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Consumer Inventory and the Cost of Living Index: Theory and Some Evidence from Japan

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Abstract

This paper examines the implications of consumer inventory for cost-of-living indices (COLIs) and business cycles. We begin by providing stylized facts about consumer inventory using scanner data. We then construct a quasi-dynamic model to describe consumers' purchase, consumption, and inventory behavior. A key feature of our model is that inventory is held by household producers, not by consumers, which enables us to construct a COLI in a static manner even in an economy with storable goods. Based on this model, we show that stockpiling during temporary sales generates a substantial bias, or so-called chain drift, in conventional price indices, which are constructed without paying attention to consumer inventory. However, the chain drift is greatly mitigated in our COLI, which is based on consumption prices (rather than purchase prices) and quantities consumed (rather than quantities purchased). We provide empirical evidence supporting these theoretical predictions. We also show empirically that consumers' inventory behavior tends to depend on labor market conditions and the interest rate.

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1 Introduction

Storable goods are abundant in the real world (e.g., pasta, toilet rolls, shampoos, and even vegetables and milk), although most economic models deal with perishable goods for the sake of simplicity. Goods storability implies that purchases (which are often observable) do not necessarily equal consumption (which is often unobservable), and the difference between the two serves as consumer inventory. In particular, temporary sales and the anticipation of an increase in the value-added tax rate often lead to a greater increase in purchases than consumption. Moreover, the COVID-19 outbreak in 2020 caused many products, such as pasta and toilet rolls, to disappear from supermarket shelves, which would not have happened if these products were not storable. The stockpiling behavior by consumers poses challenges for economists, for example in the construction of price indices. It is well known that the Törnqvist price index provides a good approximation to the cost-of-living index (COLI) (Diewert, 1976). However, the Törnqvist price index has a substantial downward bias, or so-called chain drift, when applied to high-frequency data, which stems from goods storability. This is illustrated in the left-hand panel of Figure 1, which shows the results when the Törnqvist price index is applied to Japanese scanner data compiled on a daily basis (details will be explained later). Specifically, doing so for the past quarter century, the results imply that Japan experienced annual deflation of 60% and the price level over this period fell to 10^{-10} of its value at the start of the period. Furthermore, stockpiling matters for the measurement of price elasticity (Erdem, Imai, and Keane, 2003; Hendel and Nevo, 2004; Cashin and Unayama, 2016).

In this study, we investigate how consumers' stockpiling behavior influences the price index and the macroeconomy. First, we document stylized facts associated with storable goods by employing both retailer and home scanner data for Japan. The latter are unique in that they provide household- and product-level information at a daily frequency on when households purchase individual products, when they consume them, and when they have used up all their inventory. We obtain the following stylized facts: (1) chained price indices entail extremely large biases, with the Paasche and Törnqvist indices having a downward and the Laspeyres index having an upward bias; (2) there is a considerable difference in the amount of purchases before, during, and after a sale period; and (3) consumption increases with inventory.

Second, we construct a simple model of storable goods to reproduce these stylized facts, infer consumption from data on purchases, and construct a COLI. The model incorporates stockpiling behavior in an environment where prices exogenously take either a high or a low value. The quantity purchased increases during a sale, with part of the increase in purchases being for future use when the price has increased again. The novel feature of the model is that it is quasi-dynamic in that it is household producers instead of consumers that hold inventories and solve a dynamic optimization problem. This helps simplify the calculation of the COLI, because consumers' cost minimization problem is static. This ease of calculation is an advantage over so-called dynamic COLIs, which are much more complex and difficult to calculate and, in addition, harder to interpret.[1] Using the model, we successfully reproduce the three stylized facts mentioned above.

Furthermore, we show that the downward bias in the Törnqvist price index does not disappear even if the index is constructed using consumption weights (i.e., weights based on the quantity consumed times the consumption price) instead of purchase weights (i.e., weights based on the quantity purchased times the purchase price). As pointed out by Feenstra and Shapiro (2003) and Ivancic, Diewert, and Fox (2011) among others, the Törnqvist price index entails a bias, which stems from using quantities purchased rather than quantities consumed. However, we find that this is not the sole reason. Even if we appropriately use consumption weights, the downward bias does not disappear because the path of consumption prices is asymmetric. Specifically, we show that the consumption price decreases quickly when the purchase price drops at the start of a sale but then increases again only gradually when the purchase price returns to the regular price after the sale ends, as a result of consumer inventories. This asymmetric response of the consumption price and the fact that a temporary price change almost always consists of a decrease (i.e., a sale) mean that although the Törnqvist price index is a good approximation of the COLI up to the second order, the third-order approximation error cannot be eliminated, so that the Törnqvist price index continues to entail a downward bias even when it is constructed based on consumption weights.

Next, we propose a new method to infer consumption and inventory relying only on retailer scanner data. The inferred consumption is then used to calculate the COLI based on consumption weights rather than purchase weights. We show that consumers' stockpiling behavior can be conveniently summarized by a single variable: the degree of stockpiling during sales, which captures how long consumers' inventories will last after

[1]For example, Ueda (2020) considers two types of dynamic COLIs and shows that they do not satisfy monotonicity and the time reversal test simultaneously.

a sale ends. Applying our method to the Japanese retail scanner data, we show that the chain drift in the Törnqvist price index based on consumption weights is much smaller than that based on purchase weights and that some chain drift nevertheless remains due to the asymmetric path of consumption prices that arises from stockpiling. We also show that in the COLI based on consumption weights the chain drift found in the Törnqvist price index is considerably mitigated.

Finally, we investigate the macroeconomic implications of goods being storable. We empirically show that, when hours worked are long, the degree of stockpiling tends to be small. This implies that longer hours worked decrease shopping time and prevent consumers from searching for products on sales or shops with sales. We also show that the degree of stockpiling decreases when the real interest rate increases, which suggests that the opportunity cost of stockpiling increases with the real interest rate. Such endogenous responses of consumer inventory to exogenous shocks have non-negligible effects on both the macroeconomy and prices through changes in the quantity purchased during sales. Specifically, fluctuations in the quantity purchased would be several percent greater if households' stockpiling behavior did not depend on exogenous shocks.

Our study is related to at least four strands of research. The first strand concerns household inventories of storable goods. Boizot, Robin, and Visser (2001), Griffith et al. (2009), Hendel and Nevo (2006a), and Kano (2018) empirically document patterns of household purchases and consumption. Among these studies, those by Boizot, Robin, and Visser (2001) and Kano (2018) are noteworthy in that they are part of a small group of studies that, like ours, use household inventory data. In terms of the theoretical approach, our model is closest to that developed by Hendel and Nevo (2006a). The novelty of our study is that it focuses on price indices (COLIs) and the macroeconomic implications. Moreover, to do so, we develop a model that is original in that it is quasi-dynamic and separates households into consumers in a narrow sense and household producers that can hold inventories. It should also be noted that, in this strand of literature, consumption is often assumed to be constant (e.g., Boizot, Robin, and Visser, 2001, and the empirical part of Hendel and Nevo, 2006a), although Hendel and Nevo (2006a) and Kano (2018) highlight the possibility that consumption may be state dependent.[2] Our approach takes the possibility of state-dependent consumption into account and, having confirmed that consumption is state-dependent, we use this fact for constructing the

[2]Also see Hendel and Nevo (2004), Cashin and Unayama (2016), Erdem, Imai, and Keane (2003), Hendel and Nevo (2006b), and Osborne (2018).

COLI.

The second strand of literature our study is related to is that on chain drift or the time aggregation problem in chained price indices. Notable studies in this strand include Frisch (1936), Reinsdorf (1999), Feenstra and Shapiro (2003), ILO et al. (2004, 2020), and Ivancic, Diewert, and Fox (2011). In this literature, it has been pointed out that the chain drift is due to stockpiling (which is sometimes called "quantity bouncing"), and using a model incorporating storable goods to address this issue is not an entirely new idea. For instance, Ivancic, Diewert, and Fox (2011) and de Haan and van der Grient (2011) propose using the GEKS index (originally proposed by Gini, Eltetö, Köves, and Szulc). This is constructed to satisfy the circularity test, which is one of several desirable axioms for price indices. A drawback of this approach is that it is not based on consumers' optimization problem.[3] Moreover, the GEKS index is complex and revised each time new data become available. Feenstra and Shapiro (2003) construct a simple model incorporating storable goods and propose using a virtually fixed base price index.[4] However, they introduce restrictive assumptions such as that households have perfect foresight and that households' expenditure is expressed in a specific functional form that is independent of inventories. Moreover, in calculating weights for the fixed base price index, they use a yearly mode price and average sales at that price, although we find that consumption is not constant, changing with inventories.

Using lower frequency (such as monthly or quarterly) data may help mitigate the chain drift, because inventories of most products, as shown in our study, last for less than a month. However, the use of lower frequency data has three shortcomings. First, as pointed out in many previous studies, the use of lower frequency data alone does not completely eliminate the chain drift. This can be seen in the right-hand panel of Figure 1, which shows that the chained price index at a monthly frequency ($dt = 30$ days) falls at an annual rate of 1%. The figure also shows that the drift becomes almost negligible only once dt exceeds 150 days. Given that in almost all industrial countries the consumer price index (CPI) is published on a monthly basis, this suggests that using a frequency that is sufficiently low to avoid chain drift is not a realistic solution (see also Ivancic, Diewert, and Fox, 2011). Second, some of the high frequency (e.g., daily)

[3]See, for example, Zhang, Johansen, and Nygaard (2019) and ILO et al. (2020). Zhang, Johansen, and Nygaard (2019) show that the GEKS index fails to satisfy four of five axiomatic tests, such as the identity test.

[4]See ILO et al. (2004) for details on fixed base and chain indices.

fluctuations in prices and quantities may be closely related to business cycle fluctuations. Specifically, as we show later, consumers change their stockpiling behavior depending on the macroeconomic environment such as hours worked and interest rates. This suggests that it may not be a good idea to discarding information from high frequency data in constructing price indices, because this will yield biased price indices where the bias changes with the business cycle. Third, some goods are short lived or new, and the use of lower frequency data means that those goods are potentially ignored in the construction of price indices, which may not be appropriate.

