JSPS Grants-in-Aid for Creative Scientific Research
Understanding Inflation Dynamics of the Japanese Economy
Working Paper Series No.65

On the Nonstationarity of the Exchange Rate Process

Takaaki Ohnishi
Hideki Takayasu
Takatoshi Ito
Yuko Hashimoto
Tsutomu Watanabe
and
Misako Takayasu

November 16, 2010

Research Center for Price Dynamics
Institute of Economic Research, Hitotsubashi University
Naka 2-1, Kunitachi-city, Tokyo 186-8603, JAPAN
Tel/Fax: +81-42-580-9138
E-mail: sousei-sec@ier.hit-u.ac.jp
http://www.ier.hit-u.ac.jp/~ifd/

On the Nonstationarity of the Exchange Rate Process

Takaaki Ohnishi[a,b,*], Hideki Takayasu[c], Takatoshi Ito[b], Yuko Hashimoto[d], Tsutomu Watanabe[e,a], Misako Takayasu[f]

[a] *The Canon Institute for Global Studies, 11F, Shin-Marunouchi Bldg., 1-5-1 Marunouchi, Chiyoda-Ku, Tokyo 100-6511, Japan.*
[b] *Graduate School of Economics, University of Tokyo, 7-3-1 Hongo, Bunkyo-Ku, Tokyo 113-0033, Japan.*
[c] *Sony Computer Science Laboratories, 3-14-13 Higashigotanda, Shinagawa-ku, Tokyo 141-0022, Japan.*
[d] *Statistics Department, International Monetary Fund, 700 19th Street, N.W. Washington, D.C. 20431.*
[e] *Institute of Economic Research, Hitotsubashi University, 2-1 Naka, Kunitachi-city, Tokyo 186-8603, Japan*
[f] *Department of Computational Intelligence and Systems Science, Interdisciplinary Graduate School of Science and Engineering, Tokyo Institute of Technology, 4259-G3-52 Nagatsuta-cho, Midori-ku, Yokohama 226-8502, Japan.*

Abstract

We empirically investigate the nonstationarity property of the dollar-yen exchange rate by using an eight year span of high frequency data set. We perform a statistical test of strict stationarity based on the two-sample Kolmogorov-Smirnov test for the absolute price changes, and the Pearson's chi-square test for the number of successive price changes in the same direction, and find statistically significant evidence of nonstationarity. We further study the recurrence intervals between the days in which nonstationarity occurs, and find that the distribution of recurrence intervals is well-approximated by an exponential distribution. Also, we find that the mean conditional recurrence interval $\langle T|T_0\rangle$ is independent of the previous recurrence interval T_0. These findings indicate that the recurrence intervals is characterized by a Poisson process. We interpret this as reflecting the Poisson property regarding the arrival of news.

Keywords: Econophysics, Foreign exchange market, Strict stationarity, nonstationarity, two-sample Kolmogorov–Smirnov test, Pearson's

*Corresponding author. E-mail address: ohnishi.takaaki@canon-igs.org

chi–square test, Poisson process

1. Introduction

Financial time series data have been extensively investigated using a wide variety of methods in econophysics. These studies tend to assume, explicitly or implicitly, that a time series is stationary, since stationarity is a requirement for most of the mathematical theories underlying time series analysis. However, despite its nearly universal assumption, there is little previous studies that seek to test stationarity in a reliable manner. (Tóth1a et al. (2010)).

For low frequency financial data (i.e. monthly or daily data), a number of procedures to test stationarity have been advocated and applied to various time series processes in econometrics. Most of them focus on the first two moments of a process; namely, they test covariance stationarity. These tests work well for normally distributed random variables. However, for high frequency financial data, such as tick by tick data, it is well known that price change distributions are fat-tailed, and substantially deviate from a normal distribution. These fat-tail distributions cannot be dealt with by the stationarity tests focusing on the first two moments.

In this paper, we advocate a test for strict stationarity, which considers the entire distribution of a process rather than the first two moments of the process, and apply this test to the dollar-yen exchange rate.

We describe the data used in this paper in Sec. 2. In Sec. 3, we explain our procedure to test stationarity, which is based on the two-sample Kolmogorov–Smirnov test and the Pearson's chi–square test. In Sec. 4, we present empirical results. In Sec. 5, we discuss some implications of our results.

