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# The bursting of housing bubble as jamming phase transition

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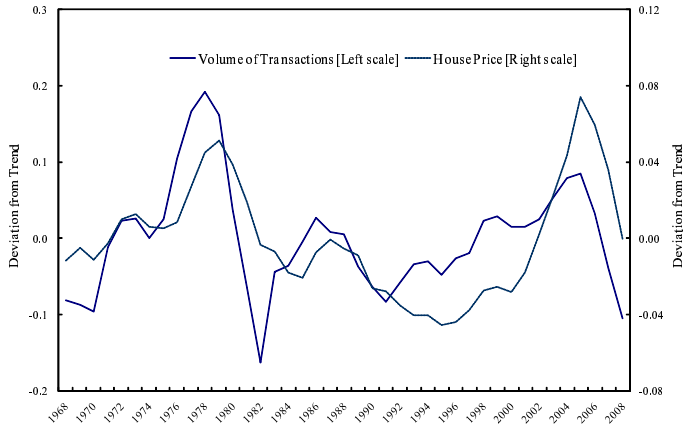
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May 3, 2009

**Abstract** Recently housing market bubble and its burst attracts much interest of researchers in various fields including economics and physics. Economists have been regarding bubble as a disorder in prices. However, this research strategy has overlooked an importance of the volume of transactions. In this paper, we have proposed a bubble burst model by focusing on transaction volume incorporating a traffic model that represents spontaneous traffic jam. We find that the phenomenon of bubble burst shares many similar properties with traffic jam formation on highway by comparing data taken from the U.S. housing market. Our result suggests that transaction volume could be a driving force of bursting phenomenon.

## 1 Introduction

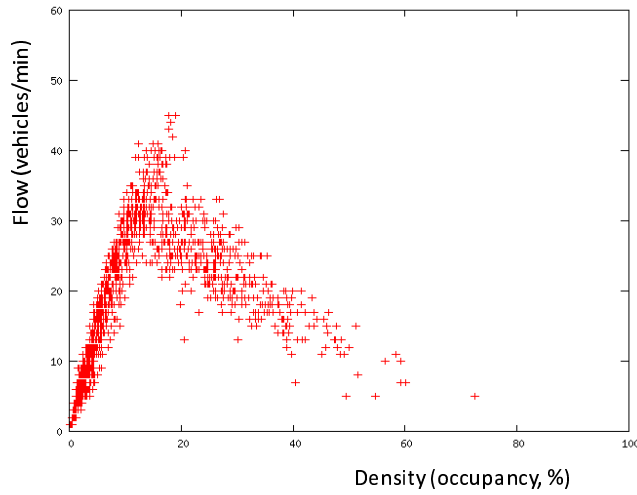
Fluctuations in real estate prices have substantial impacts on economic activities. For example, land prices in Japan exhibited a sharp rise in the latter half of the 1980s, and its rapid reversal in the early 1990s. This large swing had led to a significant deterioration of the balance sheets of firms, especially those of financial firms, thereby causing a decade-long stagnation of the Japanese economy, which is called Japan's "lost decade". A more recent example is the U.S. housing market bubble, which started somewhere around 2000 and is now in the middle of collapsing. This has already caused substantial damages to financial systems in the U.S. and the Euro area, and it is expected that it may spread worldwide as in the case of the Great Depression in the 1920s and 30s.



**Fig. 1** Fluctuations in price and transaction volume in the U.S. single-family house market. Both variables represent deviations from a linear trend.

These recent episodes have rekindled researchers' interest on the issue of bubbles. Economists have been regarding this phenomenon as a disorder in prices. Specifically, they define bubbles as a temporary deviation of asset prices from the corresponding fundamental values, which are basically determined by investors' expectations about future dividend stream and appropriate discount rates. Interestingly, even researchers in other areas, such as econophysics, share this empirical strategy in the sense that they look for abnormal behaviors in prices[1, 2]. However, this research strategy has overlooked an important aspect of bubbles; namely, fluctuations in asset prices tend to be closely correlated with those in the volume of transactions.