The third strand of literature that our study is related to is that on COLIs. This literature dates back to Könus (1924), and of more recent contributions, studies to which ours is most closely related include those by Feenstra and Shapiro (2003), Chevalier and Kashyap (2019), Reis (2009), Gowrisankaran and Rysman (2012), Wang (2013), Osborne (2018), and Ueda (2020). The studies by Feenstra and Shapiro (2003) and Chevalier and Kashyap (2019) propose a proxy for COLIs to deal with sales, in an environment where consumers' optimization problem is static. The other studies examine dynamic COLIs, which incorporate intertemporal substitution by households. For example, Osborne (2018) constructs a model that, like ours, incorporates both storable goods and sales. He then computes a dynamic COLI by calculating the sequence of taxes on or subsidies to households such that their period utility is kept constant over time. While Osborne's dynamic COLI is indeed a major contribution to the literature, interpreting the short-run movements of dynamic COLIs is often difficult. Because our model is quasi-dynamic and consumers' optimization problem is static, it is much simpler to define, interpret, and construct the COLI, which we believe is beneficial to practitioners working for national statistical offices as well as economists.[5]

Finally, the fourth strand our study is related to is that on macroeconomic implications of goods storability. Hansman et al. (2020) investigate how sticky prices influence households' stockpiling behaviors during disasters or supply disruptions. Cashin and Unayama (2016) examine the effects of preannounced increase in Japan's consumption tax rate on stockpiling to estimate the intertemporal elasticity of substitution. While these papers use a particular event to study implications of goods storability, we use

[5]Osborne (2018) finds that the price indices proposed by Feenstra and Shapiro (2003) and Chevalier and Kashyap (2018) approximate the dynamic COLI. However, his analysis is limited to particular products (canned tuna and canned soup) for particular periods, so there is a possibility that this finding is just a coincidence.

frequently observed events, that is, temporary sales, to examine how consumers' inventory behavior during sales depend on labor market conditions and the interest rate. In this respect, our study is also related to the literature on macroeconomic implications of temporary sales. Such studies include Klenow and Willis (2007), Sudo et al. (2018), and Kryvtsov and Vincent (2020), which show that business cycles influence firms' sales decisions.[6]

The remainder of our study is organized as follows. Section 2 provides stylized facts based on two kinds of scanner data for Japan. Section 3 develops a quasi-dynamic model to incorporate stockpiling behavior. Section 4 extends the model to infer consumption and consumption prices from retailer scanner data, while Section 5 shows the results on the COLI and developments in households' stockpiling behavior. Section 6 concludes.

2 Stylized Facts Based on Japanese Scanner Data

2.1 Two Kinds of Scanner Data

We use two sets of scanner data for Japan. The first set consists of retailer-side data, namely, the point-of-sale (POS) scanner data collected by Nikkei Inc. The data include the number of units sold and the sales amount (price times the number of units sold) for each product and retailer on a daily basis. The observation period runs from March 1, 1988 to October 31, 2013. Products recorded consist of processed food and daily necessities, covering 170 of the 588 categories in the CPI and making up about 20 percent of households' expenditure. See Appendix A as to how we aggregate variables of interest over days, products, and retailers, and Abe and Tonogi (2010), Sudo, Ueda, and Watanabe (2014), and Sudo et al. (2018) for a detailed description of the data.

[6]While there are many reasons to explain the existence of sales, goods storability is regarded as one of main reasons, as discussed in the survey by Hendel and Nevo (2004), the empirical studies by Blattberg and Neslin (1989), Neslin and Schneider Stone (1996), and Hendel and Nevo (2003), and the theoretical studies by Salop and Stiglitz (1982), Hong, McAfee, and Nayyar (2002), and Hendel, Lizzeri, and Roketskiy (2014). These studies are often conducted from the perspective of firms, and models are constructed to explain why firms set dispersed prices for the same goods. In contrast, our study focuses on households rather than firms. It should also be noted that, unlike these other studies, our study ignores household heterogeneity in order to simplify the analysis and concentrate on the macroeconomic implications.

The other set consists of household-side data, namely, "*Shoku-map*"[7] scanner data collected by Lifescape Marketing Co. Respondents are mainly housewives, and the data cover about 400 households in each period (about 4,000 households in total). The data record the number of units purchased and the date of consumption for each product and household on a daily basis. Moreover, they record when consumption ends (i.e., when a product is used up or has gone off, etc.) for each product and household. The data cover the period from September 1998 to February 2019. Note that products recorded are food only and there is no information on purchase prices. Another limitation is that there is no information on how much of a product (e.g., in terms of weight) is consumed each time it is consumed. The data record both the number of units purchased and consumed, which is sufficiently useful if products are consumed in discrete units, such as a cup of instant noodles or a can of beer. However, for products like salt, we do not know how much a household uses, although we do know the dates when they are used.[8] For example, Figure 2 shows the consumption pattern for salt of a particular household. In the figure, each vertical line represents a consumption flag. The figure indicates that the household purchased salt on day $t = 19$, started using it on day $t = 22$, and used it up on day $t = 144$. The inventory duration thus is $144 - 19 + 1 = 126$ days. See Appendix B for the basic statistics of the *Shoku-map* data.

In both sets of data, all products are identified by the Japanese Article Number (JAN) code, which enables us to merge the datasets. Further, we classify products into groups using the 3-digit product categories provided by Nikkei Inc. There are 214 categories in total, such as instant cup noodles, yogurt, beer, and toothbrushes.

Figure 3 shows the purchase and consumption pattern for the 3-digit product categories of beer and low-malt beer and a particular household, using the *Shoku-map* data. The horizontal axis represents days, while the vertical axis represents the cumulative number of items purchased. The left and right ends of each horizontal line show the days when products are purchased and consumed, respectively. This figure shows that the household purchased more than one bottle of beer (six or a dozen) when it had consumed its entire inventory, and that beer was consumed almost every day.

Figure 4 shows the density of consumption periods using the *Shoku-map* data. For each product and household, we look at three dates: the date of purchase (t_p), the date

[7] "*Shoku-map*" translates as "food map."

[8] If a household uses salt N times a day, the data record N, where N is an integer equal to or greater than zero.

a household starts consuming the product (t_f), and the date the household finishes the product (t_l). We then calculate the periods between the various dates.[9] Starting with the period from the date of purchase until the date a product is used up, $t_l - t_p + 1$, the figure shows that about 9% of products are fully consumed on the date of purchase, while the mean period is 27 days. Meanwhile, the mean period from the date of purchase to the date a household starts consuming a product, $t_f - t_p + 1$, and the mean period it takes for a household to consume a product once it has started, i.e., $t_l - t_f + 1$, are both 17 days.

In the following analyses using the POS data, we identify temporary sales by employing a sales filter. Specifically, we follow the procedure explained in Nakamura and Steinsson (2010) using their sales filter A with a window of $L = K = J = 42$ days.[10] Product k is classified as being on sale on date t if and only if its price p_t^k deviates from its regular price $\overline{p}_t^k$ by more than two yen.

When no sales are recorded for a particular product at a particular retailer on a particular date, earlier studies usually treated this simply as a missing observation, as if the product disappeared from shelves at the retailer on the date. However, the POS data quite often show no sales records for a particular product, retailer, and date after a sale, suggesting that stockpiling by households during a sale results in zero purchases after the sale ends. Since this conveys important information on consumer inventory, in this study, we interpolate missing observations by setting $p_t^k = \overline{p}_t^k$ and $x_t^k = 0$ if we have observations after t (i.e., unless the product permanently exits from the market). That is, we set the quantity purchased to zero and the price to the regular price on the nearest past date.[11]

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[9] We ignore some records because we cannot identify consumption periods in the following two cases. First, we cannot identify t_f if the flag for the initial state of a product is “inventory they already have.” Second, we cannot identify t_l if the flag for the final state of a product takes “gone off,” “given to others,” “unused,” “a household ceases to answer the survey before the product is used up,” or “unknown.” We only include products with the flag “used up” in our calculation.

[10] The window length is chosen following Eichenbaum, Jaimovich, and Rebelo (2011) and Kehoe and Midrigan (2015). For a detailed examination of the robustness of various filters to identify sales, see Sudo et al. (2018).

[11] However, for the right-hand panels of Figures 1 and 5, we do not employ such interpolation.

2.2 Three Stylized Facts

Using the two datasets, we present three stylized facts that are closely related to goods storability.

Fact 1. When weights are based on purchases, changes in price indices are characterized by the following inequality: $\pi^P < \pi^T < 0 < \pi^L$.

Let us denote the price and the quantity of product k in period t by p_t^k and x_t^k. Changes in chained price indices from $t - dt$ to t, π_t^X $(X = L, P, T)$, are defined by

$$\pi_t^L = \sum_{k \in K_{t-dt} \cap K_t} W_{t-dt}^k (K_{t-dt} \cap K_t) \log\left(\frac{p_t^k}{p_{t-dt}^k}\right), \tag{1}$$

$$\pi_t^P = \sum_{k \in K_{t-dt} \cap K_t} W_t^k (K_{t-dt} \cap K_t) \log\left(\frac{p_t^k}{p_{t-dt}^k}\right), \tag{2}$$

$$\pi_t^T = \sum_{k \in K_{t-dt} \cap K_t} \frac{W_{t-dt}^k (K_{t-dt} \cap K_t) + W_t^k (K_{t-dt} \cap K_t)}{2} \log\left(\frac{p_t^k}{p_{t-dt}^k}\right), \tag{3}$$

based on the Laspeyres, Paasche, and Törnqvist approach, respectively.[12] The weight share $W_t^k(K_{t-dt} \cap K_t)$ equals $p_t^k x_t^k / \sum_{k' \in K_{t-dt} \cap K_t} p_t^{k'} x_t^{k'}$, where $k \in K_{t-dt} \cap K_t$ represents a domain of products that exist both in $t - dt$ and t (such a common set is called a *matched sample*). When $dt = 1$, we can construct the chained price indices using the cumulative sum of the past price changes: $P_t^X = \exp\left(\sum_{s=1}^t \pi_s^X\right)$ for $X = L, P, T$.

It should be noted that the choices of p_t^k and x_t^k play a very important role. For the time being, we use conventional p_t^k and x_t^k that are observable from the POS data. That is, p_t^k and x_t^k represent the purchase price and quantity purchased, respectively. Later, we try to infer the consumption-based price r_t^k and consumption c_t^k.

Using the POS data, we calculate the time series of the price level (P_t^X), normalizing the initial price level to one. The results are striking. The left-hand panel of Figure 1 shows that the price level increases by almost 10^{72} over our 25-year observation period when based on the Laspeyres index, while it decreases to almost 10^{-90} when based on the Paasche index. The price decline is milder but nevertheless remains large when based on the Törnqvist index. The price level still decreases to almost 10^{-10}. Thus, the following inequality holds: $\pi^P < \pi^T < 0 < \pi^L$, indicating that the Paasche and Törnqvist indices have a downward bias and the Laspeyres index has an upward bias.