2. Data description

The tick-by-tick data we study is the USD-JPY exchange rate provided by ICAP EBS with a recording frequency of every one second, for the period of January 1998 through December 2005. The foreign exchange market is the most liquid and largest financial market in the world. Most of spot interbank transactions are executed through the global electronic broking systems such as ICAP EBS and Reuters. In the USD-JPY exchange rate, the ICAP EBS has a strong market share.

We exclude observations for special days such as Mondays, weekends, holidays, official intervention days (i.e. the government and/or the central bank intervenes in the foreign exchange market in order to stabilize the rate), which are obviously different from regular business days. We analyze the time series of 1-tick price changes of the mid-quote price, which is defined as the average of the best bid and the best ask. The best bid and the best ask, representing lowest sell offer and highest buy offer, are recorded at the end of one second time slice.

In this paper, we focus on the following two time series. The first one is the time series for the absolute price changes. We refer to this time series as G. The second one is the time series for the number of successive price changes in the same direction. We refer to this time series as D. Note that, in producing this time series, we drop observations with no price changes. For example, a particular sequence of 16 1-tick price changes

$$\{0.01, 0.02, 0.01, -0.02, 0, -0.03, -0.01, 0.02, 0, 0.02, -0.04, 0.01, -0.02, -0.03\}$$

is represented by

$$\{0.01, 0.02, 0.01, 0.02, 0, 0.03, 0.01, 0.02, 0, 0.02, 0.04, 0.01, 0.02, 0.03, \}$$

in G sequence and

$$\{3, 3, 2, 1, 1, 2\}$$

in D sequence.

3. Stationary test

For Gaussian time series processes, one can test stationarity by measuring any number of simple statistics, such as the mean or standard deviation, and employing a standard statistical test. But such an approach is not particularly good for high frequency financial time series, because from seminal work by Mantegna and Stanley (1995) we know that the distributions of price changes have fat-tails often approximated by a power law (Ohnishi et al. (2008)). Therefore the procedure for Gaussian processes cannot be applied to high frequency financial data.

Our analysis is based on a precise definition of stationarity: that is, the joint distribution of any two segments of data of the same length should be identical. Formally, a stochastic process X_t is called strictly stationary if for any set of times $t_1, t_2, \cdots, t_n$ and any k the joint probability distributions of $\{X_{t_1}, X_{t_2}, \cdots, X_{t_n}\}$ and of $\{X_{t_1+k}, X_{t_2+k}, \cdots, X_{t_n+k}\}$ coincide. That is, it requires that the joint distribution depends only on time lags. It follows that the mean remains constant, and that the autocorrelation function depends only on time lags, and not on time index.

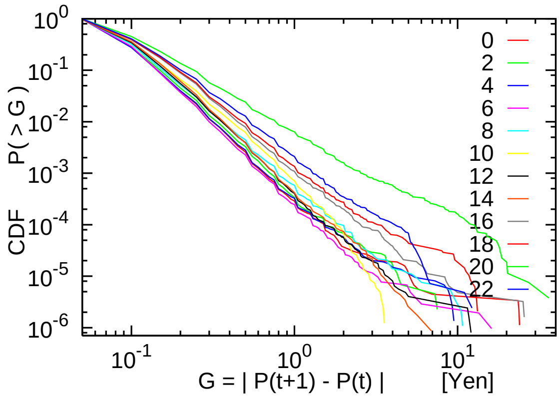

Figure 1: Cumulative probability distributions of absolute price changes G. Colors represent different hour of a day.

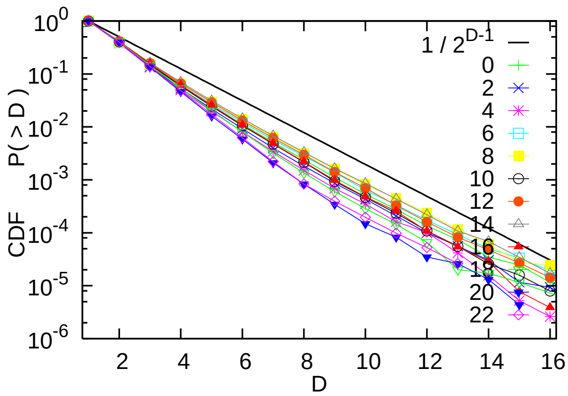

Figure 2: Cumulative probability distributions of the number of successive price changes in the same direction D. Colors represent different hour of a day.

