in the interdealer bond market.

problem for a spot transaction between a young investor and a dealer, Eq. (15), yields

$$p_{j,t} = v_t, \tag{32}$$

for all j and $t = 0, 1, ..., T-1$. Substituting $x_{O,j,t} = a_{i,t-1}$ and Eqs. (3), (14), and (32) into Eq. (31) yields

$$v_t a_{j,t} = (d + v_t)a_{j,t-1} + 0.5v_t(1 - a_{j,t-1}), \tag{33}$$

for all j and $t = 0, 1, ..., T-1$ in a symmetric equilibrium.[19]

Overall, Eqs. (27) and (33) determine the dynamics of $a_{j,t}$ and v_t for $t = 0, 1, ..., T-1$, given Eq. (26). The following conditions determine the values of T and $a_{j,t}$ and v_t for $t = T, T+1, T+2, ...$: [20]

$$a_{j,t} \begin{cases} < 1 & \text{for } t = 0, 1, ..., T-1, \\ = 1 & \text{for } t = T, T+1, T+2, ..., \end{cases} \tag{34}$$

for all j; and

$$v_t = \begin{cases} \min\left\{\dfrac{2da_{j,t-1}}{1-a_{j,t-1}}, \dfrac{d}{r}\right\} \geq \dfrac{d + 0.5v_{t+1}}{1+r} & \text{for } t = T, \\ \dfrac{d}{r} & \text{for } t = T+1, T+2, T+3, ..., \end{cases} \tag{35}$$

Eq. (34) contains the definition of T such that dealers hold all of the bonds in the economy

[19] Eq. (3) implies that $a_{i,t-1} = 1 - a_{j,t-1}$ in a symmetric equilibrium, given that $a'_{i,t-1} = 0$ for all i and t without the matching for repo transactions.

[20] Thus, T is a fixed point. Suppose $v_T = d/r$. Given a conjectured value of T, Eqs. (27) and (33) yield a sequence of $a_{j,t}$ and v_t for $t = 0, 1, 2, ..., T-1$. If $a_{j,t} < 1$ is not satisfied for any $t = 0, 1, ..., T-1$, then update the conjectured value of T downward. Otherwise, check if Eq. (35) is satisfied for $t = T$ with the value of $a_{j,T-1}$. If it is satisfied, then the equilibrium value of T is identified. Otherwise, update the conjectured value of T upward. The routine for the case in which $v_T = 2da_{j,T-1}/(1 - a_{j,T-1})$ is similar, given $v_{T+1} = d/r$. Identify the equilibrium value of T by considering both cases for the value of v_T.

(i.e., $a_{j,t} = 1$ for all j) after period T.[21] Thus, the economy converges to the steady state with Eqs. (9) and (11), with which young investors do not buy bonds from dealers in spot transactions. Eq. (35) indicates that v_t and rr_t satisfy Eqs. (9) and (11) from period $T+1$ onward, given Eq. (29).

In period T, dealers buy the last bonds held by investors, $1-a_{j,T-1}$, given $a_{j,T-1} < 1$ for all j. If $v_T = d/r$, however, dealers may not have enough cash to buy all of the bonds. In such a case, v_T drops from d/r so that dealers can buy all of the bonds in the economy. Substituting $a_{j,t} = 1$ into Eq. (33) for $t = T$ yields such value of v_T, that is, $2da_{j,T-1}/(1 - a_{j,T-1})$.[22] This value must not be less than $(d+0.5v_{T+1})/(1+r)$ because otherwise young investors buy bonds in spot transactions in period T as implied by Eq. (15), which contradicts the definition of T.

Eqs. (27) and (35) indicate that for $t = 0, 1, ..., T-1$, the interdealer bond price, v_t, drops from the steady-state value with repo transactions, d/r. Thus, the bond yield, d/v_t, increases for some periods due to a decline in bond demand if repos become unavailable from period 0 onward. Even though the model does not incorporate other investors than short-term investors, the result will be robust if other investors do not have enough capital to absorb excess supply of bonds. Given the fact that the bond yield affects long-term interest rates, such as mortgage rates, in practice, the result suggests that the disappearance of repos would negatively affect aggregate economic activity.