[12] There are various types of Laspeyres and Paasche indices. Here, we use the logarithmic Laspeyres and logarithmic Paasche indices.

Next, the right-hand panel shows the average price change over the observation period based on the Törnqvist index, where we employ different time intervals dt from 1 day to 365 days. The average deflation rate is about 60% annually if $dt = 1$. This downward bias becomes smaller as dt increases. When $dt = 30$, the average annual deflation rate decreases to about 1%. However, this size of deflation is still not negligible. When $dt = 365$, prices are compared with the same day a year earlier. Although the average annual inflation rate continues to be negative, it is only about -0.3%. As Figure 5 shows, the time-series developments in the Törnqvist index are very similar to those in the CPI for groceries, i.e., the CPI for the same product category as the POS data.

The large upward and downward biases in the different indices arise from quantity bouncing or stockpiling. To see this, let us consider the simple case of sales shown in Table 1.[13] There are three periods: $t = 1$, 2, and 3, and two products: A and B. In period $t = 2$, the price of product A drops temporarily (due to a sale). Under these circumstances, and if the elasticity of substitutions is greater than one, the share of product A in households' total purchases will be greater in period $t = 2$ than that in period $t = 1$ ($W_1 < W_2$). Furthermore, because households stockpile, the share of product A in households' total purchases in period $t = 3$ should be smaller than that in period $t = 1$ ($W_3 < W_1$). That is, households have sufficient inventory on the day after a sale, so that they do not need to purchase as much of the product as before the sale. Thus, it can be immediately seen that $\pi^P < \pi^T < 0 < \pi^L$ from equations (1) to (3).[14]

Fact 2. The quantity purchased just before a sale tends to be greater than that purchased just after a sale. The quantity purchased during the first half of the sale tends to be greater than that purchased during the second half of the sale.

We examine whether the hypothetical pattern shown in Table 1 can be observed in practice using the POS data. To do this, we collect the following variables for each

[13] A similar argument was made, for example, by Haan and van der Grient (2011).

[14] In the extreme case, product A is not purchased at all in period $t = 3$, that is, $W_3 = 0$. This causes the missing observation problem. If we do not interpolate the price and quantity in $t = 3$, for example, by setting $p^A_{t=3} = p^A$ and $x^A_{t=3} = 0$, the price increase is fully neglected from $t = 2$ to 3 because product A is not in the matched sample ($k \in K_{t=2} \cap K_{t=3}$) and, therefore, is excluded from the calculation of the changes in chained price indices in equations (1) to (3). Therefore, without interpolation, the downward bias of the Törnqvist price increases further.

product k and sales event s: T denotes the number of days a product is on sale, P_H and X_H^1 denote the price and the quantity purchased just before a sale (say, in period t), respectively, P_L^1 and X_L^1 denote the average price and quantity purchased during the first half of the sale, respectively (i.e., from $t+1$ to $t+\lfloor T/2 \rfloor - 1$), P_L^2 and X_L^2 denote the average price and quantity purchased during the second half of the sale, respectively (i.e., from $t + \lfloor T/2 \rfloor$ to $t + T$), and P_H and X_H^2 denote the price and the quantity purchased just after the sale, respectively (period $t + T + 1$).[15]

Comparison of the quantity purchased just before a sale and just after a sale
We estimate the following equation to compare the quantity purchased just before a sale and just after a sale:

$$\log(X_{H,ks}^2 / X_{H,ks}^1) = c + \omega WeekendDays_{ks} + \varepsilon_{ks}. \tag{4}$$

To be precise, we add 0.1 to both $X_{H,ks}^2$ and $X_{H,ks}^1$ on the left-hand side of the equation, because especially after a sale the quantity purchased may be zero. Since the quantity purchased tends to increase on weekends, we control for this effect by including the variable $WeekendDays_{ks}$, which is defined as $(WE_{ks}^2 - WE_{ks}^1)/(WE_{ks}^1 + WE_{ks}^2)$, where WE_{ks}^1 and WE_{ks}^2 represent the number of weekend days in the first and the second half of the sale, respectively. If both WE_{ks}^1 and WE_{ks}^2 are zero, we set $WeekendDays_{ks}$ to zero. This variable indicates the ratio of weekend days in the second half of the sale to the first half of the sale.

We expect the intercept c to be negative, indicating that the quantity purchased just after a sale is smaller than that just before a sale, although the price, at P_H, is identical. The first column of Table 2 shows that c is indeed significantly negative. On the other hand, the coefficient on $WeekendDays_{ks}$ is significantly positive, showing that the quantity purchased is larger on weekends than on weekdays.

Since there exists large heterogeneity across product categories, we estimate the above equation for each product category j, where j represents the product category to which product k belongs. This allows the coefficients c and ω to have different values for different j, that is, we estimate c_j and ω_j. The left-hand panel of Figure 6 shows the distribution of the t-statistic for coefficient c_j. The vertical axis represents the number of categories for which the the t-statistic is smaller than t^*, while the horizontal axis

[15] A sales event is identified when the price after a sale returns to within 2 yen of the price before the sale.

represents t^*. For the majority of the 214 categories in total, the t-statistic for c_j is smaller than -1.96, which confirms that the quantity purchased just after a sale is smaller than that just before a sale.

Further, to examine whether a longer sales duration and larger sales discount leads to a larger decrease in the quantity purchased after a sale, we add fixed effects[16] and check whether α and β are negative and positive, respectively, by estimating the following equation:

$$\log(X^2_{H,ks}/X^1_{H,ks}) = c_j + \omega WeekendDays_{ks} + \alpha \log(T_{ks}) + \beta \log((P^1_{L,ks} + P^2_{L,ks})/(2P_{H,ks})) + \varepsilon_{ks}.$$

The second column of the table shows that α is indeed significantly negative and β significantly positive.

Comparison of the quantity purchased during the first and the second half of a sale Next, we estimate the following equations to compare the quantity purchased during the first half and the second half of a sale:

$$\log(X^2_{L,ks}/X^1_{L,ks}) = c + \omega WeekendDays_{ks} + \varepsilon_{ks}. \tag{5}$$

If intercept c is negative, this suggests that the quantity purchased in the second half of a sale is smaller than that in the first half a sale. Because prices are not necessarily constant during a sale, we compare the quantity purchased during a sale only when $P^1_{L,ks}$ and $P^2_{L,ks}$ differ by less than one yen.

The third column of Table 2 shows that c is significantly negative, while the coefficient on $WeekendDays_{ks}$ is significantly positive. Thus, in aggregate, the quantity purchased tends to be larger at the beginning of a sale than at the end.

However, this pattern does not appear to be robust. The right-hand panel of Figure 6 shows the results when estimating the above equation at the product category level. It shows that for less than one third of categories coefficient c_j is significantly negative. On the other hand, for another third of categories, coefficient c_j is significantly positive. The reason for the difference between aggregate and product category levels seems to be that categories with a negative c_j tend to be on sale more frequently and the $|c_j|$ tends to be larger than in the case of categories with a positive c_j. It may also be worth pointing out that many daily necessities categories exhibit a positive c_j, whereas many processed food categories exhibit a negative c_j.

[16] It should be noted that as a result c no longer has a meaningful interpretation.

The last two columns of Table 2 show further estimation results. Here, we do not limit the sample to observations for which $|P^1_{L,ks} - P^2_{L,ks}| < 1$ yen. This enables us to include the log price difference during the first and the second half of a sale, $\log(P^2_{L,ks}/P^1_{L,ks})$, in the explanatory variables, because it can take widely different values. To ensure that the intercept c can still be interpreted, we subtract the mean across all products and sales events from $\log(P^2_{L,ks}/P^1_{L,ks})$ for product k and sales event s. The fourth column of the table shows that c is significantly positive. The coefficient on $\log(P^2_{L,ks}/P^1_{L,ks})$ is significantly negative, which is consistent with a standard demand response to price changes.

The last column of the table shows the estimation results obtained when the category fixed effect and $\log(T_{ks})$ are included in the explanatory variables in addition to $\log(P^2_{L,ks}/P^1_{L,ks})$. While the coefficient on $\log(P^2_{L,ks}/P^1_{L,ks})$ is hardly changed, the coefficient on $\log(T_{ks})$ is negative, suggesting that the quantity purchased in the second half of a sale tends to decrease relative to that in the first half of the sale as the sales duration lengthens.

Fact 3. Consumption tends to decrease as household inventories decrease.

While earlier studies on household inventories often assume that consumption is constant, Hendel and Nevo (2006a) and Kano (2018) highlight that consumption is state dependent. To examine whether consumption is indeed state dependent, we investigate whether consumption decreases until the next purchase as inventories decrease, using the *Shoku-map* data. Suppose that household i uses product k on date $t_l \in t_1, t_2, \cdots, t_{n_{ikt}}$ and t represents the purchase date. If the household uses product k twice on date t_{l_0}, we record t_{l_0} twice. Thus, n_{ikt} represents the number of times product k is used. We define the inventory on date $t^{'}$ ($t \leq t^{'} \leq t_{n_{ikt}}$) as $\lambda_{ikt'} = n_{ikt} - n^*$, where n^* is the maximum integer n that satisfies $t_n < t^{'}$. Note that $\lambda_{ikt'}$ is an integer between 0 and n_{ikt}. For example, $\lambda_{ikt'} = n_{ikt}$ for $t \leq t^{'} \leq t_1$. We further define the sum of inventory Λ_{ijt} for household i in product category j at the beginning of date t as

$$\Lambda_{ijt} = \sum_{k \in j} \lambda_{ikt}. \tag{6}$$

We then estimate the following linear probability model when $\Lambda_{ijt} > 0$:

$$y_{ijt} = c_i + d_j + \alpha \Lambda_{ijt} + \varepsilon_{ijt}, \tag{7}$$

where y_{ijt} is a binary variable for consumption and takes a value of one if household i uses products in product category j on date t and zero otherwise. As an alternative, we also use y_{ijt} defined as the sum of the times that products in product category j are used by household i on date t.

As Table 3 shows, the coefficient on inventories is significantly positive, regardless of which dependent variable is used. This indicates that consumption is state dependent and decreases as inventories decrease. For the estimation presented in the table, we did not use instrumental variables but employed ordinary least squares (OLS). This means that the estimates are biased if, for example, $\text{Cor}(\varepsilon_{ijt}, \varepsilon_{ijt-1}) > 0$, that is, if high consumption demand at time $t-1$ not only decreases inventories Λ_{ijt} at time t but also increases consumption at time t. If such endogeneity exists, the coefficient on inventories is underestimated when using OLS. Thus, the fact that we obtained a significantly positive estimate using OLS indicates that the state dependency of consumption continues to hold or is stronger than our estimates suggest.