Traders behave differently even within a day, depending on, say, whether it is in the morning session, or it is in the afternoon session. Specifically, we know that the absolute price changes (Ohnishi et al. (2008)) and activities (Ito and Hashimoto (2006)) display an intraday pattern. Figure 1 and 2

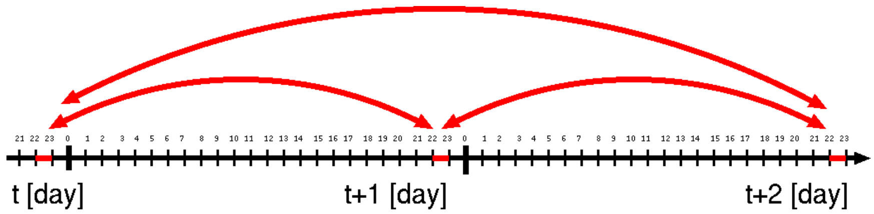

Figure 3: We compare two sub-series on the same hour of different days.

show the cumulative distribution of G and D, respectively. These distributions differ depending on the hour of a day. To remove the intraday pattern, we assume that the time series can be regarded as approximately stationary at least during the one-hour period. Based on this assumption, we decompose the entire observations into the subsets, each of which is identified by hour h and day t. We then compare the subset (h, t) (i.e. the set of observations belonging to hour h of day t) with the subset $(h, t^{'})$ (i.e. the set of observations belonging to hour h and day $t^{'}$), as illustrated schematically in Fig. 3.

To test for stationarity, we take the approach of comparing the distribution of observations belonging to the subset (h, t) and the distribution of observations belonging to the subset $(h, t^{'})$ to see if the two distributions are identical. Tests are performed by using the two-sample Kolmogorov–Smirnov test for G, and the Pearson's chi-square test for D. The stationarity is determined at a 5 percent significance level. The two-sample Kolmogorov–Smirnov test compares two cumulative distribution functions of G, then maximum difference between these two cumulative distribution functions yields P-value. The Pearson's chi-square test is performed by considering a histogram of D having 4 bins, that is $D = 1, D = 2, D = 3$ and $D \geq 4$. These two tests have the advantage of being nonparametric, and without making assumptions about the distribution function of the data, it returns the probability that two sets of data are drawn from the same distribution.

4. Results

First, we compare the distribution of observations in the subset (h, t) and the distribution of observations in the subset $(h, t^{'})$ for every pair of t and $t^{'}$. This exercise is repeated $N_0 \sim 10^6$ times. Then, we count the number of times, which is denoted by N, in which we reject the null hypothesis that the two distributions are identical. Figure 4 shows the results of this exercise.

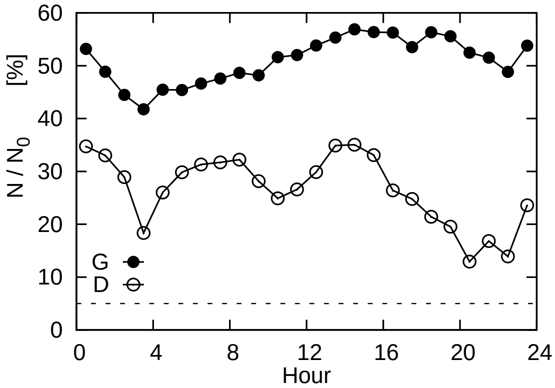

Figure 4: Percentage of all pairs whose difference lies beyond the expected value at a 5 percent significance level. Closed symbols give the results for G; open symbols give results for D. This percentage is greater than 5 percent, displaying clear evidence of nonstationarity.

