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A New Method for Identifying the Effects of Central Bank Interventions*

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Abstract

Central banks react even to intraday changes in the exchange rate; however, in most cases, intervention data is available only at a daily frequency. This temporal aggregation makes it difficult to identify the effects of interventions on the exchange rate. We propose a new method based on Markov Chain Monte Carlo simulations to cope with this endogeneity problem: We use "data augmentation" to obtain intraday intervention amounts and then estimate the efficacy of interventions using the augmented data. Applying this method to Japanese data, we find that an intervention of one trillion yen moves the yen/dollar rate by 1.7 percent, which is more than twice as large as the magnitude reported in previous studies applying OLS to daily observations. This shows the quantitative importance of the endogeneity problem due to temporal aggregation.

JEL Classification Numbers: C11, C22, F31, F37
Keywords: foreign exchange intervention; intraday data; Markov-chain Monte Carlo method; endogeneity problem; temporal aggregation

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1 Introduction

Are foreign exchange interventions effective? This issue has been debated extensively in the 1980s and 1990s, but no conclusive consensus has emerged. A key difficulty faced by researchers in answering this question is the endogeneity problem: the exchange rate responds "within the period" to central bank interventions *and* the central bank reacts "within the period" to fluctuations in the exchange rate.[1] As an example, consider the case of Japan. The monetary authorities of Japan, which are known to be one of the most active interveners, started to disclose intervention data in July 2001, and this has rekindled researchers' interest in the effectiveness of interventions.[2] However, the information disclosed is limited: only the total amount of interventions on a day is released to the public at the end of a quarter, and no detailed information, such as on the time of the intervention(s), the number of interventions over the course of the day, and the market(s) (Tokyo, London, or New York) in which the intervention(s) were executed, is disclosed.[3] Most importantly, the low frequency of the disclosed data poses a serious problem for researchers because it is well known that the Japanese monetary authorities often react to intraday fluctuations in the exchange rate.[4]

In this paper, we propose a new methodology, which is based on Gibbs sampling, to eliminate the endogeneity problem caused by the fact that data on foreign exchange market interventions is available only on an aggregate daily basis. Consider a simple two-equation system. Hourly changes in the exchange rate, $\triangle s_h$, satisfy $\triangle s_h = \alpha I_h +$ disturbance, where I_h is the hourly amount of yen-buying interventions. On the other hand, the central bank policy reaction function is given by $I_h = \beta \triangle s_{h-1} +$ disturbance. Suppose that this two-equation system represents the true structure of the economy, and that s_h is observable at the hourly frequency but I_h is not: researchers are able to observe only the daily sum of I_h, and in that sense, intervention data suffers from temporal

[1] Note that this difficulty would not arise if the central bank responded only slowly to fluctuations in the exchange rate, or if the data sampling interval were sufficiently fine. In the context of fiscal policy, for example, the government reacts to changes in variables like output and employment only slowly due to the political processes involved, so that researchers can identify the impact of fiscal policy on these variables using available quarterly data. Blanchard and Perotti (1999) used this property together with other detailed information on fiscal institutions in identifying their structural VAR model.

[2] Recent studies reflecting this renewed interest includes Ito (2003), Fatum and Hutchison (2006), Dominguez (2003), Chaboud and Humpage (2003), Galati et al (2005), Fratzscher (2005), and Fatum (2008), among others.

[3] This is true for monetary authorities in most industrial countries. For example, the Bundesbank and other euro-zone central banks do not disclose intervention data to the public; neither does the European Central Bank. The monetary authorities of the UK started to disclose information about their interventions in 2000, but the only information disclosed is the daily amount of interventions. An important exception is the Swiss National Bank (SNB), which discloses all transactions it carried out in the Swiss franc/US dollar market. Fischer and Zurlinden (1999) and Payne and Vitale (2003) use the SNB transactions data to evaluate the efficacy of interventions.

[4] Chang and Taylor (1998), for example, counting the number of reports by Reuters about Japanese central bank interventions from October 1, 1992 to September 30, 1993, find that there were reports of 154 intervention in 69 days, implying that the Japanese central bank intervenes, on average, two or three times a day.

aggregation. Given this environment, our task is to estimate α and β.

The key idea of methodology we propose is as follows. Suppose we have a guess about the values of α and β. Then the exchange rate equation and the policy reaction function allow us to recover the hourly amount of intervention, subject to the constraint that the sum of hourly amounts equals the daily amount, which is observable. In an extreme case in which the variance of the disturbance term in the first equation is very small, we estimate I_h as $I_h = \alpha^{-1} \triangle s_h$ using the first equation. In the other extreme case in which the variance of the disturbance term in the second equation is tiny, then we have $I_h = \beta \triangle s_{h-1}$ from the second equation. In more general cases, one can guess (and we will verify this later) that the estimate of I_h is a weighted average of the two, with the weights being determined by the relative importance of the two disturbance terms. Once we obtain an estimate for the hourly amount of intervention in this way, we can estimate α and β without encountering an endogeneity problem. By repeating this procedure, we are able to estimate the two parameters as well as the hourly amount of intervention.

More precisely, we are able to obtain the distributions of α and β, given the hourly amount of intervention and the hourly exchange rate. At the same time, given the two parameters, the hourly exchange rate, and the daily amount of intervention, we are able to obtain the distribution of the hourly amount of intervention. Combining these two conditional distributions, we are able to obtain the joint and marginal distributions of the two parameters as well as the hourly amount of intervention through Gibbs sampling.

Our method can be seen as an application of “imputation” or “data augmentation” techniques based on Markov Chain Monte Carlo (MCMC) simulations to the endogeneity problem. The idea of applying MCMC methods to data augmentation was first proposed by Tanner and Wong (1987). Similar MCMC methods were used by Pedersen (1995) and Eraker (2001), among others, in the context of estimating parameters in continuous diffusion processes when only discrete, and sometimes low-frequency, data are available. However, this paper is the first attempt to make use of MCMC methods to solve the endogeneity problem. Note that the endogeneity problem we discuss in this paper occurs simply because the frequency of intervention data is not sufficiently high. In this sense, our paper deals with the issue of estimation biases caused by “temporal aggregation,” which has been discussed by Sims (1971), Chow and Lin (1971), Christiano and Eichenbaum (1987), and McCrorie and Chambers (2006), among others, in a closely-related but different context.

The remainder of the paper is organized as follows. Section 2 provides a detailed explanation of our methodology to address the endogeneity problem, while Section 3 presents simulation results to demonstrate how the methodology works. In Section, 4 we apply our methodology to Japanese data. We find that an exchange rate intervention (e.g., a sale) of one trillion yen leads to 1.7 percent change in the value of the yen (depreciation): this is more than twice as large as the magnitude reported in previous studies such as Ito (2003) and Fratzscher (2005), which apply ordinary least squares to daily intervention and exchange rate data. This result is consistent with the prediction that endogeneity creates a bias toward zero for the intervention coefficient, as long as the central bank follows a leaning-against-the-wind policy. It also shows the quantitative importance of the endogeneity problem due to temporal aggregation. Section 5 concludes the paper, while the Appendix provides the technical details of our methodology.

2 Methodology

2.1 The endogeneity problem in identifying the effects of central bank interventions

In this section, we present a detailed description of our methodology to address the endogeneity problem in identifying the effects of central bank interventions on the exchange rate. Consider a simple model of the following form:

$$s_{t,h} - s_{t,h-1} = \alpha I_{t,h} + \epsilon_{t,h} \quad (1)$$

$$I_{t,h} = \beta(s_{t,h-1} - s_{t-1,h-1}) + \eta_{t,h} \quad (2)$$

where $s_{t,h}$ is the log of the yen/dollar rate at hour h of day t ($t = 1, ..., T$ and $h = 1, ..., 24$), $I_{t,h}$ is the purchase of yen (and the selling of US dollars) implemented by the Japanese central bank between $h-1$ and h of day t, $\epsilon_t \sim i.i.d.N(0, \sigma_\epsilon^2)$, and $\eta_{t,h} \sim i.i.d.N(0, \sigma_\eta^2)$. Equation (1) represents the exchange rate dynamics, while equation (2) is the central bank's policy reaction function. We assume that α is negative, implying that yen-selling interventions ($I_{t,h} < 0$) lead to a depreciation of the yen ($s_{t,h} - s_{t,h-1} > 0$) and vice versa. We also assume that the exchange rate is observable at the hourly frequency, while interventions are observable only at the daily frequency: namely, we observe $I_t \equiv \Sigma_{h=1}^{24} I_{t,h}$. Note that if we were able to observe $I_{t,h}$ at the hourly frequency, we could obtain unbiased estimators of α and β by applying OLS to each of the two equations separately.