Fig.1 depicts fluctuations in housing prices and the volume of house transactions in the U.S., which shows a positive correlation between the two variables over the business cycles. More importantly, it shows that a change in transaction volume tends to lead a change in prices by one year or two[3,4]. It is reported that similar relationships were observed for other real estate markets in other countries, including Japan, U.K., Hong Kong, and Singapore[5–8]. These evidences suggest that some sort of interaction between prices and the volume of transactions plays an important role in the process of bubble and its bursting[3,9,10], and that fluctuations in transaction volume, rather than those in prices, could be its driving force. Given this understanding, we focus more on transaction volume in this paper, and seek to propose a model which explains an emergence of temporary deviation of transaction volume from its appropriate level, as well as its reversal.



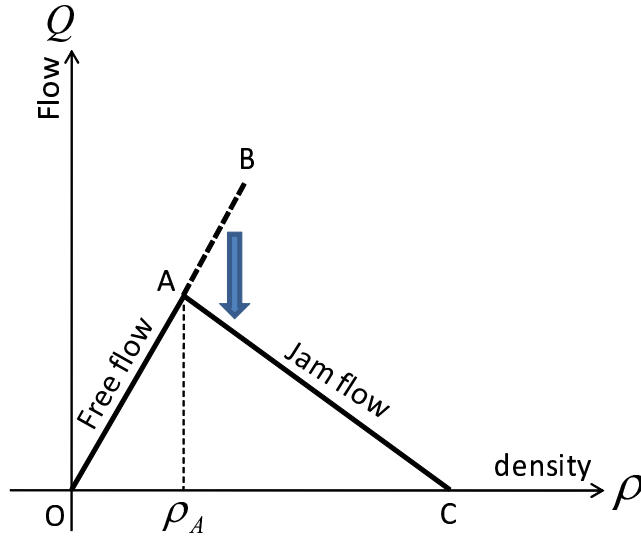
**Fig. 2** Typical flow-density diagram taken from Tokyo Metropolitan highway. Below (above) the critical density flow increases (decreases) according to the increase of density. Around the critical density 15% – 20% we see metastable state which corresponds unstable free flow.

## 2 Traffic jam perspective

The idea of our modeling stems from traffic jam on highway. Each vehicle on highway usually tries to move faster if there is enough space in front of it. However, such flow becomes unstable and soon changes into congested state if the density of vehicles becomes large enough. Fig.2 is a typical observed data of flow-density relation of vehicles on highway, which is called the *fundamental diagram*. Note that flow is defined as the multiplication of density and average velocity, and density is the percentage of the occupied area of vehicles divided by the road area.

From this figure we see that flow increases according to the increase of density up to 15%, which corresponds to free flow. Instead, flow decreases above the density 20%, which correspond to jam flow. Thus we have phase transition from free to jam state at critical density 15%, and we find that free flow overlaps jam flow between the range 15%-20% in the diagram. A schematic picture of this diagram is given in Fig.3.

Free flow on  $A - B$  is dangerous and unstable, since the average velocity of vehicles is high although the headway between successive vehicles is small. Thus small perturbation, e.g., weak brakes or slow down due to slope is enough to change the free flow into jamming state. Thus the branch  $A - B$  is called metastable state, whose lifespan is less than 10 minutes in reality and it suddenly changes into jam flow. This scenario is indicated by an arrow in Fig.3. The existence of the metastable state is crucial for accounting so-called phantom jam, i.e., traffic jam on highway without bottlenecks. Drivers' tendency for moving faster will lead to small headway with high



**Fig. 3** Schematic flow-density diagram of observed data. There is metastable state  $A - B$  above the critical density  $\rho_A$ .

velocity, which eventually causes metastable flow. Then due to small perturbation traffic jam is endogenously formed spontaneously. This has been confirmed by a simple model called the slow-to-start (SIS) model[11,12], as well as an experiment[13].