The y-axis represents N/N_0. Note that if the entire time series is stationary, the rejection rate would be 5 percent. The x-axis is the hour of a day. The closed symbols represent the result of this exercise for G, while the open symbols represent the result for D. We see that, for each h, the rejection rate is much larger the critical value, i.e. 5 percent, indicating that the null hypothesis of stationarity is clearly rejected. We suspect that this is the result of long time correlations in the time series of the absolute value of price changes (volatility clustering). Turning to the results for D, we again find that, for each h, the rejection rate is significantly above the critical value. Therefore, we conclude from these exercises that the exchange rate process is not a stationary process.

Next, we repeat the same test, but this time we compare observations in the subset (h, t) and observations in the subset $(h, t+\tau)$ for different values of τ. Figure 5 shows the results of this exercise. The y-axis represents the rejection rate, which is averaged over different h and t. The x-axis represents time lags τ. The closed symbols are the results for G. For $\tau = 1$ day as many as 34% are nonstationary. That number rises to more than 45% for $\tau \geq 60$ days. The open symbols are the results for D, showing similar feature. For $\tau = 1$ day as many as 18% are nonstationary. The percentage monotonically increases as time lag τ increases. As time lag increases, there is more opportunity for different distributions to emerge, and stationarity is lost.

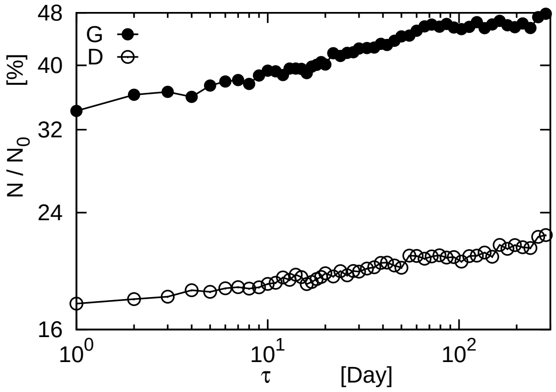

Figure 5: Percentage of all pairs whose difference lies beyond the expected value at a 5 percent significance level. Closed symbols give the results for G; open symbols give results for D. This percentage increases with the time lag τ, suggesting that the data become more nonstationary as the time lag increases.

To investigate the properties of nonstationality in more details, we focus on the interval (in days) between two different distributions emerge. Specifically, we pick up every event that the distribution changes. The series of the time intervals $\{T\}$ between those event are generated. The cumulative probability distributions as a function of the scaled recurrence intervals $T/\langle T\rangle$ is shown in Fig. 6 and 7 for G and D, respectively. For each hour of the day, these distributions follow an exponential distribution, where $\langle T\rangle$ is about 10 days for G and 3 days for D.

The independence of T is also verified in the mean conditional interval $\langle T_{i+1}|T_i\rangle$, which is defined as the mean of recurrence intervals T_{i+1} conditional on the preceding interval T_i as shown in Fig. 8, where we plot $\langle T_{i+1}|T_i\rangle / \langle T\rangle$ as a function of $T_{i+1}/\langle T\rangle$. It is seen clearly that for both G and D $\langle T_{i+1}|T_i\rangle / \langle T\rangle$ fluctuates around a horizontal line close to 1, indicating that T_{i+1} independent of the previous T_i. Thus, there is no memory effect in recurrence intervals. The numbers of occurrences counted in disjoint periods are independent from each other (i.e. independent increments).

Finally, Fig. 9 and 10 show the cumulative number of occurrence of nonstationarity as a function of time (t days). For both G and D, the cumulative number exhibits an almost linear increase, that is, the slope of the cumulative number is constant. Thus, the probability distribution of the number of occurrences counted in any time period depends only on the length of the period (i.e. stationary increments).

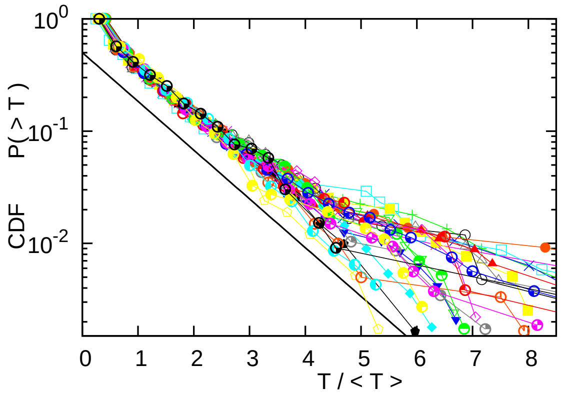

Figure 6: Cumulative probability distributions of recurrence intervals T for G. Symbols represent different hour of a day varying from 0 to 23 hour. The straight line is for demonstration and have a slope of -1.