Taking partial sums of both sides of the equations leads to a daily model of the following form:

$$s_{t,24} - s_{t-1,24} = \alpha I_t + \epsilon_t \tag{3}$$

$$I_t = \beta \sum_{h=1}^{24} (s_{t,h-1} - s_{t-1,h-1}) + \eta_t \tag{4}$$

where $s_{t,24} - s_{t-1,24} = \sum_{h=1}^{24} (s_{t,h} - s_{t,h-1})$, $I_t = \sum_{h=1}^{24} I_{t,h}$, $\epsilon_t \equiv \Sigma_{h=1}^{24} \epsilon_{t,h}$, and $\eta_t \equiv \Sigma_{h=1}^{24} \eta_{t,h}$. This shows that the endogeneity problem arises in this daily model, so that a simple application of OLS to each of the two equations separately no longer works. To illustrate this, suppose that the central bank adopts a leaning-against-the-wind policy, so that β takes a positive value. Then an increase in $\epsilon_{t,h}$ leads to an increase in $s_{t,h} - s_{t,h-1}$ through equation (1), and to an increase in $I_{t,h+1}$ through equation (2). This means that I_t and ϵ_t in equation (3) are positively correlated, so that an OLS estimator of α has an upward bias. On the other hand, an increase in $\eta_{t,h}$ increases $I_{t,h}$ through equation (2), thereby creating an appreciation of the yen as long as α is negative. This implies that the error term in equation (4), η_t, and the regressor, $\sum(s_{t,h} - s_{t,h-1})$, are negatively correlated and, as a result, an OLS estimate of β has a downward bias.

2.2 MCMC simulations

We propose a method for estimating equations (1) and (2) using the daily data for interventions and the hourly data for the exchange rate. The set of parameters to be estimated is α, β, σ_ϵ^2, and σ_η^2. We first introduce an auxiliary variable, $I_{t,h}$, to substitute missing observations. Then we obtain a conditional distribution of each parameter, given the other parameters and the values of the auxiliary variable. Similarly, we obtain a conditional distribution of the auxiliary variable, given the parameters. Finally, we use the Gibbs sampler to approximate joint and marginal distributions of the entire parameters and the auxiliary variable from these conditional distributions.[5]

2.2.1 Prior distributions

We choose the following priors for the unknown parameters. We adopt a flat prior for α and β. On the other hand, we assume that the priors for σ_ϵ^2 and σ_η^2, are more informative than the flat ones but still relatively diffused. Specifically, we assume that the prior of σ_ϵ^2 is given by

$$IG\left(\frac{\nu_1}{2}, \frac{\delta_1}{2}\right)$$

[5]See Kim and Nelson (1999) for more on Gibbs sampling.

with $\nu_1 = 10$ and $\delta_1 = 0.001$, implying that the mean of σ_ϵ is 0.011 and that the 95 percent confidence interval is 0.007 to 0.018. The prior of σ_η^2 is given by

$$IG\left(\frac{\nu_2}{2}, \frac{\delta_2}{2}\right)$$

with $\nu = 10$ and $\delta = 0.1$, implying that the mean of σ_η is 0.118 and that the 95 percent confidence interval is 0.071 to 0.176.

2.2.2 Computational algorithm

The above assumptions about the priors and the data generating process, which is given by equations (1) and (2), provide us with posterior conditional distributions that are needed to implement Gibbs sampling. The following steps 1 through 5 are iterated to obtain the joint and marginal distributions of the parameters and the auxiliary Variables. The summations are taken from $(t, h) = (1, 1)$ to $(T, 24)$, unless otherwise stated.

Step 1 Generate α conditional on $s_{t,h}$, $I_{t,h}$, and σ_ϵ^2. We have the regression $s_{t,h} - s_{t,h-1} = \alpha I_{t,h} + \epsilon_{t,h}$. Hence, the posterior distribution is $\alpha \sim N(\phi_s, \omega_s)$ where $\phi_s = \sum I_{t,h}(s_{t,h} - s_{t,h-1}) / \sum I_{t,h}^2$ and $\omega_s = \sigma_\epsilon^2 / \sum I_{t,h}^2$.

Step 2 Generate σ_ϵ^2 conditional on $s_{t,h}$, $I_{t,h}$, and α. The posterior is $\sigma_\epsilon^2 \sim IG\left(\frac{\nu_s}{2}, \frac{\delta_s}{2}\right)$ where $\nu_s = \nu_1 + T$ and $\delta_s = \delta_1 + RSS_s$ with $RSS_s = \sum(s_{t,h} - s_{t,h-1} - \alpha I_{t,h})^2$.

Step 3 Generate β conditional on $s_{t,h}$, $I_{t,h}$, and σ_η^2. We have the regression $I_{t,h} = \beta(s_{t,h-1} - s_{t-1,h-1}) + \eta_{t,h}$. Hence, the posterior distribution is $\beta \sim N(\phi_I, \omega_I)$ where $\phi_I = \sum I_{t,h}(s_{t,h-1} - s_{t-1,h-1}) / \sum(s_{t,h-1} - s_{t-1,h-1})^2$ and $\omega_I = \sigma_\eta^2 / \sum(s_{t,h-1} - s_{t-1,h-1})^2$.

Step 4 Generate σ_η^2 conditional on $s_{t,h}$, $I_{t,h}$, and β. The posterior distribution is $\sigma_\eta^2 \sim IG\left(\frac{\nu_I}{2}, \frac{\delta_I}{2}\right)$ where $\nu_I = \nu_2 + T$ and $\delta_I = \delta_2 + RSS_I$ with $RSS_I = \sum(I_{t,h} - \beta(s_{t,h-1} - s_{t-1,h-1}))^2$.

Step 5 Generate $I_{t,h}$ conditional on I_t, α, β, σ_ϵ^2, and σ_η^2. Consider the case in which the aggregated intervention amount is not known. Then, the posterior distribution is as follows:

$$(I_{t,1}, ..., I_{t,24})' \sim N(\Xi_t, \Psi)$$

where $\Xi_t = (\xi_{t,1}, ..., \xi_{t,24})'$ and $\Psi = diag(\varphi, ..., \varphi)$ with $\xi_{t,h} = \frac{1}{\sigma_\eta^2}\beta(s_{t,h-1} - s_{t-1,h-1}) + \frac{\alpha^2}{\sigma_\epsilon^2}\alpha^{-1}(s_{t,h} - s_{t,h-1})$ and $\varphi = (\frac{1}{\sigma_\eta^2} + \frac{\alpha^2}{\sigma_\epsilon^2})^{-1}$. We consider the posterior distribution of $(I_{t,1}, ..., I_{t,23}, I_t)$.