Our result holds even when we repeat the estimation at the category level. Specifically, we estimate the above equation for each category allowing for variations in α across different categories j. Figure 7 shows the distribution of the t-statistic for coefficient α_j. The vertical axis indicates the number of categories for which the t-statistic is larger than t^*, while the horizontal axis represents t^*. In 144 out of the 157 categories, the t-statistic for α_j is larger than 1.96 when the dependent variable is a binary variable for consumption. The number of categories is 146 when the dependent variable is the number of times products are used.

3 Quasi-Dynamic Model

3.1 Setup

In this section, we construct a simple partial-equilibrium model to explain the stylized facts described in the previous section, to infer inventories and consumption, and to construct the COLI. We assume that product $k \in K_t$ is storable and, for simplicity, that it does not depreciate. Time t is a discrete day.

The novelty of our model is that we assume that households comprise consumers and household producers and distinguish between the two. That is, household producers are a special type of household member that has technology to hold inventories, although there is a cost associated. The household producer provides inventory services consisting of purchasing storable goods from manufacturers (the quantity purchased denoted by x_t^k), holding inventory (denoted by i_t^k), and selling the goods to consumers (the quantity sold denoted by y_t^k). Market entry is free, so that the expected firm value is zero. In this respect, there is no loss of consumer surplus. On the other hand, consumers in a narrow sense cannot hold inventory: their purchases always equal their consumption, c_t^k. They purchase goods from household producers and/or manufacturers at the *consumption price* r_t^k.

This framework enables us to solve the COLI in a conventional static manner. All we need to know is two variables: c_t^k and r_t^k. We do not need to use a complex dynamic COLI. Although the economy is hypothetical, many key properties continue to hold. For example, the three stylized facts can be explained not only by a dynamic model like the one used by Hendel and Nevo (2006a) but also by our quasi-dynamic model. On the other hand, it would be difficult to explain the three stylized facts using a completely static model ignoring storability.

We consider the optimization problems of household producers and consumers. The price of storable goods, p_t^k, can take one of two different values, a high (regular) value and a low (sales) value, which are determined stochastically and exogenously.

Consumers

There are a unit mass of consumers. Consumers' cost minimization problem is given by

$$\min_{c_t^k} \left\{ \sum_{k \in K_t} r_t^k c_t^k + \lambda_t \left\{ U - \left[\sum_{k \in K_t} b_t^k \left(c_t^k \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \right\} \right\}, \tag{8}$$

where U is the target utility, $\sigma(> 0)$ denotes the elasticity of substitution, and $b_t^k = b^k + \varepsilon_t^k$ is a taste or quality parameter for product k in period t with time-varying fluctuations of ε_t^k.[17] We extensively use the following relation with respect to the optimal quantity purchased:

$$c_t^k = \left(\frac{r_t^k / b_t^k}{r_{t'}^k / b_{t'}^k} \right)^{-\sigma} c_{t'}^k. \tag{9}$$

[17] There is no consumer heterogeneity in the model. Many previous studies, such as Boizot, Robin, and Visser (2001) and Hendel and Nevo (2004, 2006a), investigate the frequency of purchases conditional on past purchase behavior. To obtain quantitatively plausible values for this, we need household heterogeneity, because otherwise the frequency of purchases would be either zero or one. In our study, we ignore consumer heterogeneity to focus on macroeconomic implications rather than individual behavior.

Household producers

Household producers maximize their "firm" value:

$$V(i_{t-1}, p_t) = \mathrm{E}_t \left[\sum_{j=0}^{\infty} \beta^j \left\{ \sum_{k \in K_{t+j}} \left(r_{t+j}^k y_{t+j}^k - p_{t+j}^k x_{t+j}^k - C(i_{t+j}^k) \right) \right\} \right], \tag{10}$$

subject to the cost of inventory, $C(0) > 0$, $C(i)' > 0$, $C(i)'' \geq 0$, and the evolution of inventory:

$$i_t^k = i_{t-1}^k - y_t^k + x_t^k. \tag{11}$$

Household producers sell amount y_t^k of product k to consumers at consumption price r_t^k. Furthermore, purchases and inventories must be nonnegative:

$$x_t^k,\ i_t^k \geq 0. \tag{12}$$

The first-order conditions with respect to x_t^k and i_t^k are

$$0 = r_t^k - p_t^k + \psi_t^k, \tag{13}$$

$$C'(i_t^k) = \beta \mathrm{E}_t[r_{t+1}^k] - r_t^k + \mu_t^k, \tag{14}$$

where ψ_t^k and μ_t^k represent the Lagrange multipliers with respect to x_t^k and i_t^k, respectively. Note that ψ_t^k is strictly positive when x_t^k is zero, and zero when x_t^k is positive. Likewise, μ_t^k is strictly positive when i_t^k is zero, and zero when i_t^k is positive.

The free entry condition leads to a nonpositive value for an entering household producer with zero inventory holdings:

$$V(i_{t-1} = 0, p_t) \leq 0. \tag{15}$$

Prices

The prices of storable goods follow a Markov process. They take one of the following two values: P_H when there is no sale and P_L ($P_H > P_L$) during a sale. Moreover:

$$\mathrm{Prob}(P_L|P_H) = \overline{q}$$

$$\mathrm{Prob}(P_L|P_L) = \underline{q}. \tag{16}$$

Market clearing

Goods market clearing is given by

$$\int_0^{N_t} y_{t,j}^k dj + \int_0^{M_t} z_{t,j}^k dj = \int_0^1 c_{t,j}^k dj, \tag{17}$$

where $z_{t,j}^k$ represents the direct supply of storable product k by manufacturers to consumers. Household consumption c_t equals consumers' purchases from household producers, y_t, and manufacturers, z_t. In the market, there are a unit mass of consumers, N_t represents household producers, and M_t represents manufacturers.

Note that the household producers we consider in the model are still part of the households, although we separate them to simplify our analysis. Thus, the quantity purchased that is recorded in the POS data, X_t, should equal the sum of the quantity purchased by household producers $\int_0^{N_t} x_{t,j}^k dj$ and the quantity purchased directly by consumers $\int_0^{N_t} z_{t,j}^k dj$. Clearly, this is not equal to aggregate consumption $\int_0^1 c_{t,j}^k dj$.

The COLI

As highlighted, consumers' optimization problem is static, so our COLI is identical to the conventional COLI. Consumers' cost minimization problem subject to constant utility yields the following equation for the optimal quantity consumed:

$$c_t^k = \left(r_t^k / b_t^k\right)^{-\sigma} \lambda_t^\sigma U. \tag{18}$$

The unit cost function, $\lambda_t = \mathcal{C}(r_t)$ for $U = 1$, is given by

$$\mathcal{C}(r_t) = \sum_{k \in K_t} r_t^k c_t^k = \left[\sum_{k \in K_t} \left(b_t^k\right)^\sigma \left(r_t^k\right)^{1-\sigma}\right]^{1/(1-\sigma)}. \tag{19}$$

Although b_t^k is unobservable, equation (9) tells us that we need to know only two variables for each period, the consumption price r_t^k and the consumption share $r_t^k c_t^k$, to calculate the change in the COLI between period t and period t', $C(r_t)/C(r_{t'})$.[18]

3.2 Equilibrium Properties

We discuss the equilibrium properties of the model in relation to the aforementioned stylized facts. In the following discussion, we omit the superscript for product k for

[18]Given that household producers form part of households, it would clearly be desirable to incorporate the effect of changes in their "firm" value on consumers' utility and the COLI. However, this effect likely is small, because the free entry condition means that the firm value is close to zero.

simplicity. Further, we denote aggregate inventories at the end of period $t-1$, by $I_{t-1} \equiv \int_0^{N_t} i_{t-1,j} dj$.

The first lemma states the property of the consumption price, the price at which consumers make their purchase, r_t. Unless household producers hold excessive inventories due to an unexpected shock, r_t equals P_L when p_t equals P_L (sales price). That is, consumers purchase goods directly from manufacturers at $r_t = P_L$. When $p_t = P_H$ (regular price), consumers may purchase goods from household producers at a price below P_H. Price r_t is lower the larger inventories I_{t-1} are.

Lemma 1 *Consumption price r_t satisfies $0 < r_t \leq P_H$. Suppose there is no large unexpected shock to b_t. Then r_t satisfies $P_L \leq r_t \leq P_H$. When $p_t = P_L$, $r_t = P_L$. Furthermore, $r_t = r(I_{t-1}, p_t, b_t)$ is nondecreasing in p_t and b_t and nonincreasing in I_{t-1}.*

The proofs of this and the lemmas that follow are provided in Appendix C. The next lemma states the stockpiling behavior of household producers. Only when $p_t = P_L$ do household producers purchase goods and hold inventories with the aim of selling the goods at a higher price after the sale has ended.

Lemma 2 *Suppose there is no large unexpected shock to b_t. If $p_t = P_H$, household producers do not purchase goods, that is, $x_t = 0$. If $p_t = P_L$, household producers purchase goods and hold inventories. Inventories are independent of i_{t-1}, I_{t-1}, and b_t.*

The left-hand panel of Figure 8 illustrates the pattern of price and quantity changes during a sales event when inventories are held just for one period. The top and bottom panels show prices and quantities, respectively. The sales event takes place in periods $t = 2$ and 3, when the price is lower than during other periods. The bottom panel shows that, in period $t = 2$, quantities purchased by household producers and consumers, X_t, represented by the solid dot, increase. The reason is not only that households consume more but also that they stockpile. Thus, X_t is higher than consumption, c_t, represented by the circle, with the difference representing stockpiling. In period $t = 3$, X_t coincides with c_t, since there is no additional need for stockpiling. Then, in period $t = 4$ when the sale ends, household producers sell their inventories to consumers. The consumption price r_t, at which the household producers and consumers transact, lies between P_L and P_H. Since consumption decreases with the consumption price, its level in period $t = 4$ is lower than in periods $t = 2$ and 3 but higher than in periods $t = 1$ and 5. At the beginning of $t = 5$, household producers hold no inventories, so consumers purchase goods at price P_H and consume less.

Stylized Fact 2

The following lemma explains Stylized Fact 2.

Lemma 3 *Suppose there is no shock, i.e.,* $b_t = b$*. Then the quantity purchased by household producers and consumers just before a sale is greater than or equal to that just after a sale.*

The quantity purchased by household producers and consumers on the first day of a sale is greater than or equal to that on the final day of a sale.