In sum, we have found that the occurrence of nonstationarity possesses the following properties: Poisson distribution of the recurrence intervals, independent increments, and stationary increments. Hence, the occurrence of nonstationarity is well-modeled by the Poisson process. Nonstationarities occur at random instants of time at an average rate of λ per day, where $\lambda = 1/\langle T\rangle$ is about 0.1 for G and 0.33 for D.

5. Discussion

We have studied the time series of the absolute price changes and the number of successive price changes in the same direction. First, we have tested nonstationarity of the time series based on the two-sample Kolmogorov-Smirnov test and Pearson's chi-square test. We have found that both the absolute price changes and the number of successive price changes deviate substantially from a stationary process. Second, we have investigated the properties of nonstationarity. We have found that the recurrence intervals between the days nonstationarity occurs is modeled by the Poisson process.

For the absolute price changes, long-term correlations are likely to reflect nonstationarity. However, the origin of nonstationarity might be different from long-term memory of the volatility, because our observations of the recurrence intervals disagree with the results reported by the previous studies based on the return intervals between price volatility above a certain threshold, where the return intervals strongly depend on the previous return inter-

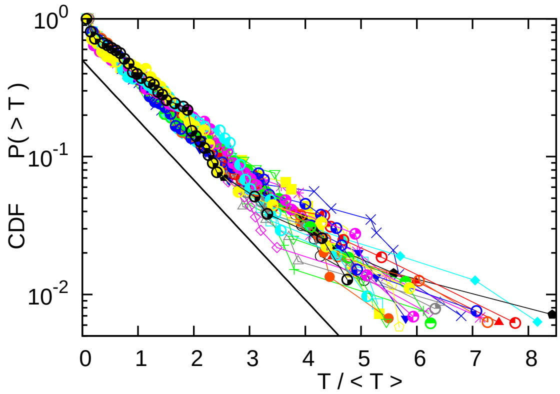

Figure 7: Cumulative probability distributions of recurrence intervals T for D. Symbols represent different hour of a day varying from 0 to 23 hour. The straight line is for demonstration and have a slope of -1.

val (Yamasaki et al. (2005); Wang et al. (2006, 2007); Vodenska-Chitkushev et al. (2008); Jung et al. (2008)).

The results for the number of successive price changes are very interesting, because the number depends not on the size of price changes, but on the signs of price changes. The previous papers have found some evidence for the presence of the memory effect in the sign of price changes

(Mizuno et al. (2003); Hashimoto et al. (2008)), but the length of memory reported in those papers is about several ten seconds, clearly too short-lived to account for the nonstationarity observed in the previous section.

Despite both time series capture different features of price changes, the recurrence intervals of nonstationarity is characterized by the Poisson process for both cases. Therefore, the most possible explanation for nonstationarity is the arrival of news, which is considered to be well described by the Poisson process. The efficient market hypothesis claims that prices reflect all news coming into the markets very rapidly. It may be the case that nonstationarity of price changes is a reflection of nonstationarity of the arrival of news.

Nonstationarity poses a serious problem for the calculation of auto-correlation function of the time series. The correlation coefficients themselves may not be constant, which gives an unexpected estimation error. These findings could lead to a better understanding and modeling of the price dynamics.

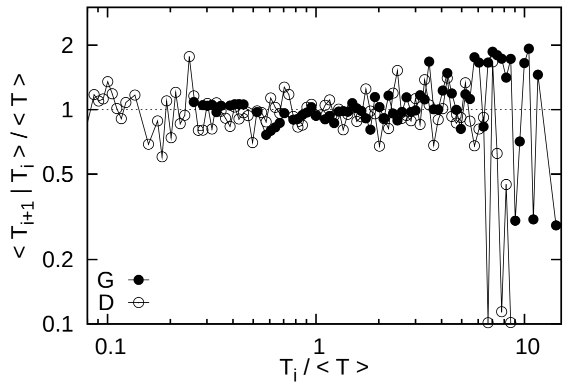

Figure 8: Scaled mean conditional interval $\langle T_{i+1}|T_i\rangle$ / $\langle T\rangle$ as a function of scaled preceding interval $T_{i+1}/\langle T\rangle$. Closed symbols give the results for G; open symbols give results for D.