Notice that when we know $(I_{t,1}, ..., I_{t,23}, I_t)$, the intervention in the last hour, $I_{t,24}$, is already determined. The posterior distribution is as follows:

$$(I_{t,1}, ..., I_{t,23}, I_t)' \sim N\left(\Xi_t^*, \Psi^*\right)$$

where $\Xi_t^* = B\Xi_t$ and $\Psi^* = B'\Psi B$ with

$$B = \begin{bmatrix} 1 & 0 & ... & 0 \\ 0 & 1 & ... & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & ... & 1 \end{bmatrix}. \tag{5}$$

We can partition the matrices Ξ_t^* and Ψ^* as follows:

$$\Xi_t^* = \begin{bmatrix} \Xi_{t,1}^* \\ \Xi_{t,2}^* \end{bmatrix}, \quad \Psi^* = \begin{bmatrix} \Psi_{11}^* & \Psi_{21}^* \\ \Psi_{12}^* & \Psi_{22}^* \end{bmatrix},$$

where $\Xi_{t,1}^*$ is 23×1, $\Xi_{t,2}^*$ is 1×1, Ψ_{11}^* is 23×23, Ψ_{12}^* is 1×1, Ψ_{21}^* is 1×1, and Ψ_{22}^* is 1×1. Finally, we can construct the posterior distribution of $(I_{t,1}, ..., I_{t,23})$ conditional on I_t as follows:

$$(I_{t,1}, ..., I_{t,23} |\ I_t)' \sim N\left(\Xi_{t,1}^* + \Psi_{12}^*(\Psi_{22}^*)^{-1}(I_t - \Xi_{t,2}^*), \Psi_{11}^* - \Psi_{12}^*(\Psi_{22}^*)^{-1}\Psi_{21}^*\right).$$

By generating the auxiliary variables $I_{t,1}, ..., I_{t,23}$ from this posterior distribution conditional on the parameters and the aggregated intervention, we can construct the intervention in the last hour as $I_{t,24} = I_t - \Sigma_{h=1}^{23} I_{t,h}$.

We iterate steps 1 through 5 $M + N$ times and discard the realizations of the first M iterations but keep the last N iterations to form a random sample of size N on which statistical inference can be made. M must be sufficiently large so that the Gibbs sampler converges. Also, N must be large enough to obtain the precise empirical distribution. In our simulations, we set $M = 2000$ and $N = 2000$ and run 3 independent Markov chains.

3 Simulation Analysis

In this section we conduct Monte Carlo simulations to evaluate the performance of our methodology. We start by assuming that the data generating process is given by equations (1) and (2) with $s_{0,24} = \ln(100)$, $\alpha = -0.015$, $\beta = 3.2$, $\sigma_\epsilon = 0.01$, and $\sigma_\eta = 0.1$. We borrow the estimates of α and

β from Kearns and Rigobon (2005):[6] $\alpha = -0.015$ implies that a 1 trillion yen intervention by the Japanese monetary authorities moves the yen/dollar rate by 1.5 percent; on the other hand, $\beta = 3.2$ implies that a one percent deviation of the exchange rate from its target level causes the Japanese monetary authorities to intervene with 32 billion yen.

We generate bivariate time series $\{s_{t,h}, I_{t,h}\}$ by (1) and (2). The length of the time series is set at 100 days (T=100), and 500 replications of this length are generated. We repeat this for T=250 and 500. We then estimate the unknown parameters under the following three cases. The first one is what we refer to as the "infeasible estimator." We assume that the hourly amount of intervention, $I_{t,h}$, is observable to us, and we simply apply OLS to the hourly data of intervention and exchange rates. This estimator can be seen as the best one (although it is infeasible), and will be used as a benchmark. The second case we refer to as the "naive OLS estimator," where we assume that intervention data is available only at the daily frequency, and we apply OLS to the daily intervention and exchange rate data. Specifically, we estimate equations (3) and (4) separately. This estimator suffers from the endogeneity problem, as explained earlier. The third case we refer to as the "MCMC estimator," where we assume that exchange rate data is available at the hourly frequency, but intervention data is available only at the daily frequency, and we apply our MCMC method to these data.

Table 1 presents the simulation results. We evaluate the performance of the three estimators in terms of bias, which is defined to be the average deviation of an estimator from its true value, as well as the root mean squared error.[7] We can see that the infeasible estimator is unbiased and precise in the sense that both the bias and the root mean squared error are small, while the naive OLS estimator performs much worse. The naive OLS estimators of α and β have upward and downward biases, respectively, and we see no clear tendency that these biases become smaller with the sample size T. In contrast, the MCMC estimator performs as well as the infeasible estimator: the bias is almost the same as in the case of the infeasible estimator; the root mean squared error is slightly larger, but the difference tends to become smaller with T.

[6] We divide their estimate of β by 24 to convert their estimate, which is based on a daily frequency, to one based on an hourly frequency.

[7] The MCMC method provides us with a posterior distribution for each of the parameters. We use the mean of the distribution as a point estimate.

Table 1: Finite sample properties of the three estimators

		Infeasible estimator		Naive OLS estimator		MCMC estimator	
		Bias	$\sqrt{MSE}$	Bias	$\sqrt{MSE}$	Bias	$\sqrt{MSE}$
$T = 100$	α	0.0000	0.0013	0.0102	0.0238	-0.0001	0.0018
	β	0.0055	0.0602	-3.6671	6.6171	0.0039	0.0859
$T = 250$	α	0.0000	0.0009	0.0102	0.0237	0.0000	0.0012
	β	0.0091	0.0387	-4.0189	5.1603	0.0075	0.0569
$T = 500$	α	0.0001	0.0006	0.0103	0.0238	0.0001	0.0008
	β	0.0065	0.0274	-4.3055	4.8954	0.0034	0.0401

Note: "Bias" is defined to be the deviation of each estimator from the true value. For example, the bias associated with α is equal to the mean of estimators of α over 500 replications minus its true value, namely -0.015. "$\sqrt{MSE}$" represents the root mean squared error for each estimator. We estimate 3 chains from independent starting points in each replication. Each chain runs 4000 draws and the first 2000 are discarded as the burn-in-phase.

4 Application to Japanese Data

4.1 Policy reaction function with transaction costs

Figures 1 and 2 show the hourly movement of the yen/dollar rate, and the daily amount of intervention, both for the period from April 1991 to December 2002.[8] In applying our method to the Japanese data, we modify the model described by equations (1) and (2) in the following way. Equation (2) implies that interventions are every-day events: namely, the central bank intervenes (by a small amount) even on "quiet" days when the exchange rate is fairly stable. But this is not consistent with the fact that interventions were carried out only on 7 percent of the total business days, that is, 214 out of 3,055 business days, during the sample period. In this sense, Japanese interventions have an "all or nothing" property, suggesting that we need to incorporate some form of transaction costs associated with the conduct of interventions.

Specifically, following Almekinders and Eijffinger (1996) and Ito and Yabu (2007), we assume that the Japanese monetary authorities have to pay some fixed costs on intervention days, in the form of political costs. These political costs may include, for example, the costs incurred by the Japanese government in conducting negotiation with governments of relevant countries, as pointed

[8]Our sample period does not include the period of "Great Intervention" in 2003 and 2004, during which the Japanese monetary authorities aggressively purchased US dollars and sold yen as a part of their "quantitative easing" policy. Previous studies argue that the central bank's motivation for these interventions was quite different from the one in the preceding period. See Taylor (2006) for more on the intervention policy during this period.

out by Ito and Yabu (2007). The Japanese monetary authorities are assumed to compare the benefits of intervention (greater stability in the exchange rate) and the fixed costs they have to incur in implementing interventions. As is well known, a solution to this type of optimization with fixed costs is characterized by a state-dependent rule: namely, the monetary authorities carry out interventions only when the optimal level of intervention for that day exceeds a threshold. In our baseline regression we use a state-dependent rule of the form:

$$I_{t,h} = 1(|I^*_{t,1} - \mu_I| > c) I^*_{t,h} \tag{6}$$

$$I^*_{t,h} = \mu_I + \beta(s_{t,h-1} - s_{t-1,h-1}) + \rho I_{t-1} + \eta_{t,h} \tag{7}$$

Equation (7) describes how the optimal level of intervention, $I^*_{t,h}$, is determined, and equation (6) represents a state-dependent policy reaction function, where $1(\cdot)$ represents a zero-one indicator function. In equation (7), we assume that the optimal level of intervention depends on the change in the exchange rate over the last 24 hours. We also allow autocorrelation between intervention today and intervention yesterday so as to capture the tendency for interventions to be clustered, which was highlighted by Fatum and Hutchison (2003). In equation (6), we assume that intervention is carried out if the optimal level of intervention at the beginning of a day, $I^*_{t,1}$, exceeds a prespecified threshold c, which is determined by the size of the political costs. Note that $I_{t,h}$ equals $I^*_{t,h}$ for any h as long as $I^*_{t,1}$ exceeds the threshold. In other words, once the monetary authorities decide to intervene on day t at the beginning of that day, they are allowed to intervene for every hour of day t without incurring any extra political costs. In this sense, the monetary authorities' decision on whether to intervene or not is made only once a day, although the amount of intervention for every hour of the day is decided during the daytime depending on fluctuations in the exchange rate over the course of the day.