The following remark is in order. In the previous section, we found that the latter part of Stylized Fact 2 is not necessarily robust; that is, we found that, for many product categories, the quantity purchased during the first half of a sale is smaller than that purchased during the second half of a sale. Whether the quantity purchased during the first half of a sale is smaller than that purchased during the second half of a sale or not depends on whether prices are stochastic. In the above model, prices are stochastic, and neither household producers nor consumers can accurately predict future prices. This is why household producers hold inventories at the end of the first day of a sale. Suppose instead that the duration of a sale is known *ex ante*. In this case, there is no incentive for household producers to stockpile except for the final day of the sale. Thus, the quantity purchased by household producers and consumers on the first day of a sale should be smaller than or equal to that on the final day of a sale.

Stylized Fact 3

Consumption c_t is clearly state dependent; specifically, it depends on consumption price r_t.

Lemma 4 *Suppose there is no shock, i.e.,* $b_t = b$*. Then* c_t *decreases in* r_t*. After a sale ends,* r_t *and* c_t *are nondecreasing and nonincreasing over time, respectively, until the next sale begins.*

Stylized Fact 1

Our model succeeds in explaining the chain drift. We denote the change in the Törnqvist price index based on consumption weights by π_t^{T*}. Moreover, we introduce the degree of stockpiling $m(\geq 0)$, which indicates how many days' worth of inventories remain in the hands of household producers. If $m = 0$, no stockpiling occurs during a sale and hence no chain drift due to sales arises. Therefore, what we are interested in is the case where $m > 0$. In the next section, we will specify how m is determined. In Appendix C, we discuss how equilibrium is determined in the special case of $m = 1$, that is, when inventories are cleared in just one period after a sale.

Lemma 5 *Consider one sales event for product k such that $p_t = p_{t+T+1} = P_H$ and $p_{t+j} = P_L$ for $j = 1, \cdots, T$ ($T \geq 1$). Suppose that the prices and quantities of other goods remain unchanged; $\sum_{k' \in K_0 \cap K_t} p_t^{k'} x_t^{k'} = 1$; the price before and after the sale is P_H for a sufficiently long duration (i.e., $p_{t-j} = p_{t+T+1+j} = P_H$ for $j = 0, 1, \cdots, T_H$, where T_H is sufficiently large compared with m); and $b_t^k = b^k$.*

If $\sigma > (<) 1$ and $m > 0$, the change in the price index from t to $t+T+1+T_H$ satisfies $\pi^{COLI} = 0$; $\pi^L > (<) 0$; $\pi^P < (>) 0$; $\pi^T < 0$; and $\pi^{T*} < (>) 0$.

If $\sigma = 1$ or $m = 0$, then $\pi^{COLI} = \pi^L = \pi^P = \pi^{T*} = 0$ and $\pi^T < 0$.

If $\sigma > 1$, $\pi^P < \pi^T < 0 < \pi^L$. If $\sigma > 1$ and $m = 1$, $\pi^T < \pi^{T*} < 0$.

It is well known that the Törnqvist index is a good approximation of the COLI up to the second order (Diewert, 1976). While the purchase-based Törnqvist index clearly performs poorly when goods are storable, the above lemma also suggests that using consumption weights does not eliminate the chain drift from the Törnqvist index (i.e., $\pi^{T*} < 0$ if $\sigma > 1$).

The reason is the systematic asymmetry due to the non-negative constraints on inventories and purchases (equation (12)). As Figure 8 shows, the response of the consumption price to an increase and a decrease in the posted price is asymmetric. While a decrease in the posted price causes an instantaneous decrease in the consumption price, the response of the consumption price to an increase in the posted price is gradual. Such asymmetry is not a coincidence but a natural outcome of the non-negative constraints on inventories and purchases. Thus, the value of the Törnqvist index after a sale always deviates from the value before a sale if $\sigma > 1$. In other words, while the Törnqvist index is a good approximation of the COLI up to the second order, the third-order error cannot be cancelled out by the decrease and the subsequent same-sized increase in the posted price.

To resolve the chain drift and derive a better approximation of the COLI, we need a superlative index that takes the elasticity of substitution σ into account. One candidate is the order r superlative index, where we define P_r as

$$P_r(r_0, r_1, c_0, c_1) = \frac{\left\{\sum_{k \in K_0 \cap K_1} s_0^k \left(\frac{r_1^k}{r_0^k}\right)^{(1-\sigma)}\right\}^{1/\{2(1-\sigma)\}}}{\left\{\sum_{k \in K_0 \cap K_1} s_1^k \left(\frac{r_0^k}{r_1^k}\right)^{(1-\sigma)}\right\}^{1/\{2(1-\sigma)\}}}, \tag{20}$$

where s_t^k represents the consumption share of product k in period t.

The following lemma shows that P_r serves as a COLI if the unit cost function is expressed as

$$\mathcal{C}(r_t) = \left[\sum_{i \in K_0 \cap K_1} \sum_{k \in K_0 \cap K_1} \alpha^{ik} \left(r_t^i\right)^{(1-\sigma)} \left(r_t^k\right)^{(1-\sigma)}\right]^{1/\{2(1-\sigma)\}} \tag{21}$$

where $\alpha^{ik} = \alpha^{ki}$. This cost function is based on a more generalized form of utility than that given by equation (8).[19]

Lemma 6 *Given the unit cost function of (21), P_r equals $\mathcal{C}(r_1)/\mathcal{C}(r_0)$.*

4 Inference of Consumption and Consumption Prices and Calculation of the COLI

4.1 Our Approach

The previous section showed that our model can explain the stylized facts. However, many important variables such as consumption, inventories, and the consumption price remain unobservable, although we can observe the quantity purchased and posted price using retailer-side POS scanner data. From a practical perspective, consumption and the consumption price are essential for constructing the COLI. For macroeconomists, it is of great interest to see whether any changes in stockpiling behavior can be observed over time and, if so, what the determinants are. Therefore, in this section, we propose a simple and tractable methodology to infer these variables using retailer-side scanner data.

Some of the previous studies on households' inventory behavior, including Erdem, Imai, and Keane (2003) and Hendel and Nevo (2006b), structurally estimate the parameters associated with inventory cost functions, which would be helpful in identifying

[19] Another index that can serve as a COLI is the Lloyd–Moulton Index. See ILO et al. (2004).

the paths of consumption, inventories, and consumption prices. These studies employ household-side scanner data, where the quantities and prices of the goods purchased by individual households are recorded, to conduct their empirical exercises. One of the datasets we employ in the present study, the *Shoku-map* data, is similar to the datasets they use, and it contains information on purchases and consumption at the household level. However, it lacks information on purchase prices, making it difficult for us to take an approach similar to theirs. More importantly, few practitioners at national statistical offices have access to household-side scanner data, so that an approach for constructing a COLI based on consumption and consumption prices that requires household-side scanner data would not be very useful for them. Furthermore, the empirical exercises conducted in the previous studies focus on a particular product (e.g., ketchup in Erdem, Imai, and Keane (2003) and detergent in Hendel and Nevo (2006b)) to estimate the parameters associated with the inventory cost function. The task we have set ourselves in this study is quite different, in that we seek to estimate a COLI which covers numerous products that are heterogeneous in terms of their storability and the form of their inventory cost function. However, reliably estimating inventory cost functions for all products likely is next to impossible, so that we use a different approach.

Specifically, we employ retailer-side scanner data rather than household-side scanner data to construct a COLI. Retailer-side scanner data, which record transactions at the store level rather than at the household level, are widely used by national statistical offices in an attempt to construct COLIs using alternative data sources (see, for example, chapter 10 of ILO et al., 2020). Because it is almost impossible to obtain reliable estimates for the inventory cost functions of all products, we introduce an assumption about the path of consumption prices after a sale ends; that is, we assume that the increase in consumption prices after a sale ends is *linear* with time. This assumption allows us to estimate consumption, consumption prices, and inventories from retailer-side scanner data and, as explained later, is consistent with the model developed in the previous section. Note that previous studies on household inventories often assume that consumption is constant over time (e.g., Boizot, Robin, and Visser, 2001 and the empirical part of Hendel and Nevo, 2006a). The assumption of constant consumption allows us to infer consumption, consumptions prices, and inventories, but this is not consistent with the fact we showed in Section 2, that is, people consume more when they have more inventories. In contrast, our assumption that consumption prices increase linearly with time after a sale ends is consistent with state-dependent consumption.

4.1.1 Methodology

We assume that there are no changes in demand (taste or quality) for each product, i.e., $b_t = b$. Thus, the consumption price at $p_t = P_L$ equals P_L. The key variable is the consumption price at $p_t = P_H$ after a sale ends, that is, $r_H(I_{t-1}) \equiv r(I_{t-1}, P_H)$, where $r_H(0) = P_H$. Knowing this variable enables us to obtain consumption c_t after a sale using equation (9).

When $p_t = P_H$, household producers' optimization problem is given by

$$C'(i_H; I_{t-1}) = \beta \{(1-\overline{q}) r_H(I_t) + \overline{q} P_L\} - r_H(I_{t-1}) + \mu_t \tag{22}$$

from equation (14). This equation shows that household producers strike a balance between the benefits of a future consumption-price increase (the right-hand side) and the costs of holding inventories (the left-hand side). Note that μ_t is the Lagrange multiplier associated with i_t and equals zero if $i_t > 0$.

We note that when inventory cost function $C(\cdot)$ is written in a certain form (see Appendix D), $r_H(I_t) - r_H(I_{t-1})$ becomes a positive constant. In other words, the expectation of a *linear* consumption-price increase prevents household producers from selling all of their inventories instantaneously or from selling none at all. Household producers gradually sell off their inventories to consumers.

In the following analysis, we assume this linearity holds.[20] While this admittedly is a restrictive assumption, it can be interpreted as an intermediate of the following two scenarios. The first is when $C'' \to 0$. In this scenario, $r_H(I_t) - r_H(I_{t-1})$ increases in t. Put differently, if inventory costs are not convex, household producers require a greater consumption-price increase as time goes by, because they discount the future ($\beta < 1$) and expect another sale to come at some point ($\overline{q} > 0$). As for the second scenario, suppose $\beta = 1$ and $\overline{q} = 0$. Then $r_H(I_t) - r_H(I_{t-1})$ decreases in t. The cost of holding inventories decreases as household producers' inventories decrease because of $C'' > 0$, which makes household producers require a smaller consumption-price increase as time goes by.