[21] Also, Eq. (34) ensures that the aggregate amount of bonds held by dealers never exceeds the total supply of bonds in the economy. Given Eq. (26) and a positive value of v_t, Eqs. (33) and (34) imply that $a_{j,t} \geq 0$ for all j and $t = 0, 1, 2, ...$

[22] Eqs. (34) and (35) jointly ensure that a dealer's consumption is non-negative:

$$c_{j,t} = (d + v_t)a_{j,t-1} + 0.5v_t(1 - a_{j,t-1}) - v_t a_{j,t} \geq 0,$$

for $t = T, T+1, T+2, ...$ If $v_T = 2da_{j,T-1}/(1 - a_{j,T-1})$ in period T, then $c_{j,T} = 0$. In this case, the rate of return on bonds, $\beta(d + v_{T+1})/v_T$, exceeds the marginal utility of consumption, that is, 1, given $v_T < d/r = v_{T+1}$. Thus, dealers spend all of their cash on bonds. If $v_T = d/r$ in period T, then $\beta(d + v_{T+1})/v_T = 1$. Hence, dealers consume the residual of cash left after buying bonds (i.e., $c_{j,T} \geq 0$). From period $T+1$ onwards, dealers consume the dividends on their bonds (i.e., $c_{j,t} = d > 0$), given $v_t = d/r$ and $a_{j,t} = 1$ for $t = T+1, T+2, T+3, ...$

7 Behavior of a long-term investor

Thus far, I have only analyzed the behavior of short-term investors. Would a long-term investor also need a repo to buy bonds in an OTC market? The answer is no, if an investor's investment horizon equals or exceeds the maturity of bonds.

For simplicity, I add only one long-term investor to the model. A long-term investor has no mass, so that the aggregate features of the model do not change. A long-term investor's utility function is the same as Eq. (1) for a dealer, including the value of the time discount factor specified by Eq. (2). Thus, a long-term investor has the same horizon as the maturity of bonds—infinity. A long-term investor is randomly matched with a dealer for a spot transaction in each period. Hence, one of the dealers has five matches in each period. Ex-ante, the possibility of this event is zero for each dealer. A long-term investor receives an arbitrarily large cash endowment, e, in each period as each young investor does.

Suppose that the economy is in a symmetric steady-state equilibrium with repo transactions, shown above. Thus, Eqs. (9) and (11) hold in each period. The Nash-bargaining problem for a spot transaction between a dealer and a long-term investor is

$$\max_{p''_{j,t},\, a''_t \geq 0} \{[d + E_t p''_{j',t+1} - (1+r)p''_{j,t}]a''_t\}^{0.5}[(p''_{j,t} - v_t)a''_t]^{0.5}, \tag{36}$$

where j and j' denote the indices of dealers matched with the long-term investor in the current and the next period, respectively; $p''_{j,t}$ is the bond price for the long-term investor in each period; and a''_t is the amount of bonds purchased by the long-term investor in each period. The left curly brackets and the right square brackets are the trade surpluses for the long-term investor and the dealer, respectively.[23]

[23] In the left curly brackets is the expected gross return on bonds for the long-term investor minus the opportunity cost of buying bonds. As implied by Eqs. (9) and (37), $p''_{j',t+1}$ equals the present discounted value of future dividends on bonds for the long-term investor. The right square brackets contain the net cash flow for the dealer from buying bonds in the interdealer bond market and reselling the bonds to the long-term investor. Without loss of generality, the dealer obtains all of the bonds to deliver to investors from

Given Eqs. (2) and (9), this Nash-bargaining problem implies that

$$p''_{j,t} = v_t, \tag{37}$$

for all j and t. Because there is no trade surplus for a long-term investor or a dealer, a''_t can take any value.[24] Thus, a long-term investor can buy bonds in a spot transaction.

The intuition for this result is simple. If an investor's investment horizon equals or exceeds the maturity of bonds, then the investor does not have to resell bonds before maturity. Thus, the investor can buy bonds in a spot transaction without worrying about reselling the bonds with an ex-post price discount in an OTC market. This result implies that short-term investors need repos because their investment horizon is shorter than the maturity of bonds.

8 Conclusions

I have presented a model featuring dealers and short-term investors in an OTC bond market. The model illustrates that bilateral bargaining in an OTC market results in an endogenous bond-liquidation cost for short-term investors. This cost explains their need for repos. Using the model, I have discussed the cause of a repo margin specific to short-term investors, the role of repos in curbing a rise in the bond yield, and the relationship between the investment horizon of an investor and an investor's need for a repo.

In this paper, I take as given the fact that most bond markets are OTC markets. A question remains regarding the optimal market design, such as whether to introduce a centralized bond market. Also, I do not formally model the relationship between bond financing through a repo market and real economic activity, such as housing investments. These issues are left for future research.

the interdealer bond market, as described above.

[24] Given that a long-term investor has no mass, a''_t does not affect the amount of bonds to be held by short-term investors and dealers in equilibrium.