4.2 Change in intervention strategy

Figure 2 shows that there is a structural break somewhere around 1995: interventions are small in size but frequent during the former period, while they are larger in size but less frequent during the latter period. As mentioned by Ito (2003), among others, this break corresponds to a replacement of the person in charge of the conduct of interventions in June 1995.[9] The number of days when

[9] On June 21, 1995, Eisuke Sakakibara was appointed as Direct General of the International Finance Bureau of the Ministry of Finance. Regarding exchange rate interventions, he later wrote: "The market was accustomed to interventions, because they were too frequent. The interventions were taken as given. Most interventions, including joint interventions, were predictable, so that interventions, even joint ones, had only small, short-term effects, and

interventions were carried out is 165 out of 1,101 business days during the period from April 1991 to June 1995, so that the probability of intervention was 0.15. In the latter period, the corresponding probability was 0.03 (49 intervention days out of 1,954 business days). On the other hand, the average yen amount of interventions on days when such interventions were conducted was 0.05 trillion yen in the former period and 0.52 trillion yen in the latter period. Kearns and Rigobon (2005) make use of this shift in the Japanese intervention policy as a key piece of information in identifying the effects of Japanese intervention on the yen/dollar rate.

We incorporate this structural change in the policy reaction function as follows:

$$I_{t,h} = \begin{cases} 1(|I^*_{t,1} - \mu_I| > c_1) I^*_{t,h} & \text{for } t < T_B \\ 1(|I^*_{t,1} - \mu_I| > c_2) I^*_{t,h} & \text{for } t \geq T_B \end{cases} \tag{8}$$

where T_B is the break date (namely, June 1995), and c_1 and c_2 are different thresholds for the two subperiods. Here we assume that the change in the Japanese policy reaction function is represented solely by a change in threshold c, or the size of political costs, and that the other parameters are identical across the two subperiods. We make this assumption simply to obtain empirical results that are comparable to those of Kearns and Rigobon (2005), whose identification method requires such an assumption. Note that our identification method does not require us to impose this assumption.

4.3 Baseline results

In our baseline regression, we use equations (7) and (8), together with the following equation describing exchange rate dynamics:

$$s_{t,h} - s_{t,h-1} = \mu_s + \alpha I_{t,h} + \epsilon_{t,h}. \tag{9}$$

Table 2 presents the results. We run regressions with and without the lagged intervention term I_{t-1}, with the left half of the table showing the result without that term, and the right half showing that with that term. Details regarding the algorithm used are provided in the Appendix.[10]

Table 2 shows that the coefficient on the intervention variable, α, in equation (9) is negative and significantly different from zero in the sense that the 95 percent posterior interval does not include

could not change the sentiment of the market." (Sakakibara, 2000, p.119) "[A] change in intervention philosophy and technique [was introduced]. For this, all I had to do was to make a decision and convince the Vice Minister and the Minister of [its desirability]. For one, the frequency of interventions was reduced substantially, and the per-intervention amount was increased, in order to push up the level [of the dollar vis-à-vis the yen]" (Sakakibara, 2000, p.120).

[10] We iterate 20,000 times and discard the first 10,000 realizations ($M = 10,000$ and $N = 10,000$). We run 5 independent Markov chains and report the Gelman-Rubin statistic $\hat{R}$ to monitor the convergence of the Markov chains. $\hat{R} < 1.1$ is considered as a sign of convergence. See Gelman et al. (2003) for details. We confirm that convergence is accepted in every case.

Table 2: Baseline Results

	Without lagged intervention term				With lagged intervention term			
	Mean	Std. Dev.	Pr(< 0)	$\hat{R}$	Mean	Std. Dev.	Pr(< 0)	$\hat{R}$
				Equation for exchange rate dynamics				
α	-0.0174 [-0.0189, -0.0158]	0.0008	1.0000	1.063	-0.0176 [-0.0192, -0.0162]	0.0008	1.0000	1.017
				Equation for policy reaction function				
β	0.2253 [0.0828, 0.3743]	0.0740	0.0011	1.003	0.2134 [0.0751, 0.3546]	0.0713	0.0010	1.001
ρ					0.0140 [-0.0026, 0.0306]	0.0084	0.0498	1.000
c_1	0.1023 [0.0929, 0.1136]	0.0053	0.0000	1.058	0.1011 [0.0918, 0.1100]	0.0047	0.0000	1.018
c_2	0.1697 [0.1546, 0.1881]	0.0085	0.0000	1.067	0.1679 [0.1526, 0.1838]	0.0080	0.0000	1.016

Note: Constants are estimated but not reported. The columns labeled "Mean" and "Std. Dev." refer to the mean and standard deviation of the marginal distribution of a parameter. The columns labeled "Pr(< 0)" refer to the frequency of finding negative values. The columns labeled $\hat{R}$ refer to the Gelman-Rubin statistic to monitor the convergence of the Markov chains. $\hat{R} < 1.1$ is considered as a sign of convergence. The values in the brackets are the 95 percent posterior bands of the parameter. We estimate 5 chains from independent starting points. Each chain runs $20,000$ draws and the first half is discarded as the burn-in-phase.

zero. Note that the frequency of finding negative values, Pr(< 0), equals unity, indicating that we find not one positive values in 10,000 draws. The estimated value of α is equal to -0.0174, implying that a yen-selling (yen-buying) intervention of one trillion yen leads to a 1.74 percent depreciation (appreciation) of the yen. The estimate is robust to changes in the specification, i.e., whether a lagged intervention term is included or not.

Our estimate regarding the impact of foreign exchange interventions is more than twice as large as that obtained in previous studies. Ito (2003), for example, applying OLS to daily data of Japanese interventions and the yen/dollar rate, arrived at a corresponding change of 0.6 percent for the sample period of April 1991 to March 2001 and 0.9 percent for the subperiod from June 1995 to March 2001. Similarly, Fratzscher (2005), applying a similar regression as Ito (2003 using daily data for the period 1990-2003, found that Japanese interventions of ten billion dollars, which is approximately equal to one trillion yen, moves the yen/dollar rate by 0.8 percent. Our much larger estimation result suggests that these previous studies suffer from the endogeneity problem, so that the effectiveness of interventions on the exchange rate was biased toward zero.[11]

Turning to the coefficients in the policy reaction function, we find, first, that the coefficient on the change in the exchange rate, β, is positive and significantly different from zero, indicating that a leaning-against-the-wind policy was adopted by the Japanese monetary authorities during this sample period, and that they sold (purchased) about 53 billion yen in a day in response to a one percent appreciation (depreciation) of the yen. Second, the estimates of c_1 and c_2 are both positive, as predicted, and, more importantly, c_2 is significantly larger than c_1, suggesting that the fact that interventions during the latter sample period were larger but less frequent was due to the greater political costs. Third, the estimate of autocorrelation between interventions, represented by ρ, is estimated to be positive, as expected, but not significantly different from zero in terms of the 95 percent confidence interval; however, the probability of it being below zero, $\Pr(< 0)$, is slightly less than 5 percent, indicating that it is still significantly different from zero as far as the one side test is concerned.

Our MCMC approach gives us a posterior distribution for the auxiliary variable, $I_{t,h}$, for each t and h. Figure 3 shows the estimates of this variable for each hour on April 10, 1998, when the Japanese monetary authorities purchased 2.6 trillion yen, the largest yen-buying intervention in our sample period. The red solid line represents the mean of the posterior distribution of $I_{t,h}$, while the red dotted lines represent the 95 percent confidence interval. We see that the estimated hourly amount of intervention is almost always positive (i.e., almost all interventions are yen-buying interventions), and that it tends to be larger when the yen is weaker, which is consistent with equation (7). However, the estimated hourly amount takes the largest value, 0.5 trillion yen, at 6-7 am GMT (or 2-3 pm in Tokyo), and this is exactly the time when the yen exhibits a sharp appreciation and records its highest level on this day. This can be interpreted as aggressive yen-buying intervention during this hour causing a sharp appreciation, which is consistent with equation (9). Put differently, our MCMC approach does "data-augmentation" for $I_{t,h}$ so that the estimates of the hourly amount of intervention become consistent both with equations (7) and (9), given intraday fluctuations in the exchange rate.