Now we come to the point that there is similarity between spontaneous jam formation and bubble burst. Traffic jam occurs because there are too many cars relative to the available road space, while “transaction jam” in a housing market occurs because there are too many houses traded in the market relative to the total amount of liquidity (or money) supplied to the market by investors, including banks, who are outside the market. Transaction is an exchange of a house and money, and vehicle’s motion is also considered as an exchange of a vehicle and a free space in front of it. Thus we propose the following analogy: For vehicle traffic,

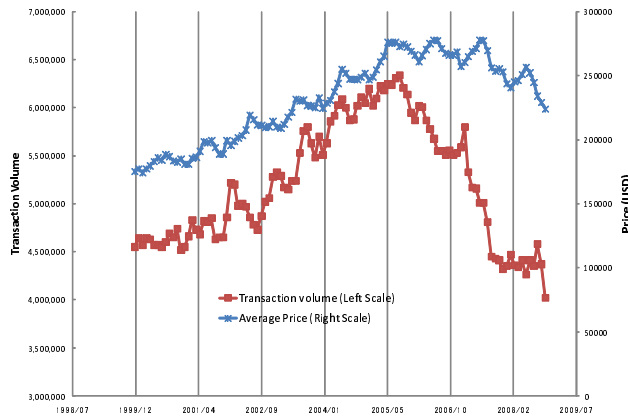
$$\text{Density} = \frac{\text{Space for cars}}{\text{Total available space}},$$

$$\text{Average velocity} = \frac{\text{Number of moved cars}}{\text{Number of cars}},$$

and for housing market transactions,

$$\text{Density} = \frac{\text{Housing inventory}}{\text{Total available liquidity}}, \quad (1)$$

$$\text{Average velocity} = \frac{\text{Transactions}}{\text{Housing inventory}}, \quad (2)$$



**Fig. 4** Monthly fluctuations in average house prices and transaction volumes in the U.S. housing market from Dec 1999 to Nov 2008.

where housing inventory and transactions are evaluated as USD or other monetary unit.

Furthermore, we conjecture that metastable state plays a great role also in bubble burst, as so in phantom jam. More precisely, we think that before the burst occurs, there is metastable transaction between real estate companies. We will show such modeling below, and compare the result with observed data taken from the US housing market.

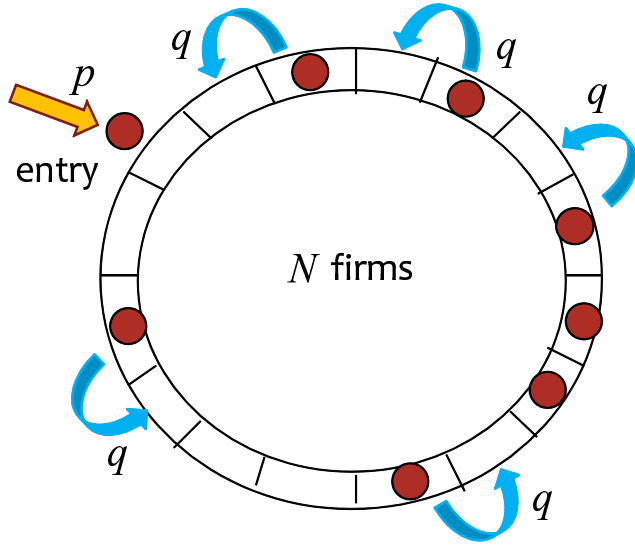
### 3 Bubble burst model

#### 3.1 Transaction and price

Before considering a model for bubble burst, let us focus on the housing sales from December 1999 to November 2008 in the U.S., during which both transaction volumes and prices showed a significant swing in our dataset. From fig.4, we immediately see that a decline in transaction volume occurred earlier than a fall in price. We also find that housing price is almost saturated around the bursting period, which is also clearly seen in the data of Japan's bubble burst in 1992. Thus in our model we focus on transaction volume instead of price in order to grasp the essence of the bursting phenomena.

#### 3.2 Model description

Now we propose a simple model for bubble burst incorporating the SIS model. Consider an economy with symmetric  $N$  firms dealing with houses, which are identified by  $i$  ( $i = 1, \dots, N$ ) and located along a circle (Fig.5). Firm  $i$  is allowed to purchase a house from firm  $i - 1$ , but not allowed to do so from any other firm. For simplicity, it is assumed that the amount