The benefit of this linearity assumption is that it greatly simplifies our analysis. Given the path of $r_H(I_{t-1})$, we compute consumption c_t as $(r_H(I_{t-1})/P_L)^{-\sigma} c_L^*$, where c_L^* represents consumption during a sale, which we specify below. Furthermore, we can calculate the degree of stockpiling, m. More precisely, we define m to denote how

[20] In Section 4.4.3, we try different approaches to infer consumption and the consumption price.

long inventories last after a sale ends. In continuous time, we can derive the following equation:

$$m_{cont} = \frac{P_H - P_L}{P_L} \frac{\sigma - 1}{1 - (P_H/P_L)^{-\sigma+1}} \frac{I_L}{c_L^*}. \tag{23}$$

The proof is provided in Appendix D. Simply put, the equation can be derived because cumulative consumption for m_{cont} periods equals the initial inventories outstanding just after a sale ends, I_L, and the consumption price linearly increases from P_L to P_H in m_{cont} periods. The right-hand panel of Figure 8 illustrates the pattern of price and quantity changes in the case of $m = 5$. The consumption price, depicted by the circles in the top panel, increases at a constant rate from $t = 3$ to 8. The bottom panel shows that while the quantity purchased falls to zero from $t = 4$ to 7, consumption does not fall to zero, but decreases gradually.

In Appendix D, we provide a detailed explanation of how we calculate $r_H(I_{t-1})$ and m as well as c_L^* and I_L for each sales event s of product k and retailer r. Briefly put, c_L^* is the lower value of the quantity purchased (observable in the POS data) during a sale. Note that, according to the model, consumption during a sale is equal to purchases except for the first day of the sale, since there is no need for further stockpiling. Inventories I_L equal the cumulative amount of purchases during a sale minus the cumulative amount of consumption, Tc_L^*, in the same period.

4.1.2 The Elasticity of Substitution

This approach is also useful for obtaining the value of the elasticity of substitution, σ.[21] Note that the estimate of the elasticity of substitution is biased unless we properly take account of stockpiling. Even if there is a clear negative relationship between observable purchases and posted prices, this does not necessarily indicate the elasticity of substitution. As argued by Hendel and Nevo (2004), price reductions influence the quantity purchased not only via the consumption effect (consumption is price sensitive) but also via the stockpiling effect (i.e., consumers stockpile for future consumption). Owing to the latter effect, which is often larger than the former, the elasticity of substitution is likely to be overestimated when we ignore stockpiling.

Our approach makes it possible to obtain the value of σ. For each 3-digit product

[21] The elasticity we measure in this study represents the short-run own price elasticity as opposed to the long-run elasticity, which ignores stockpiling, and the cross-price elasticity, which compares different products.

category, for each sales event of product k and retailer r, we collect the records of the log ratio of the quantity consumed during a sale to the quantity consumed when the product is sold at the regular price divided by the log ratio of the sale price to the regular price, that is, $\Gamma \equiv -\log\left(c_L/c_H\right)/\log\left(r_L/r_H\right)$. Equation (9) suggests that Γ equals σ on average.[22] We calculate the unweighted average of Γ across sales events, products, and retailers for each 3-digit product category, which we define as σ.

4.2 Simulation Results

We employ numerical simulations to show the paths of the consumption price, consumption, and purchases by generating randomized price paths. Instead of specifying the form of the cost of inventories $C(i)$, we specify m and calculate other variables such as inventories, consumption, and the consumption price so that they are consistent with this specification. The probability of sales is governed by $\overline{q} = 0.03$ and $\underline{q} = 0.50$ and their size is given by $P_L/P_H = 0.9$. These values are chosen so that they are consistent with our POS data. The degree of stockpiling m and the elasticity of substitution σ are not directly observable in our data, but the above approach enables us to infer them. For the simulation, we set them at 5 and 4, respectively. We generate randomized price paths for a period of $T = 405$ days and discard observations for the first and last 20 days to calculate changes in the price indices over 365 days. The number of storable goods is 100 ($k = 1, 2, \cdots, 100$). We repeat this simulation $N = 100$ times.[23]

The left-hand panel of Figure 9 depicts a typical path of the price and quantity. This confirms Stylized Fact 2. That is, the quantity purchased falls to zero for m periods after the sale ends, which is smaller than the quantity purchased just before the sale. When the sale lasts two days, the quantity purchased is greater on the first day than on the second day. Furthermore, we find that, for m periods after the sale ends, the consumption price r_t increases at a constant rate from P_L to P_H, while consumption decreases nonlinearly because $\sigma > 1$.

We calculate the price indices based on the COLI, the chained order r superlative, the chained consumption-weighted Törnqvist, the chained purchase-weighted Törnqvist,

[22] It can be assumed that prices are exogenous for households, especially over such a short time horizon. Thus, any bias from endogeneity is unlikely to be a major problem.

[23] Using consumption price r_t^k, we calculate the unit cost λ_t from equation (19) and then consumption c_t^k from equation (18). Note that c_t^k for product k can take different values even under the same value of r_t^k, becaues consumption prices for other products may differ.

the chained purchase-weighted Laspeyres, and the chained purchase-weighted Paasche. For each $n \in N$ and price index, we calculate the price level after 365 days by setting the initial price level to one. We then take the mean and the standard deviation of the price levels of the different n.

Row (1) of Table 5 shows the benchmark simulation results, while the other rows show the simulation results when we use different approaches to inferring consumption and the consumption price (see Section 4.4.3). The COLI and the chained order r superlative index do not show any chain drift and asymptotically return to their original level. The chained consumption-weighted Törnqvist index has a downward chain drift of 1% annually. The purchase-weighted price indices have much larger chain drift: the Törnqvist index decreases to $10^{-0.8}$ of its original value (decrease by 84%), the Laspeyres index increases to $10^{1.4}$ of its original value (increase by a factor of 27), and the Paasche index decreases to $10^{-3.0}$ of its original value (decrease by 99.9%). The size of the chain drift in the purchase-weighted Törnqvist index is comparable to the actual size of the chain drift in the Törnqvist index for Japan.

4.3 Application to Japanese Retailer Scanner Data

4.3.1 Paths of Prices and Quantities

We apply the approach explained in Section 4.1.1 to Japanese POS scanner data. The right-hand panel of Figure 9 shows the actual paths of the price and the quantity purchased of a particular brand of instant cup noodles at a particular retailer. While these paths are observable, the paths of the quantity consumed and the consumption price are not. Our approach enables us to infer them, as shown in the figure. The paths of the consumption price and the quantity consumed based on the POS data resemble those obtained in the simulation depicted in the left-hand panel of Figure 9.

4.3.2 The Degree of Stockpiling

Figure 10 presents a histogram of the degree of stockpiling m_{cont} at the 3-digit product category level (hereafter we simplify the notation by dropping *cont* from m_{cont}). The distribution of m ranges from 1.0 to 5.6, with the mode being around 2. Table 4 lists the top and bottom five product categories with the largest and smallest m. The top three categories are instant cup noodles, diluted beverages, and frozen meals, in that order. These products can indeed be stored for a long time. However, the storability of products does not necessarily imply a high degree of stockpiling. In fact, the three bottom categories are razors, chilled condiments (e.g., sauces, dressings, and soba soup), and cosmetic accessories (e.g., hand cream and sunscreen lotion). The products in these categories are also highly storable. What distinguishes them from storable goods such as cup noodles is that once they are opened, they can continue to be used for a long time. That is, whereas cup noodles, for example, are consumed more or less immediately after they are opened, razors, chilled condiments, or cosmetic accessories can often be used for weeks or even months. As a result, consumers tend not to purchase more than two units even when they are on sale. This suggests that the degree of stockpiling, m, may be negatively correlated with the length that a product lasts once it is opened.

The degree of stockpiling is highly heterogeneous even within products belonging to the same product category. To illustrate this, we collect m for each retailer, product, and sales event for the product categories of instant cup noodles and tofu products, and draw the cumulative distribution for the degree of stockpiling ($m < m^*$). Figure 11 shows that while around half of the observations exhibit the lowest value of m (i.e. $m = 1$), instant cup noodles have a thicker right tail than tofu products. Around one tenth of the observations of instant cup noodles exhibit a value of m higher than 100 days. On the other hand, because tofu products expire more quickly, they are stockpiled less during a sale than instant cup noodles.

4.3.3 The Elasticity of Substitution

Figure 12 shows the value of the elasticity of substitution σ for 3-digit product categories, calculated from $\Gamma = -\log(c_L/c_H)/\log(r_L/r_H)$. The left-hand panel displays the histogram of σ and shows that σ is distributed smoothly around the mode of three and most of the values are positive.

For comparison, a simple calculation of σ is possible if we ignore storability. Hendel and Nevo (2004) argue that neglecting stockpiling leads to an overestimation of σ by a factor of two to six. To confirm this, we obtain σ by simply calculating the slope of $-\Delta\log X_t/\Delta\log p_t$ using the observed series of purchases X_t and posted prices p_t, where Δ is the difference from the previous date. In this simple calculation, we use all observations as long as the posted price changes by more than two yen from the previous date. Note that, according to equation (9), the simple calculation would be valid if c_t and r_t equal X_t and p_t, respectively.

The dotted line in the left-hand panel shows the histogram of the simple estimate of σ, which is distributed to the right of the solid line (our estimate). This result is in line with Hendel and Nevo's (2004) result, suggesting that σ is overestimated if we ignore storability. The right-hand panel shows the scatter plot of the value of σ, where each dot represents a 3-digit product category. In most categories, the simple estimates are larger than our estimates. Nevertheless, the two measures are not independent of each other and exhibit a positive correlation. Finally, it should be noted that the estimates of σ are negative for some product categories (although they are not necessarily significant), whereas from a theoretical perspective we would expect them to be positive. In what follows, we aggregate the variables of interest at the 3-digit category level only when σ is greater than 1.0.

4.4 Validity Checks of Our Approach

4.4.1 Comparison of Inflation Rates: Data and Simulation

To check the validity of our approach, we first examine whether the size of the simulated chain drift is comparable to that of the actual chain drift. To obtain the latter, we calculate the daily average of the inflation rate from January 1989 to December 2011 based on the purchase-weighted Törnqvist index for each 3-digit product category j (denoted by π_j) using the POS data. The first and last 12-month periods of the data are omitted from the calculation because identifying sales events is difficult when data are censored. At the same time, we record the average of the following variables for each 3-digit product category: the degree of stockpiling, m_j; the probability that a product will go on sale on the following day given that it is not currently on sale, $\overline{q_j}$; the probability that a product will continue to be on sale on the following day given that it is currently on sale, $\underline{q_j}$; and the size of the sale discount, $(P_L/P_H)_j$. Then, using these values, we simulate the model and calculate the average size of the chain drift by employing the method discussed in Section 4.2. The elasticity of substitution σ is set to 5 for all product categories.