Figure 4 shows the movement of the yen/dollar rate before and after the hour that a yen-selling

[11] Kearns and Rigobon (2005), who identified the effects of intervention by making use of a structural change in the policy reaction function, report that an intervention of one billion dollars moves the yen/dollar rate by 1.5 percent, which is relatively close to our estimate, although it is still outside our 95 percent confidence interval.

Table 3: Intensive and Extensive Margins of Japanese Interventions

	1991-2002	1991-1995 (A)	1995-2002 (B)	B/A
Yen-amount per business day [trillion]	0.010	0.007	0.012	1.84
Probability of intervention *day*	0.070	0.149	0.025	0.16
Yen-amount per intervention *day* [trillion]	0.155	0.047	0.519	11.06
Probability of intervention *hour*	0.079	0.061	0.138	2.25
Yen-amount per intervention *hour* [trillion]	0.081	0.032	0.156	4.89

Note: "Yen-amount of intervention per business day" is defined as the total amount of intervention during the observation period divided by the number of business days. "Probability of intervention day" is defined as the number of intervention days divided by the number of business days. "Yen-amount per intervention day" is defined as the total amount of intervention during the observation period divided by the number of intervention days. "Probability of intervention hour" is defined as the number of intervention hours divided by the number of intervention days multiplied by 24. "Yen-amount per intervention hour" is defined as the total amount of intervention during the observation period divided by the number of intervention hours.

intervention is carried out. For the figure, we collect the estimates of $I_{t,h}$ for 148 business days when yen-selling interventions were reported to have been implemented. We then identify h when the estimate of $I_{t,h}$ exceeds a certain threshold, which in this case is given by the 99 percent confidence interval. Note that $\tau = 0$ in the figure represents the hour of intervention and that the yen/dollar rates are divided by the levels at the hour of intervention for normalization. The solid line represents the 50th percentile, or median, of the distribution of the exchange rate, while the two dotted lines represent the 40th and 60th percentiles, respectively. The figure shows that there is a trend of yen appreciation prior to the hour of intervention, indicating that the Japanese monetary authorities adopt a leaning-against-the-wind policy in the sense that they sell yen and purchase dollars so as to prevent the yen from appreciating further. It also shows that the value of the yen falls very quickly in response to the intervention and stays there at least twelve hours after the intervention. This indicates that interventions have a persistent effect on the level of the yen/dollar rate, even though the effect on the change in the exchange rate is only temporary. These results are all consistent with the estimated parameters reported in Table 2 and, more importantly, can be seen as indirect evidence that the timing of intervention is correctly estimated by our method.

Figure 5 shows the cumulative distribution functions of the number of interventions per day for the two subperiods.[12] In the figure, the horizontal axis represents the number of interventions (one-hour intervals in which an intervention, or interventions, occur) per day, while the vertical axis

[12] In Figure 5, we count the number of interventions exceeding the threshold given by the 90 percent confidence interval.

shows the probability that this number, or a smaller number, of interventions occur on a given day. The figure shows that multiple interventions per day are not a rare phenomenon at all; namely, the probability of two or more interventions per day is 0.33 for the pre-1995 period, and 0.77 for the post-1995 period. The figure also shows that there exists a substantial difference between the two subperiods. On average, intervention occurred 1.5 times a day during the pre-1995 period,[13] while they occurred 3.3 times a day during the post-1995 period.

Next, we decompose the yen-amount of intervention per business day into an extensive margin (i.e., the probability of intervention for a given day) and an intensive margin (i.e., the yen-amount per intervention day) and compare these for the two subperiods. The results are shown in Table 3.

As can be seen, the post-1995 period is characterized by a lower extensive margin and a higher intensive margin; this confirms what we saw in Figure 2. But in addition, we can conduct a similar decomposition at the hourly frequency using our estimate of $I_{t,h}$, and the results are shown on the two rows from the bottom. The yen amount per intervention day is decomposed into an extensive margin (the probability of interventions in a given hour on a day that interventions were conducted) and an intensive margin (the yen-amount per intervention hour).[14] Interestingly, although part of the larger yen-amount per intervention day comes from the larger extensive margin, it mostly derives from the larger intensive margin. Interestingly, the larger yen-amount per intervention day in the post-1995 period comes partly from the larger extensive margin, but mostly from the larger intensive margin. It could therefore be said that the latter period is characterized by a higher intensive margin not only at the daily frequency, but also at the hourly frequency.

4.4 Alternative specifications of the policy reaction function

An important feature of our MCMC approach is that we make use of the knowledge about the structure of the economy, which is represented by the equation for exchange rate dynamics and the equation for policy reaction function. This implies that the performance of the entire estimation process crucially depends on whether the structure of the economy is properly specified or not. In this subsection we will check the sensitivity of the baseline results to various changes in the specification of the policy reaction function.

[13]This figure is relatively close to Chang and Taylor's (1998) finding based on Reuters reports that the Japanese central bank on average intervened 2.2 times a day in 1992-1993. See footnote 4.

[14]Note that the product of the two margins equals the yen amount per intervention day divided by 24.

4.4.1 Higher political costs at night

One possible factor determining the reaction function is that political costs are higher at night. Neely (2001) presents survey results from central banks about various issues related to foreign exchange intervention. Among the questions he included is one that asked at what times of the day interventions were conducted. He provided the following optios: "prior to normal business days," "morning of the business day," "afternoon of the business day," and "after normal business hours." One of the interesting features we learn from the responses to this question is that about 56 percent of central banks answered that they *never* intervene "prior to normal business days," and similarly about 35 percent answered that they *never* intervene "after normal business hours." Yet, it is possible that the intervention strategy of the Japan's monetary authorities are quite different from that of other monetary authorities because of Japan's geographical location. However, various pieces of anecdotal evidence regarding the intervention behavior of Japan's monetary authorities suggest that they are active during hours in which the Tokyo market is open, while they are much less active during other hours, which is more or less similar to what Neely's (2001) survey results indicate.

The fact that central banks seldom intervene during night hours may be interpreted as reflecting that the political costs are higher at night than during the daytime, so that central banks hesitate to intervene at night even if the optimal level of intervention is not zero. If this is the case, our assumption regarding political costs may be inappropriate. That is, in the previous subsection we assumed that the Japanese monetary authorities incur political costs at the beginning of a day, and once such costs have been incurred at that time, they are allowed to intervene at any time of that day, including night time, *without* incurring any additional political costs. An alternative specification would be that the Japanese monetary authorities incur additional political costs, which are very high (probably prohibitively high), when they intervene at night.

Based on this line of reasoning, we assume that $I_{t,h}$ is equal to zero at night ($h = 9, \ldots, 24$, or between 6 pm and 9 am Tokyo time). Specifically, we replace equation (8) by:

$$I_{t,h} = \begin{cases} 1(|I^*_{t,1} - \mu_I| > c_1)I^*_{t,h} & \text{for} \quad h = 1, \ldots, 8, \text{ and } \quad t < T_B \\ 1(|I^*_{t,1} - \mu_I| > c_2)I^*_{t,h} & \text{for} \quad h = 1, \ldots, 8, \text{ and } \quad t \geq T_B \\ 0 & \text{for} \quad h = 9, \ldots, 24 \end{cases} \tag{10}$$

and repeat the same exercise as before. The regression result is presented in Table 4, showing that the baseline result obtained earlier is not sensitive to this change in the policy reaction function. The

Table 4: Intervention Only in the Tokyo Market

	Without lagged intervention				With lagged intervention			
	Mean	Std. Dev.	Pr(< 0)	$\hat{R}$	Mean	Std. Dev.	Pr(< 0)	$\hat{R}$
	Equation for exchange rate dynamics							
α	-0.0117 [-0.0127, -0.0107]	0.0005	1.0000	1.034	-0.0117 [-0.0127, -0.0107]	0.0005	1.0000	1.017
	Equation for policy reaction function							
β	0.5251 [0.1707, 0.8797]	0.1799	0.0019	1.004	0.5205 [0.1584, 0.8708]	0.1828	0.0021	1.004
ρ					0.0422 [0.0001, 0.0827]	0.0211	0.0247	1.002
c_1	0.1491 [0.1361, 0.1638]	0.0070	0.0000	1.045	0.1499 [0.1365, 0.1651]	0.0073	0.0000	1.017
c_2	0.2526 [0.2327, 0.2764]	0.0113	0.0000	1.043	0.2541 [0.2326, 0.2786]	0.0118	0.0000	1.020

coefficient associated with the effectiveness of intervention, α, is negative and significantly different from zero, as before, although it is now a little smaller, indicating that an intervention of one trillion yen moves the yen/dollar rate by 1.17 percent. Second, the coefficient on the change in the exchange rate in the policy reaction function, β, is positive and significantly different from zero, indicating again that the Japanese monetary authorities adopt a leaning-against-the-wind policy. Third, the coefficients related to the size of the political costs, c_1 and c_2, are both positive and significantly different from zero, as before, and the political costs are significantly larger in the latter sample period.