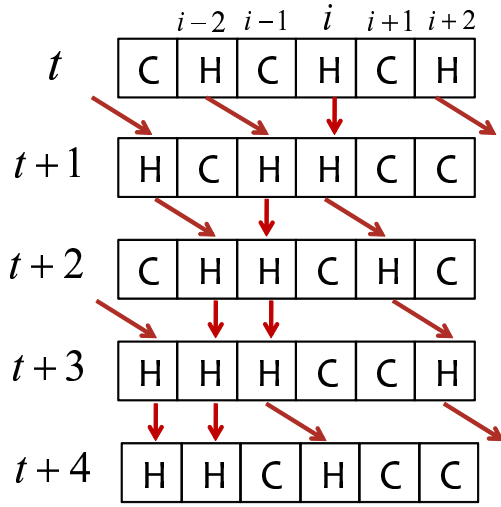


**Fig. 5** Bubble burst model on a circular market. There are  $N$  firms dealing houses represented by circles. The probability of successful transaction is given by  $q$ .

of cash each firm owns before buying a house is identical to the price of a house, so that each firm has its entire asset either in the form of cash or in the form of a house.

There are two rules governing transactions in this economy. First, all transactions must be in the form of an exchange of cash and a house, and no barter transactions (i.e. transactions between a house and another house) are allowed. This implies that transaction between firms  $i$  and  $i - 1$  never takes place in period  $t$  unless firm  $i$  holds cash at the beginning of that period. This rule corresponds to what macroeconomists call “cash-in-advance” constraint. Second, firm  $i$  becomes timid if firm  $i$  fails to make a transaction with firm  $i + 1$  in period  $t$ . Specifically, firm  $i$  hesitates to deal with firm  $i - 1$  and refuses to purchase a house in period  $t + 2$  even if firm  $i$  successfully has sold a house to firm  $i + 1$  in period  $t + 1$ , therefore holding cash at the beginning of period  $t + 2$  (Fig.6). This is because, ceteris paribus, firm  $i$  is able to increase the probability that firms behind him (firm  $i + 1, i + 2, i + 3, \dots$ ) hold cash, instead of a house, thereby reducing the probability that he will be refused to sell a house to firm  $i + 1$ . This hesitation rule represents firms’ preference to cash as a means to store value until the next period, because of its general acceptability.

In the modeling of vehicle traffic, there are also two important rules: the exclusion principle and the SIS rule which correspond to the above two rules in our model. Firms and houses correspond to the road cells and vehicles, respectively. The exclusion principle in vehicle traffic is that a cell can accommodate at most one vehicle for avoid collision. The SIS rule represents an inertia effect of vehicles, i.e., once a vehicle stops, then it



**Fig. 6** Hesitation of firm  $i$  in period  $t+2$ , which is due to the collapse of credit in the period  $t$ , leads to jam of transaction in this economy. H and C represent house and cash, respectively.

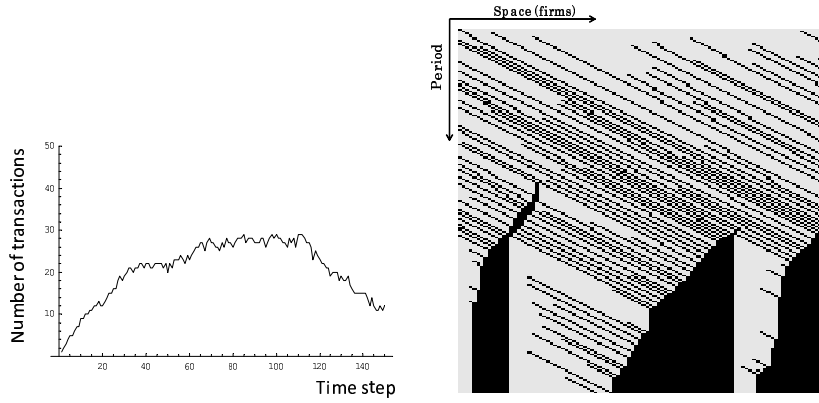
waits extra one time step to move forward after the front cell becomes empty. In our model, this wait corresponds to the hesitation to make another transaction due to the lack of confidence about the trade partner's ability to pay, and thus the fear of default. Moreover, this hesitation rule is crucial to have chain reaction of transaction fails, which leads to the bubble burst. This is similar to chain reaction of brakes on a highway under the SIS rule, which is generally observed in real traffic data.