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Appendix

A Proof for Eq. (7) as a dealer's value function

In each period, the maximization problem for each dealer j in the competitive interdealer markets can be written as the following recursive form:

$$V_t(a_{j,t-1},\ b_{j,t-1},\ q'_{j,t},\ z_{j,t-1}) = \tag{A.1}$$

$$\max_{\{c_{j,t},\ x_{ID,j,t},\ b_{j,t}\}} c_{j,t} + \beta E_t V_{t+1}(a_{j,t},\ b_{j,t},\ q'_{j,t+1},\ z_{j,t}), \tag{A.2}$$

$$\text{s.t. } c_{j,t} + v_t x_{ID,j,t} + \frac{b_{j,t}}{1+rr_t}$$

$$= da_{j,t-1} + b_{j,t-1} + p_{j,t}x_{Y,j,t} + p'_{j,t}z_{j,t} - q_{j,t}x_{O,j,t} - q'_{j,t}z_{j,t-1}, \tag{A.3}$$

$$a_{j,t} = x_{ID,j,t} + x_{O,j,t} + z_{j,t-1} - x_{Y,j,t} - z_{j,t} + a_{j,t-1}, \tag{A.4}$$

$$c_{j,t} \geq 0, \tag{A.5}$$

where V_t denotes the value function for a dealer's expected discounted utility in period t; and $x_{ID,j,t}$ is the net balance of bonds that the dealer trades in the interdealer bond market in the period. The state variables for dealer j at the beginning of period t are: the amount of bonds owned by the dealer at the end of the previous period, $a_{j,t-1}$; the balance of interdealer loans at the end of the previous period, $b_{j,t-1}$; the repurchase price of bonds in a repo transaction in the current period, $q'_{j,t}$; and the amount of bonds sold to a young investor in a repo transaction in the previous period, $z_{j,t-1}$. Given the values of these state variables, the results of bond transactions with investors in the same period, i.e., $p_{j,t}$, $x_{Y,j,t}$, $p'_{j,t}$, $z_{j,t}$, $q_{j,t}$, and $x_{O,j,t}$, are pre-determined for dealers in the competitive interdealer markets. Eq. (A.3) is the flow of funds constraint for the dealer. Eq. (A.4) is the law of motion for $a_{j,t}$. Eq. (A.5) is a non-negativity constraint on the dealer's consumption, $c_{j,t}$.

Hereafter, suppose that V_{t+1} equals $V_{j,t+1}$ implied by Eq. (7) as a function. Given this conjecture, I will show that $V_t = V_{j,t}$. Thus, I will prove that $V_t = V_{j,t}$ for all t by mathematical induction.

Given that dealers and investors take the value of η_{t+1} as exogenous, the solution for the maximization problem yields

$$1 + \eta_t = \beta(1 + rr_t)(1 + \eta_{t+1}) \geq 1, \tag{A.6}$$

$$\eta_t c_{j,t} = 0, \tag{A.7}$$

$$(1 + \eta_t)v_t - \beta(d + v_{t+1})(1 + \eta_{t+1}) = 0, \tag{A.8}$$

where η_t is the Lagrange multiplier for $c_{j,t} \geq 0$ in Eq. (A.5). Eq. (A.6) implies that the shadow value of a unit of cash for a dealer at the end of the period, $1+\eta_t$, is pinned down by the present discounted value of the rate of return on interdealer loans, $\beta(1 + rr_t)(1 + \eta_{t+1})$. Thus, dealers and investors take the value of η_t as given. This result holds because a dealer can use the interdealer loan market as a buffer for the dealer's excess cash or cash shortage at a competitive interest rate, rr_t. In equilibrium, the cost of interdealer loans (i.e., $\beta(1+rr_t)(1+\eta_{t+1})$) must be equal to, or greater than, the marginal utility from consumption (i.e., 1). Otherwise every dealer would take interdealer loans, which would violate the market clearing condition for the interdealer loan market, Eq. (4).

To confirm Eq. (A.7), note that $c_{j,t} = 0$ if $\beta(1 + rr_t)(1 + \eta_{t+1}) > 1$, because dealers are better off by postponing consumption in this case. Eq. (A.8) follows from the first-order condition with respect to $x_{ID,j,t}$, which implies that a dealer is indifferent to the value of $a_{j,t}$ if and only if Eq. (A.8) holds. Otherwise, $a_{j,t} = \infty$ or $a_{j,t} = -\infty$ for all j, with which the market clearing condition for the interdealer bond market, Eq. (3), would be violated.