Figure 13 shows that our approach does a reasonably good job in explaining the chain drift. In the figure, each circle represents a product category. The figure shows that product categories that have a large chain drift in the data tend to exhibit a large chain drift in the simulation as well. The circles tend to lie below the 45 degree line, suggesting that the actual chain drift is smaller than the simulated chain drift. However, the quantitative difference is not large except for two categories, frozen staple foods and frozen meals, which entail large chain drift in the simulation.[24]

4.4.2 Comparison with the *Shoku-map* Data

To further check the validity of our approach, we compare the degree of stockpiling (m) obtained from the POS data with related variables in the *Shoku-map* data. Specifically, we look at the duration of inventories defined as the difference between the date of purchase (t_p) and the date on which the item is used up (t_l). Using the cross-sectional dispersion of m at the 3-digit product category level, we examine if there is any significant correlation. The left-hand panel of Figure 14 shows that there is no significant correlation between $\log(m)$ in the POS data and $\log(t_l - t_p + 1)$ in the *Shoku-map* data. This suggests that m does not necessarily represent storability.

However, a significant correlation arises when we look at the change in $\log(m)$ in March 2014. On April 1st, 2014, the consumption tax rate was raised from 5% to 8%. This was a preannounced event, which prompted households to stockpile before the tax hike.[25] To examine the effect of the consumption tax hike, we extend the observation period for the POS data to March 2014 and calculate the change in $\log(m)$ in March 2014 from the same month in the previous year. The right-hand panel of Figure 14 shows that the values of this variable are mostly positive. Furthermore, the correlation coefficient between this variable and $\log(t_l - t_p + 1)$ is quite high at +0.41. This evidence supports our interpretation that m captures the degree of stockpiling. The figure implies that the degree of stockpiling increases in response to an anticipated price increase and that goods with a longer inventory duration tend to be stockpiled more.

If m captures stockpiling, it should incorporate not only storability but also the quantity purchased. For this reason, the average quantity purchased in the *Shoku-map* data, $\log(q)$, may correlate with $\log(m)$ in the POS data. Figure 15 shows that this is indeed the case. Positive correlations are observed between $\log(q)$ and $\log(m)$ both for the average of the entire observation period and in March 2014. The correlation coefficient

[24] In Appendix E, we investigate how the inflation rate, π_j, depends on m_j, $\overline{q_j}$, $\underline{q_j}$, and $(P_L/P_H)_j$.

[25] In Japan, increases in the consumption tax rate—from 3 to 5% in April 1997, from 5 to 8% in April 2014, and from 8 to 10% in October 2019—have always been a major political and economic event, because they have been accompanied by large demand increases before the tax hike and persistent weak demand (recessions) after the hike. For this reason, Prime Minister Shinzo Abe postponed the latest hike, from 8% to 10% twice, once in 2014 and once in 2016.

is greater for March 2014 (+0.40) than the observation period overall (+0.31).

4.4.3 Robustness of Our Approach

It should be noted that the fact that goods are storable means that price indices will always be subject to chain drift. This is the case regardless of whether a simplifying assumption such as the one we introduce is employed. In fact, the model in the previous section succeeded in qualitatively explaining the chain drift in various price indices, including its sign, without relying on this simplifying assumption.

However, additional assumptions are needed for quantitative purposes such as investigating the size of the chain drift. Since the linearity assumption is only one of many possible assumptions, it is important to examine how much the size of the chain drift changes in response to different assumptions about the path of consumption or the consumption price after a sale ends. We do so using simulations. Specifically, we keep the same model setup as that explained in Section 4.2, including the size of the sale discount (P_L/P_H) and inventories outstanding during a sale (I_L), but vary the paths of consumption and the consumption price after a sale ends and the degree of stockpiling (m).

We employ two kinds of alternative approaches to inferring consumption and the consumption price. The first approach is to assume a linear consumption decrease instead of a linear consumption-price increase after a sale ends. It can then be shown that the consumption price follows

$$r(x) = \left\{ \frac{x}{m_{cont}} \frac{c_H - c_L^*}{c_L^*} + 1 \right\}^{-1/\sigma} P_L, \tag{24}$$

where x represents the time elapsed after a sale ($0 \leq x \leq m_{cont}$). Clearly, the pace of consumption-price increase is no longer constant unless $\sigma = 1$. The second approach is to set m slightly lower or higher than its benchmark value ($m = 5$), and to find the value of the concavity/convexity parameter γ_0 that describes the pattern of increase in the consumption price:

$$r(x) = \left(P_L^{1/\gamma_0} + \gamma_1 x \right)^{\gamma_0}. \tag{25}$$

The value of γ_0 is one when the consumption price increases linearly. See Appendix F for details.

Rows (2) to (6) of Table 5 show the simulation results when we use the alternative approaches. The assumption of a linear consumption decrease causes the chained purchase-weighted Törnqvist index to slightly increase from 0.163 to 0.168 (row (2)). The concave consumption-price increases with $m = 3$ and 4 also cause the chained purchase-weighted Törnqvist index to slightly increase from 0.163 to 0.172 and 0.168, respectively (rows (3) and (4)). The convex consumption-price increases with $m = 6$ and 7 decrease the chained purchase-weighted Törnqvist index slightly from 0.163 to 0.159 and 0.154, respectively (rows (5) and (6)). From these results, we can safely conclude that the alternative approaches leave our previous quantitative results more or less unchanged.

5 Empirical Results

In this section, we present our main results regarding price indices and developments in households' stockpiling behavior.

5.1 Price Indices

Using the POS data, we calculate the time-series of the price level based on the following three definitions. The first is the Törnqvist index based on the purchase weight. This is the conventional approach and we showed the results in Figure 1. The second definition is the Törnqvist index based on the consumption weight. Here, we use the consumption price as well as consumption to calculate the price index. The third definition is the order r superlative index, where we use the estimated elasticity of substitution σ at the category level.

Figure 16 shows the time-series paths of the three price indices, where the initial price level is normalized to one. The upper part of Table 6 shows the means and standard deviations for inflation rates based on the three price indices from 1990 to 2012. The mean of the inflation rates for the Törnqvist index based on the purchase weight is around -50% annually, which is by far the lowest.[26] In the consumption-based Törnqvist and order r superlative indices, the substantial downward bias in the conventional consumption-based Törnqvist index is mitigated. However, the downward bias is not completely

[26] The size of deflation based on the Törnqvist index in Table 6 is slightly smaller than that shown in the right-hand panel of Figure 1 (when $dt = 1$). This is because we did not interpolate missing observations to calculate the price index for the right-hand panel and the aggregation methodology for drawing the right-hand panel, which follows the procedure employed in by Watanabe and Watanabe (2014), is different.

eliminated. While the rate of deflation is slightly smaller in the case of the order r superlative index, both consumption-based indices exhibit deflation of approximately 10% annually. Thus, both indices still appear to exaggerate price changes. However, recall that our data frequency is daily. Thus, cumulatively, even a tiny chain drift in the daily data becomes large over a longer time horizon. For example, an annual price change of -10% annually equates to a daily price change of only -0.03%.

Moreover, large deflation does not necessarily mean our approach fails to eliminate the chain drift due to stockpiling. Other factors might cause a bias in those indices. One such factor could be product turnover (product creations and destructions). Product turnover entails changes in product quality, which likely influence price indices. Ueda, Watanabe, and Watanabe (2019) show that product prices tend to decline over the lifespan of a product. In particular, new products with a short life span often experience a large price decrease during the first few months. Such a pattern leads to a downward trend in price indices.[27]

To take this into account, we examine how much the chain drift decreases when we exclude the effect of product turnover. To do so, we calculate the time-series paths of the three price indices as before but now use only long-selling products. Long-selling products are defined as products that are recorded in the POS data both before January 1, 1990 and after December 31, 2012. Figure 17 and the lower part of Table 6 show that while the purchase-based Törnqvist index continues to exhibit a considerable downward bias, the consumption-based Törnqvist and the order r superlative indices have much smaller drift when using long-selling products than when using all products.[28] In particular, the order r superlative index appears to have no downward chain drift. The index moves around the original level of one, and the average inflation rate is only around 1.8% annually. This result suggests that our approach succeeds in eliminating the chain drift that arises from stockpiling.

[27]Another possible factor is that we do not consider changes in purchases over the course of a week or month. For example, quantities purchased tend to be larger on weekends than on weekdays. Moreover, the timing of wage or pension payments may well generate cyclicality in purchases over the course of a month if financial constraints matter. If purchases are subject to weekly or monthly cyclicality, consumption may also have a similar cyclicality.

[28]The share of these long-selling products in all products is about 1.2% in terms of the amount of consumption (consumption price r times the quantity consumed c). Note also that for all the price indices we consider in this study, products need to be recorded in the POS data for at least two days, since otherwise we cannot compare price changes from the previous date.

Finally, Figure 18 shows the annualized inflation rates of the following three price indices. The first index is the Törnqvist index using consumption weights and all products. The second is the Törnqvist index using consumption weights and only long-selling products. For both these indices, we calculate the annualized inflation rate by cumulating the daily log inflation (x_t) over the past 365 days. The last index is the CPI for groceries, and we calculate the inflation rate for each month as the change in the index from the same month in the previous year. Although the three indices occasionally show similar changes—for example, all three indices indicate an increase in the inflation rate around 2007–08—, their ups and downs are very different in most of our observation period. For example, around 2007–08, the first index appears to have increased before the other two indices. This suggests that the Törnqvist index using consumption weights and all products provides a different, and possibly more timely and useful, perspectives on inflation developments.