### 3.3 Simulations

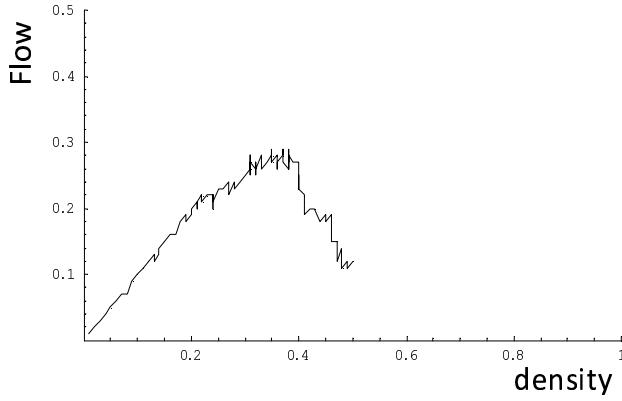
Given the above rules, we conduct simulations. We assume that there exists no housing inventory in this market at the beginning of period 0, but in each period, firm  $i$ , which is randomly chosen, purchases a new house from someone outside this market with probability  $p$  if (1) firm  $i$  does not hold a house (and therefore owns cash), and (2) firms ahead and behind firm  $i$  (namely, firms  $i-1$  and  $i+1$ ) do not hold a house either. In words, firm  $i$  wants to purchase a house from firm  $i-1$ , but cannot do that because firm  $i-1$  does not own a house. At the same time, firm  $i$  expects that firm  $i+1$  will be able to buy a house from him because firm  $i$  owns cash. It is only in this situation that firm  $i$  brings in a new house to the market from outside. We also assume that the probability of successful transaction between any two adjacent firms (the one with cash and the one ahead with a house) is given by  $q$  (Fig.5).

Fig.7 (left) shows fluctuations in transaction volume over time. An important thing to note is that transaction volume exhibits an abrupt decline





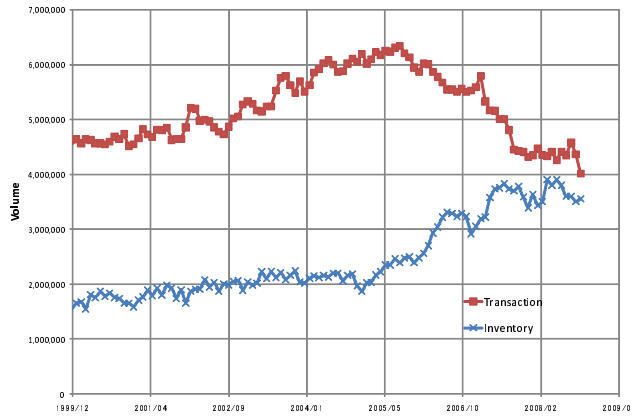
**Fig. 7** (left) Fluctuations in transaction volume over time. (right) Corresponding spatiotemporal figure of transaction. Small black squares represent houses which move to right due to the success of deal. Parameters  $N$  and  $q$  are set at  $N=100$  and  $q=0.99$  in the simulation.



**Fig. 8** Flow-density diagram Parameters  $N$  and  $q$  are set at  $N=100$  and  $q=0.99$ .

in  $t=115$  after a quiet period in which transaction volume is kept fairly stable. This abrupt decline is an endogenous event, and can be seen as phase transition phenomenon emerging from metastable state. Fig.7 (right) shows an example of spatiotemporal figure of transaction flow. We see jam of transaction emerges and rapidly grows endogenously, which is very similar to an abrupt change from free to congested flow in highway traffic.

The fundamental diagram is given in Fig.8. Flow is defined by multiplying eq.(1) and eq.(2). Due to the entry of houses we see linear increase of flow as density increases. The critical density is about  $1/3$  in this model [14], so we see the metastable transaction between 0.3 and 0.4. Then this market suddenly breaks due to the collapse of credit, and bubble bursts.