4.4.2 Alternative forms of the optimal intervention function

Next, we consider alternative forms of the optimal intervention function. Equation (7), which is basically identical to the intervention function adopted by Kearns and Rigobon (2005), may be too simple to capture details of Japanese intervention policy. Ito and Yabu (2007) propose a policy reaction function that can be regarded as a better approximation to the Japanese policy reaction function. Specifically, they assume that the optimal amount of intervention depends on the deviation of the actual exchange rate from its target level, which is determined by the weighted average of

Table 5: Alternative Specification of Optimal Intervention Function

	Without lagged intervention				With lagged intervention			
	Mean	Std. Dev.	Pr(< 0)	$\hat{R}$	Mean	Std. Dev.	Pr(< 0)	$\hat{R}$
				Equation for exchange rate dynamics				
α	-0.0175 [-0.0190, -0.0159]	0.0008	1.0000	1.059	-0.0174 [-0.0188, -0.0159]	0.0008	1.0000	1.029
				Equation for policy reaction function				
β_1	0.1494 [-0.0178, 0.3197]	0.0858	0.0400	1.002	0.1572 [-0.0106, 0.3274]	0.0863	0.0333	1.004
β_2	0.0757 [-0.0295, 0.1819]	0.0544	0.0823	1.003	0.0690 [-0.0356, 0.1766]	0.0548	0.1041	1.003
β_3	-0.0038 [-0.0499, 0.0432]	0.0236	0.5639	1.005	-0.0057 [-0.0510, 0.0388]	0.0229	0.5975	1.002
ρ					0.0128 [-0.0041, 0.0297]	0.0086	0.0689	1.001
c_1	0.1017 [0.0920, 0.1125]	0.0053	0.0000	1.054	0.1027 [0.0933, 0.1132]	0.0051	0.0000	1.028
c_2	0.1689 [0.1538, 0.1865]	0.0083	0.0000	1.057	0.1704 [0.1556, 0.1870]	0.0081	0.0000	1.025

$s_{t-1,h-1}$, $s_{t-21,h-1}$, and $s^{MA}_{t-1,h-1}$, where $s^{MA}_{t,h}$ is defined as the moving average of the exchange rate over the last one year. To incorporate this idea into our model, we replace equation (7) by:

$$I^*_{t,h} = \mu_I + \beta_1(s_{t,h-1} - s_{t-1,h-1}) + \beta_2(s_{t,h-1} - s_{t-21,h-1}) + \beta_3(s_{t,h-1} - s^{MA}_{t,h-1}) + \rho I_{t-1} + \eta_{t,h} \tag{11}$$

Note that equation (7) is a special case of the above equation with both of β_2 and β_3 being equal to zero.

We conduct the same exercise as before and the results are presented in Table 5. We confirm two features of the baseline results: the coefficient associated with the effectiveness of intervention, α, is negative and significantly different from zero; the coefficients related to political costs, c_1 and c_2, are both positive and significantly different from zero. Turning to the new coefficients in the policy reaction function, β_1 and β_2 are both positive as before, but β_3 is almost zero, indicating the Japanese monetary authorities take a leaning-against-the-wind posture with respect to changes in the exchange rate at the daily and monthly frequency, but not at the annual frequency.[15]

[15]Ito (2003) estimates a policy reaction function that is similar to equation (11), but with simple OLS, and obtains

5 Conclusion

Estimating the effects of central bank interventions is not an easy task because the central bank reacts even to intraday changes in the exchange rate, while intervention data at best is available at the daily frequency. In this paper, we therefore proposed a new methodology based on Markov Chain Monte Carlo simulation to cope with this endogeneity problem. We first conduct "imputation" or "data augmentation" to obtain intraday amounts of intervention and then estimate the efficacy of interventions using the augmented data. Applying this method to Japanese intervention data, we found that an intervention of one trillion yen moves the yen/dollar rate by 1.7 percent, which is more than twice as large as the magnitude reported in previous studies applying OLS to daily observations. We interpreted this difference as highlighting the quantitative importance of the endogeneity problem due to temporal aggregation. Some previous studies pursued the idea of augmenting the observed low-frequency data with simulated high-frequency data, especially in the area of finance, but this paper is the first attempt to apply this idea in addressing the endogeneity problem.

Our method should work well in an environment in which the true structure of the economy, including the policy reaction function, is well known to researchers. Our empirical analysis demonstrated that this is indeed the case for Japanese foreign exchange interventions. However, given that not much research has been conducted on central banks' intraday behavior, our knowledge about this is still limited. Specifically, in our empirical section, we needed to make guesses regarding the form of the reaction function at the hourly frequency, assuming that this was similar to the reaction function at the daily frequency, which has been extensively investigated in previous studies. However, it cannot be ruled out that monetary authorities behave differently depending on the time frame (i.e., interday or intraday). It is our future task to accumulate knowledge about this.

$\beta_1 = 1.0422$; $\beta_2 = 0.1369$; $\beta_3 = 0.0632$.

A Estimation procedure of the model with political costs

The methodology for estimating the model without political costs given by equations (1) and (2) is presented in Section 2.2. The purpose of this appendix is to provide details regarding the estimation of the model with political costs represented by equations (6) and (7). The parameters to be estimated are μ_s, α, μ_I, β, c_1, c_2, σ_ϵ^2, and σ_η^2. In addition to these parameters, we estimate auxiliary variables, $I_{t,h}$ and $I^*_{t,h}$, for each h and t. A flat prior is adopted for μ_s, α, μ_I, β, c_1, and c_2. As for σ_ϵ^2 and σ_η^2, priors are the same as those used in Section 3.

The posterior conditional distributions, which are needed to implement Gibbs Sampling, are obtained from the priors and the assumptions of the data generating process. The following steps 1 through 6 are iterated to obtain joint and marginal distributions of the parameters and the auxiliary variables.

Step 1 Generate μ_s and α conditional on $s_{t,h}$, $I_{t,h}$, and σ_ϵ^2. We have the regression $s_{t,h} - s_{t,h-1} = \mu_s + \alpha I_{t,h} + \epsilon_{t,h}$. Hence, the posterior distribution is $(\mu_s, \alpha)' \sim N(\phi_s, \omega_s)$ where $\phi_s = (X_s'X_s)^{-1}X_s'Y_s$ and $\omega_s = (X_s'X_s)^{-1}\sigma_\epsilon^2$ with the matrices $X_s = \{1, I_{t,h}\}$ and $Y_s = \{s_{t,h} - s_{t,h-1}\}$.

Step 2 Generate σ_ϵ^2 conditional on $s_{t,h}$, $I_{t,h}$, μ_s, and α. The posterior is $\sigma_\epsilon^2 \sim IG\left(\frac{\nu_s}{2}, \frac{\delta_s}{2}\right)$ where $\nu_s = \nu_1 + T$ and $\delta_s = \delta_1 + RSS_s$ with $RSS_s = \sum(s_{t,h} - s_{t,h-1} - \mu_s - \alpha I_{t,h})^2$.