Substituting Eqs. (A.3), (A.4) and (A.6)-(A.8) into Eq. (A.2) yields

$$
\begin{aligned}
V_t(a_{j,t-1},\ b_{j,t-1},\ q'_{j,t},\ z_{j,t-1}) \\
= (1+\eta_t)\big[(d+v_t)a_{j,t-1} + b_{j,t-1} + (p_{j,t}-v_t)x_{Y,j,t} + \left(p'_{j,t}-v_t\right)z_{j,t} \\
+ (v_t - q_{j,t})x_{O,j,t} + (v_t - q'_{j,t})z_{j,t-1}\big] \\
+ \beta E_t\left\{V_{t+1}(a_{j,t},\ b_{j,t},\ q'_{j,t+1},\ z_{j,t}) - (1+\eta_{t+1})[(d+v_{t+1})a_{j,t} + b_{j,t}]\right\}. \quad \text{(A.9)}
\end{aligned}
$$

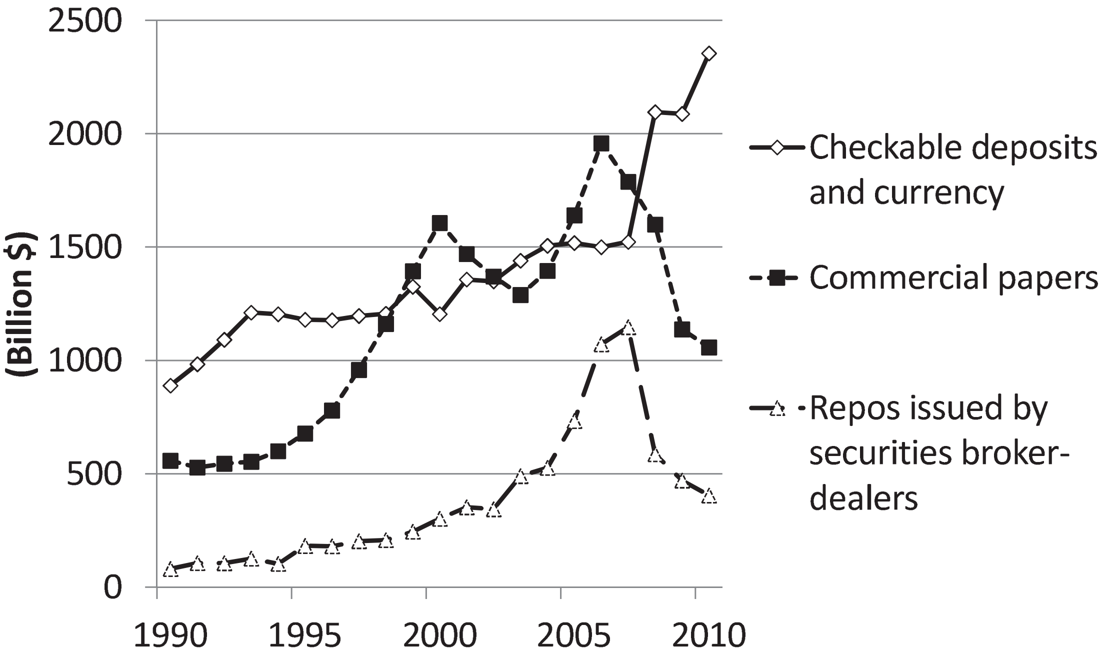
Substituting $V_{j,t+1}$ implied by Eq. (7) into V_{t+1} yields $V_t = V_{j,t}$. Eqs. (A.6)-(A.8) imply that η_t satisfies Eq. (6).

Table 1: Composition of securities traded in the U.S. tri-party repo market

	Share (%)
Fed-eligible securities	82.7
US Treasury securities	28.9
Agency MBS	36.3
Agency debentures	11.4
Agency REMICs	4.6
Ginnie Mae MBS Pools	1.2
Ginnie Mae REMICs	0.3
Non-Fed eligible securities	17.5

Source: Copeland, Martin and Walker (2010).
Notes: The sample period is from July 2009 to January 2010. Fed-eligible securities are the securities acceptable for the Federal Reserve in its market operations.

Figure 1: Outstanding values of money market instruments in the U.S.



Source: Flow of Funds Accounts of the U.S.

Figure 2: Bond market structure in the model

	Period t	$t+1$ ...
Cohort-$(t-1)$ investors:	sell bonds and consume cash ("old")	
Bilateral $\cdots$	Bonds $\downarrow$ <Repo> $\uparrow$ Cash Bonds $\downarrow$ <Spot> $\uparrow$ Cash	
	$\boxed{\text{Dealers}}$	
	Bonds $\downarrow\uparrow$ Cash	
Competitive $\cdots$	$\boxed{\text{Interdealer market}}$	
	Bonds $\downarrow\uparrow$ Cash	
	$\boxed{\text{Dealers}}$	
Bilateral $\cdots$	Bonds $\downarrow$ <Repo> $\uparrow$ Cash Bonds $\downarrow$ <Spot> $\uparrow$ Cash	
Cohort-t investors:	born with cash and buy bonds ("young")	sell bonds and consume cash ("old")