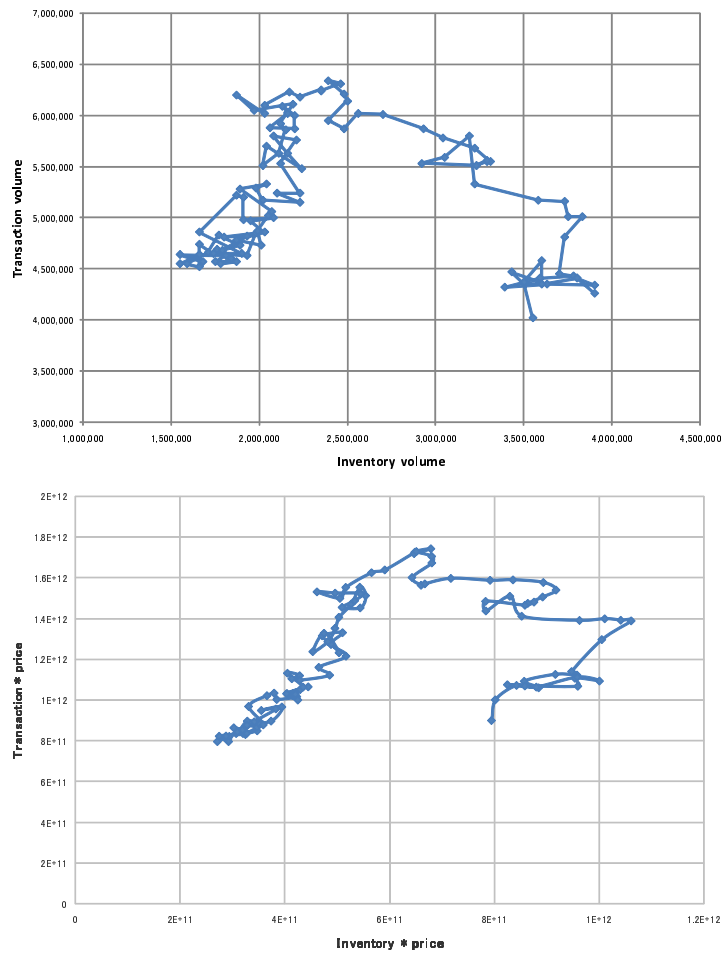
**Fig. 9** Transaction volume and inventory in the U.S. housing market from Dec 1999 to Nov 2008.

### 3.4 Comparison with data

The variation of transaction volume and inventory is given in Fig.9. We see that they both increase first, but at the middle of 2006 transaction begins to fall and inventory increases. Fig.10 is the fundamental diagram of this data, i.e., we depict it by taking inventory as the horizontal axis and transaction volume as the vertical axis. Because we cannot observe the total amount of liquidity supplied to the market, we make two alternative assumptions in depicting the figures. The upper one is that the total amount of liquidity changes in proportion to changes in housing prices. This represents a situation, for example, that housing prices continue to rise, and investors have an optimistic expectation about the future course of prices, and thus willingly provide additional liquidity to the market. This assumption is adopted in the upper figure where transaction and inventory are both measured in the number of housing units. Alternatively, we may assume that the total liquidity supplied to the market from the outside investors is exogenously determined (i.e. it does not depend on the evolution of prices); more specifically, we assume that the total amount of liquidity does not change at all during the entire sample period. This assumption is adopted in the lower figure where transaction and inventory are evaluated in dollars. The two figures should be compared with Fig.8, showing similar properties such as increases of flow at early stage and its collapse due to the decrease of transaction.

## 4 Concluding discussions

In this paper, we have proposed a bubble burst model by using a traffic model that represents spontaneous traffic jam. We find that the phenomenon



**Fig. 10** Fundamental diagram, i.e., inventory versus transaction volume depicted from the data shown in Fig.9. The horizontal and vertical axes represent the number of housing units in the upper figure and dollars in the lower figure.

of bubble burst shares many similar properties with traffic jam formation. Especially we would like to stress on the importance of the SIS rule which will be recast as hesitation in our model. The simulation results obtained in our model is similar to those of the data taken from the U.S. housing market. We focus on transaction in our model instead of the price, and this has not been considered up to now in studying bubble burst phenomena. Our result suggests that the transaction could be a driving force of bursting phenomenon.

In our theoretical model we know the value of the critical density, then we can predict whether the market is now in a dangerous metastable state or not by checking the amount of inventory. Of course we don't know this

critical value in the real market, we hope that our analysis will help to judge the danger and avoid the burst and crash of market. Studying other bubble phenomena, e.g., oil market and extensions of our model such as introducing price change and various dealing network are ongoing and future works.

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