Step 3 Generate μ_I and β conditional on $s_{t,h}$, $I^*_{t,h}$, and σ_η^2. We have the regression $I^*_{t,h} = \mu_I + \beta(s_{t,h-1} - s_{t-1,h-1}) + \eta_{t,h}$. Hence, the posterior distribution is $(\mu_I, \beta)' \sim N(\phi_I, \omega_I)$ where $\phi_I = (X_I'X_I)^{-1}X_I'Y_I$ and $\omega_I = (X_I'X_I)^{-1}\sigma_\eta^2$ with the matrices $X_I = \{1, s_{t,h-1} - s_{t-1,h-1}\}$ and $Y_I = \{I^*_{t,h}\}$.

Step 4 Generate σ_η^2 conditional on $s_{t,h}$, $I^*_{t,h}$, μ_I, and β. The posterior is $\sigma_\eta^2 \sim IG\left(\frac{\nu_I}{2}, \frac{\delta_I}{2}\right)$ where $\nu_I = \nu_2 + T$ and $\delta_I = \delta_2 + RSS_I$ with $RSS = \sum(I^*_{t,h} - \mu_I - \beta(s_{t,h-1} - s_{t-1,h-1}))^2$.

Step 5 Generate $I_{t,h}$ and $I^*_{t,h}$ conditional on $s_{t,h}$, I_t, μ_s, α, μ_I, β, c_1, c_2, σ_ϵ^2, and σ_η^2. Consider the case without the political costs. The posterior distribution without knowing I_t is as follows:

$$(I_{t,1}, ..., I_{t,24})' \sim N(\Xi_t, \Psi)$$

where $\Xi_t = (\xi_{t,1}, ..., \xi_{t,24})'$ and $\Psi = diag(\varphi, ..., \varphi)$ with $\xi_{t,h} = \frac{1}{\sigma_\eta^2}(\mu_I + \beta(s_{t,h-1} - s_{t-1,h-1})) + \frac{\alpha^2}{\sigma_\epsilon^2}\alpha^{-1}(s_{t,h} - s_{t,h-1} - \mu_s)$ and $\varphi = (\frac{1}{\sigma_\eta^2} + \frac{\alpha^2}{\sigma_\epsilon^2})^{-1}$. Hence, the posterior distribution of $(I_{t,1}, ..., I_{t,23}, I_t)$

is as follows:

$$(I_{t,1},...,I_{t,23},I_t)^{'} \sim N\left(\Xi^*,\Psi^*\right)$$

where $\Xi_t^* = B\Xi_t$ and $\Psi^* = B'\Psi B$ with B defined by (5). We can partition the matrices Ξ_t^* and Ψ^* as follows:

$$\Xi_t^* = \begin{bmatrix} \Xi_{t,1}^* \\ \Xi_{t,2}^* \end{bmatrix}, \quad \Psi^* = \begin{bmatrix} \Psi_{11}^* & \Psi_{21}^* \\ \Psi_{12}^* & \Psi_{22}^* \end{bmatrix}$$

where $\Xi_{t,1}^*$ is 23×1, $\Xi_{t,2}^*$ is 1×1, Ψ_{11}^* is 23×23, Ψ_{12}^* is 1×1, Ψ_{21}^* is 1×1, and Ψ_{22}^* is 1×1. Then we can construct the posterior distribution of $(I_{t,1},...,I_{t,23})$ conditional on I_t as follows:

$$(I_{t,1},...,I_{t,23}|I_t)^{'} \sim N\left(\widehat{\Xi},\hat{\Psi}\right)$$

where $\widehat{\Xi} = \Xi_1^* + \Psi_{12}^*(\Psi_{22}^*)^{-1}(I_t - \Xi_2^*)$ and $\hat{\Psi} = \Psi_{11}^* - \Psi_{12}^*(\Psi_{22}^*)^{-1}\Psi_{21}^*$. We can partition the matrices $\widehat{\Xi}$ and $\hat{\Psi}$ as follows:

$$\widehat{\Xi}_t = \begin{bmatrix} \widehat{\Xi}_{t,1} \\ \widehat{\Xi}_{t,2} \end{bmatrix}, \quad \hat{\Psi} = \begin{bmatrix} \hat{\Psi}_{11} & \hat{\Psi}_{21} \\ \hat{\Psi}_{12} & \hat{\Psi}_{22} \end{bmatrix}$$

where $\widehat{\Xi}_{t,1}$ is 1×1, $\widehat{\Xi}_{t,2}$ is 22×1, $\hat{\Psi}_{11}$ is 1×1, $\hat{\Psi}_{12}$ is 1×22, $\hat{\Psi}_{21}$ is 22×1, and Ψ_{22}^* is 22×22. Then the posterior distribution of $I_{t,1}$ conditional on I_t is $I_{t,1}|\ I_t \sim N(\widehat{\Xi}_{t,1},\hat{\Psi}_{11})$. The posterior distribution of $(I_{t,2},...,I_{t,23})^{'}$ conditional on I_t and $I_{t,1}$ is as follows:

$$(I_{t,2},...,I_{t,23}|\ I_t,I_{t,1})^{'} \sim N\left(\widehat{\Xi}_{t,2} + \hat{\Psi}_{21}(\hat{\Psi}_{11})^{-1}(I_t - \widehat{\Xi}_{t,1}),\hat{\Psi}_{22} - \hat{\Psi}_{21}(\hat{\Psi}_{11})^{-1}\hat{\Psi}_{12}\right) \tag{12}$$

Since we have the political costs, $I_{t,h}$ and $I_{t,h}^*$ are generated from the following:

$t \in \{\ I_t \neq 0,\ t < T_B\}$: Generate $I_{t,1}$ from a truncated normal distribution such as $N(\widehat{\Xi}_{t,1},\hat{\Psi}_{11})$ conditional on $|I_{t,1} - \mu_I| > c_1$. Then generate $(I_{t,2},...,I_{t,23})^{'}$ from (12) and construct $I_{t,24} = I_t - \sum_{h=1}^{23} I_{t,h}$. Set $I_{t,h}^* = I_{t,h}$ for $h = 1,...,24$.

$t \in \{I_t = 0,\ t < T_B\}$: Generate $I_{t,1}^*$ from a truncated normal distribution such as $N(\mu_I + \beta_1(s_{t-1,24} - s_{t-2,24}),\sigma_\eta^2)$ conditional on $|I_{t,1}^* - \mu_I| < c_1$. Then generate $I_{t,h}^*$ from $N(\mu_I + \beta_1(s_{t,h-1} - s_{t-1,h-1}),\sigma_\eta^2)$ for $h = 2,...,24$. Set $I_{t,h} = 0$ for $h = 1,...,24$.

$t \in \{I_t \neq 0,\ t \geq T_B\}$: Generate $I_{t,1}$ from a truncated normal distribution such as $N(\widehat{\Xi}_{t,1},\hat{\Psi})$ conditional on $|I_{t,1} - \mu_I| > c_2$. Then generate $(I_{t,2},...,I_{t,23})^{'}$ from (12) and construct $I_{t,24} = I_t - \sum_{h=1}^{23} I_{t,i}$. Set $I_{t,h}^* = I_{t,h}$ for $h = 1,...,24$.

$t \in \{I_t = 0,\ t \geq T_B\}$: Generate $I^*_{t,1}$ from a truncated normal distribution such as $N(\mu_I + \beta_1(s_{t-1,24} - s_{t-2,24}), \sigma^2_\eta)$ conditional on $|I^*_{t,1} - \mu_I| < c_2$. Then, generate $I^*_{t,h}$ from $N(\mu_I + \beta_1(s_{t,h-1} - s_{t-1,h-1}), \sigma^2_\eta)$ for $h = 2, ..., 24$. Set $I_{t,h} = 0$ for $h = 1, ..., 24$.