6. Acknowledgements

Numerical calculations were performed by the Hitachi HA8000 at Information Technology Center, the University of Tokyo. T.O. acknowledges grant from the Ministry of Education, Culture, Sports, Science, and Technology of Japan (Grant-in-Aid for Young Scientists (B) No. 20760053).

References

B. Tóth1a, F. Lillo, J. Farmer, arXiv:1001.2549v1 (2010).

R. Mantegna, H. Stanley, Nature 376 (1995) 46–49.

T. Ohnishi, H. Takayasu, T. Ito, Y. Hashimoto, T. Watanabe, M. Takayasu, Journal of Economic Interaction and Coordination 3 (2008) 99–106.

T. Ito, Y. Hashimoto, Journal of the Japanese and International Economies 20 (2006) 637–664.

K. Yamasaki, L. Muchnik, S. Havlin, A. Bunde, H. Stanley, Proceedings of the National Academy of Sciences 102 (2005) 9424–9428.

F. Wang, K. Yamasaki, S. Havlin, H. Stanley, Physical Review E 73 (2006) 26117.

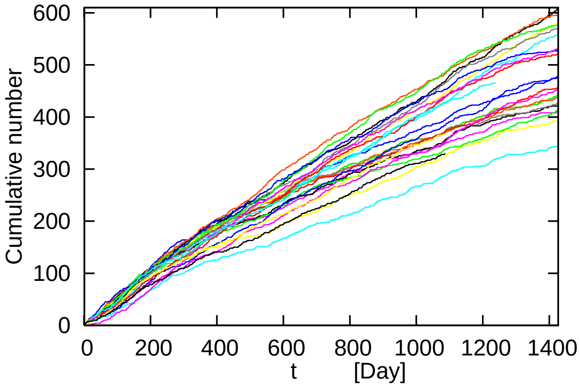

Figure 9: The cumulative number of occurrence of nonstationarity for G. Symbols represent different hour of a day varying from 0 to 23 hour. Almost linear increases suggest nonhomogeneous Poisson process.

F. Wang, P. Weber, K. Yamasaki, S. Havlin, H. Stanley, The European Physical Journal B-Condensed Matter and Complex Systems 55 (2007) 123–133.

I. Vodenska-Chitkushev, F. Wang, P. Weber, K. Yamasaki, S. Havlin, H. Stanley, The European Physical Journal B-Condensed Matter and Complex Systems 61 (2008) 217–223.

W. Jung, F. Wang, S. Havlin, T. Kaizoji, H. Moon, H. Stanley, The European Physical Journal B 62 (2008) 113–119.

T. Mizuno, S. Kurihara, M. Takayasu, H. Takayasu, Physica A: Statistical Mechanics and its Applications 324 (2003) 296–302.

Y. Hashimoto, T. Ito, T. Ohnishi, M. Takayasu, H. Takayasu, T. Watanabe, NBER Working Paper (2008).

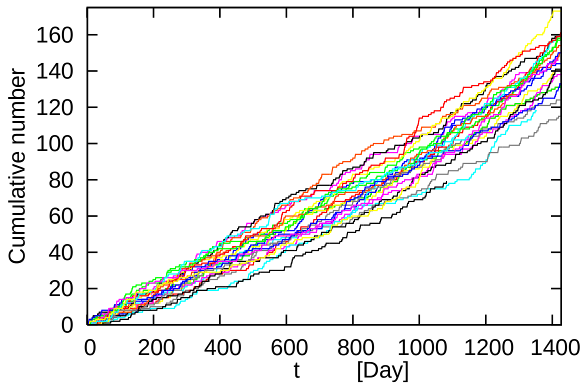

Figure 10: The cumulative number of occurrence of nonstationarity for D. Symbols represent different hour of a day varying from 0 to 23 hour. Almost linear increases suggest nonhomogeneous Poisson process.