Step 6 Generate c_1 and c_2 conditional on μ_s, α, μ_I, β, σ^2_ϵ, and σ^2_η. Define the cumulative distribution functions of $N(\widehat{\Xi}_{t,1}, \widehat{\Psi}_{11})$ and $N(\mu_I + \beta_1(s_{t-1,24} - s_{t-2,24}), \sigma^2_\eta)$ as $\Phi_t^{I_t \neq 0}$ and $\Phi_t^{I_t = 0}$, respectively. The posterior distribution of c_1 is

$$\Pi_{t<T_B} \left[\Phi_t^{I_t=0}(c_1 + \mu_I) - \Phi_t^{I_t=0}(-c_1 + \mu_I)\right]^{1(I_t=0)} \left[1 - \Phi_t^{I_t\neq 0}(c_1 + \mu_I) + \Phi_t^{I_t\neq 0}(-c_1 + \mu_I)\right]^{1(I_t\neq 0)}.$$

Similarly, the posterior distribution of c_2 is

$$\Pi_{t\geq T_B} \left[\Phi_t^{I_t=0}(c_2 + \mu_I) - \Phi_t^{I_t=0}(-c_2 + \mu_I)\right]^{1(I_t=0)} \left[1 - \Phi_t^{I_t\neq 0}(c_2 + \mu_I) + \Phi_t^{I_t\neq 0}(-c_2 + \mu_I)\right]^{1(I_t\neq 0)}.$$

These densities are intractable and hence we implement the Metropolis-Hastings algorithm to draw from them.

We iterate steps 1 through 6 $M + N$ times and discard the realizations of the first M iterations but keep the last N iterations to form a random sample of size N on which statistical inference can be made. We set $M = 10,000$ and $N = 10,000$.

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Figure 1: Hourly fluctuations in the yen-dollar rate

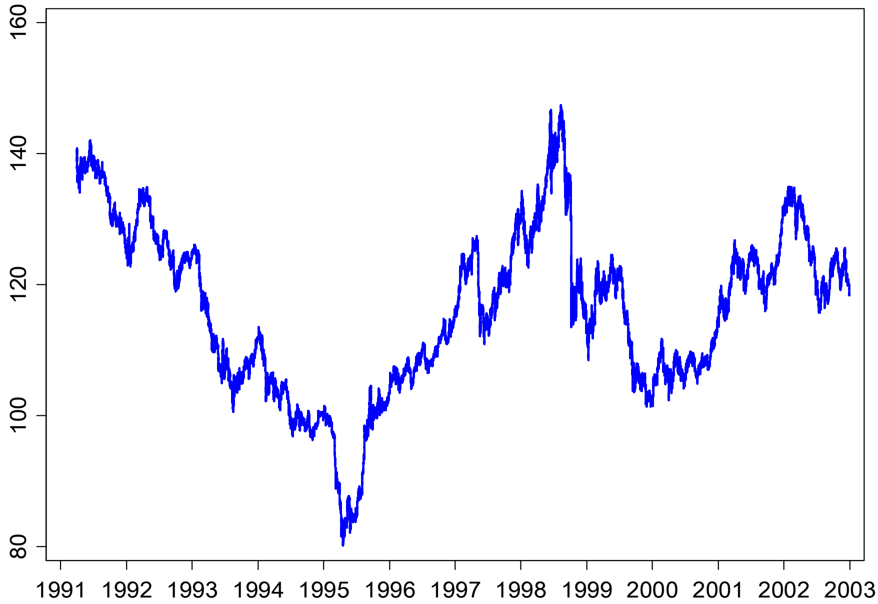

Figure 2: Daily amounts of Japanese interventions

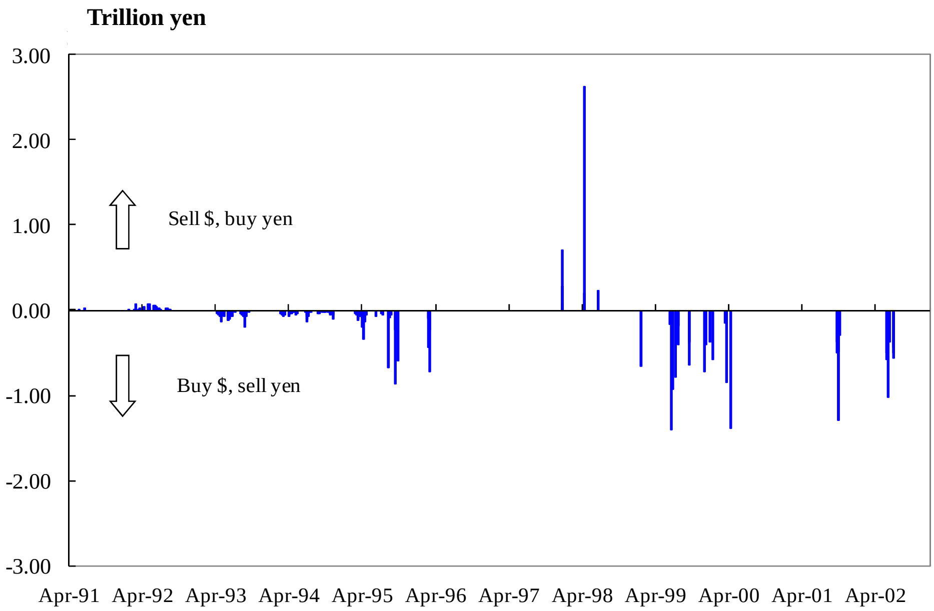

Figure 3: Estimated hourly amounts of intervention on April 10, 1998

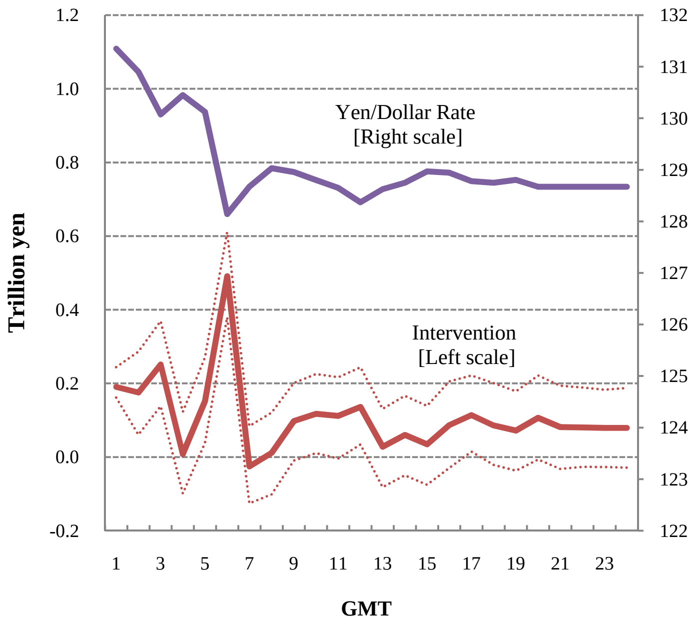

Figure 4: Exchange rates before and after yen-selling intervention

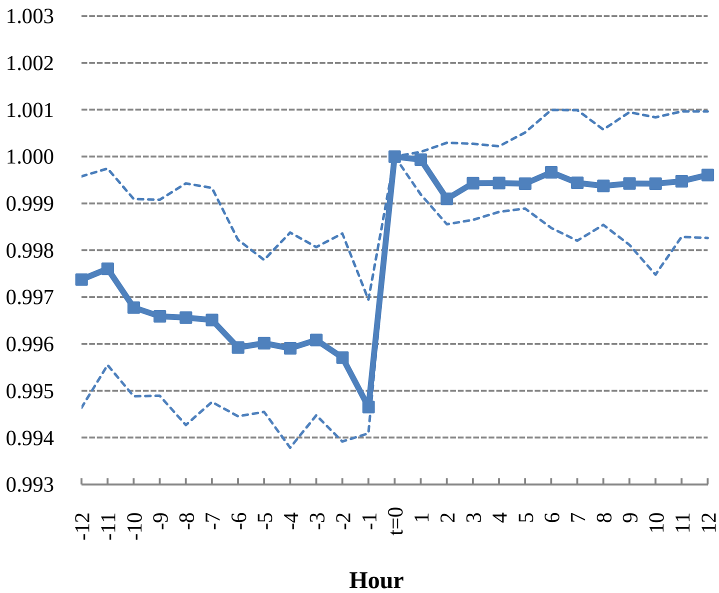

Figure 5: Frequency of interventions per day