5.2 Developments in Households' Stockpiling Behavior

5.2.1 Changes in Stockpiling Behavior

The degree of stockpiling m is not only heterogeneous but also time-varying. The line with dots in Figure 19 shows the time-series developments in aggregate $\log(m_t)$ from January 1989 to December 2011. The line indicates that there has been a secular decrease in the last two decades.[29]

It should be noted that m can change as a result of changes not only in households' intrinsic behavior but also in prices, which are exogenous to households. To show this formally, we use equation (14) when $p_t = P_L$. Household producers optimize their inventories during a sale to satisfy

$$C'(i_L; I_{t-1}) = \beta \left\{(1-\underline{q}) r_H(I_t) + \underline{q} P_L\right\} - P_L + \mu_t. \tag{26}$$

If a sale ends in period t, the consumption price increases in period $t+1$. This provides household producers with a profit if they hold inventories, as the right-hand side of the

[29]The aggregate $\log(m)$ is obtained as follows. First, we calculate semi-aggregate $\log(m_j)$ for 3-digit product category $j = 1, 2, \cdots, 145$ by aggregating $\log(m_{ks})$ for product $k \in j$ and sales event s, assigning equal weights. Second, we take their unweighted average across categories. This aggregation method is slightly different from that explained in Appendix A, where we aggregate $\log(m_{ks})$ for product k and sales event s, assigning equal weights. However, this difference has little effect on the size of aggregate $\log(m)$.

equation shows. However, household producers incur a cost when holding inventories, as shown in the left-hand side of the equation. Suppose $C'(i_t; I_{t-1}) = C > 0$ and linear consumption-price increases. Then we have

$$C = \beta(1-\underline{q})\left\{\frac{1}{m}\frac{P_H - P_L}{P_L} + 1\right\} P_L - (1-\beta\underline{q})P_L,$$

and, in turn,

$$m = \beta(1-\underline{q})\frac{P_H - P_L}{P_L}\left(1 - \beta + \frac{C}{P_L}\right)^{-1}. \tag{27}$$

This equation suggests that the degree of stockpiling m negatively depends on the probability that a sale will continue to occur at $t+1$ given that a sale occurs at t, positively depends on the size of the sale discount, and negatively depends on the cost of holding inventories. In other words, more stockpiling occurs the sooner a sale is expected to end and the larger the sale discount is. More generally, $\overline{q}$ is also likely to influence r_H and, in turn, m. Thus, $\overline{q}$, $\underline{q}$, and $(P_H - P_L)/P_H$ should constitute explanatory variables for m.

Therefore, we regress the following equation for $\log(m_{jt})$:

$$\log(m_{jt}) = c_j + d_t + AX_{jt} + \varepsilon_{jt}, \tag{28}$$

using the vector of explanatory variables $X_{jt}= \{\log(1-\overline{q_{jt}}), \log\left(1-\underline{q_{jt}}\right), \log((P_H - P_L)/P_H)_{jt}\}$, where j and t represent the 3-digit product category and the month (January 1989 to December 2011), respectively. Time fixed effect d_t captures the aggregate, demand-side, time-varying component of m_{jt}. Some of the variables we use are persistent and close to an I(1) process. Thus, to avoid spurious regression, we also estimate the above equation using the time differences of the variables such as $\Delta\log(m_{jt}) \equiv \log(m_{jt}) - \log(m_{jt-1})$.

Table 7 presents the estimation results. In the table, column (1) shows the result when we use $\log(m_{jt})$ as the dependent variable and do not include the time fixed effect. Column (2) shows the result when we use $\log(m_{jt})$ as the dependent variable and include the time fixed effect. Column (3) shows the result when we use the time difference of $\log(m_{jt})$ as the dependent variable and do not include the time fixed effect.

The coefficient on $\log((P_H - P_L)/P_H)$ is significantly positive in all columns. The positive relationship between $(P_H - P_L)/P_H$ and m is consistent with the relationship derived from equation (27). The coefficient on $\log(1-\overline{q})$ is insignificant when the time fixed effect is included (column (2)), while it is significantly positive without the time fixed effect (column (1)) and negative in the regression that uses time differences (column (3)).

The coefficient on $\log\left(1-\underline{q}\right)$ is significantly negative in all columns, even though equation (27) suggests it should be positive. One possible reason is that $\underline{q}$ might be endogenous. It should be noted that in this regression, the cost of inventories is not controlled for. For product categories with low inventory costs, m is likely to be high. If firms hold longer sales (high $\underline{q}$) for these products, we would expecte to observe a negative coefficient on $\log\left(1-\underline{q}\right)$ rather than a positive one. Another reason is that in equation (23) m is proportional to I_L, which tends to increase as the duration of a sale T increases. Owing to this construction, our measure of m tends to increase as $\underline{q}$ increases.

The solid line in Figure 19 shows the time-series of time fixed effect d_t (column 2 in the table), which represents changes in the degree of stockpiling after controlling for the effects of price changes. Developments in d_t differ from those in aggregate $\log(m_t)$. Specifically, d_t exhibits a steady increase from the early 2000s, while aggregate $\log(m_t)$ does not. Although we do not show it here, we find that this deviation is explained by both the decrease in the probability of sales ($\underline{q}$) and the decrease in the size of sale discounts ($\log\left((P_H - P_L)/P_H\right)$).

5.2.2 Effects of Macroeconomic Variables on Stockpiling Behavior

What brought about the secular decrease in household stockpiling behavior (m_t and d_t) in the 1990s and then the reversal since the early 2000s (d_t, but not m_t)? Household stockpiling behavior likely is influenced by a number of factors, which we consider in this subsection.

Possible Channels First, according to equation (27), an increase in β increases stockpiling because households put greater weight on future consumption. Possible factors that may bring about changes in β are preference shocks, which are often incorporated in dynamic stochastic general equilibrium models as part of demand shocks and also cause changes in real interest rates. Specifically, preference shocks generate a negative relationship between stockpiling and real interest rates. Furthermore, it is also thought that higher inflation expectations promote stockpiling, which also yields a negative correlation between real interest rates and stockpiling. Thus, investigating developments in real interest rates should provide a clue as to how and why stockpiling has changed in Japan.[30] Over the last two decades, Japan has seen successive waves of monetary accommodation, leading to a decline in nominal interest rates, although, due to the zero lower bound on nominal interest rates, it is debatable whether real interest rates have declined as a result of monetary policy.

Second, stockpiling behaviour may be influenced by factors that change the cost of holding inventories C. According to equation (27), an increase in C decreases the incentive for stockpiling. Possible factors that may influence C include, for example, the size of houses and Japan's demographic structure (population aging). However, looking at data for the size of houses from the Housing and Land Survey conducted by the Ministry of Internal Affairs and Communications every five years shows that there has been a steady increase in both the average housing area and the average housing area per household member from 1993 to 2013 from $88.4m^2$ to $93.0m^2$ and from $29.8m^2$/person to $38.5m^2$/person, respectively. This suggests that C should have decreased monotonically and d_t increased monotonically. However, such a monotonic decrease did not occur, as shown in Figure 19. Population aging is also a monotonic development in Japan. Meanwhile, interest rates may influence C. For example, a higher interest rate increases borrowing costs, which may prevent households from stockpiling.

The third factor concerns labor market conditions. Consider the following two opposing hypotheses. Suppose that labor market conditions are unfavorable for households, that is, low labor demand brings about high unemployment, low hours worked, and low income. One hypothesis is that households face stricter financial (liquidity) constraints and are therefore unable to purchase as much as they would like when prices are low. In that case, unemployment has a negative effect on stockpiling, while hours worked have a positive effect. The other hypothesis is that when unemployment is high and hours worked are low, households have more time for shopping, which allows them to find products that are on sale and stockpile inventories. Also, a decrease in income may make households more price-sensitive. In that case, unemployment has a positive effect on stockpiling and hours worked have a negative effect.

Regression Bearing these factors in mind, we examine whether the degree of stockpiling m_t depends on the macroeconomic environment. We estimate the following equation:

[30] Similarly, interest rates and the cost of holding inventories are also important determinants of firm inventory investment (see, e.g., Kahn, 2016).

$$\Delta\log(m_{jt}) = c_j + B\Delta Z_t + A\Delta X_{jt} + \mu_{jt}, \tag{29}$$

or

$$\Delta\log(m_{jt}) = c_j + B\Delta Z_t + A\widetilde{\nu_{jt}} + \mu_{jt}, \tag{30}$$

where

$$\Delta X_{jt} = e_j + D\Delta Z_t + \nu_{jt}, \tag{31}$$

$$\widetilde{\nu_{jt}} \equiv \Delta X_{jt} - (\hat{e}_j + \hat{D}\Delta Z_t). \tag{32}$$

Here, ΔX_t is the time difference of the price variables used above from month $t-1$ to t, and ΔZ_t is the time difference of exogenous variables consisting of the unemployment rate, log hours worked, and the real interest rate from month $t-1$ to t and $t-2$ to $t-1$. The real interest rate in period t is defined as the overnight call rate in period t minus the actual inflation rate based on the CPI from t to $t+12$ (all in percent). We estimate the equation using the time difference to avoid spurious regression. Lagged variables for Z_t are added to incorporate the possibility that it takes time for labor market conditions and the real interest rate to influence household stockpiling behavior.[31]

In the first regression, we estimate the degree of stockpiling using ΔX_t as the independent variable. In the second regression, we use $\widetilde{\nu_{jt}}$, the residuals of equation (31) defined as (32), as the independent variable. Labor market conditions and the real interest rate ΔZ_t likely influence firms' pricing ΔX_{jt} as well. We examine this effect by estimating equation (31) and then calculate $\widetilde{\nu_{jt}}$ so that it is orthogonal to ΔZ_t. Using $\widetilde{\nu_{jt}}$, we evaluate the overall effect of ΔZ_t on $\Delta\log(m_{jt})$, which incorporates the indirect effect through ΔX_{jt}.

Table 8, particularly column (5), shows the main estimation results, while column (1) shows the estimation results when we simply use ΔX_{jt}. The effect of the unemployment rate on $\Delta\log(m_{jt})$ is small, because the two coefficients on the unemployment rate at t and $t-1$ more or less cancel each other out. The two coefficients on hours worked at t and $t-1$ are significantly negative, suggesting that longer hours worked decrease the degree of stockpiling. This result supports the hypothesis that longer hours worked

[31]Note that m_{jt} is not likely to influence Z_t because the former is a variable at the product category level.

decrease households' time for shopping, which prevents them from stockpiling inventories during sales, rather than the hypothesis focusing on households' financial constraints and predicting the opposite effect. The coefficient on the real interest rate at t is significantly negative, suggesting that a higher real interest rate decreases the degree of stockpiling. This is consistent with the reasoning mentioned above.

Columns (2) to (4) show the estimation results for equation (31). They imply that, in response to longer hours worked, firms change their pricing so that the frequency of sales decreases (low $\overline{q}$), the duration of sales increases (high $\underline{q}$), and the size of sale discounts increases (high $(P_H - P_L)/P_H$). The effects of the changes in the unemployment rate and the real interest rate on pricing are unclear, however.

5.2.3 Implications for the Macroeconomy and the Price Index

The previous section has shown that stockpiling in Japan depends on hours worked and real interests rates – in other words, it fluctuates with the business cycle. This result raises the question how quantitatively important the business-cycle dependence of stockpiling is for the macroeconomy. In Japan, it can be regarded as very important, since temporary sales make up about 30% of retailers' total revenue (see Sudo et al., 2018).

To examine this issue, we examine the extent to which stockpiling by households and pricing by firms is affected by business cycle fluctuations. Furthermore, we calculate the extent to which business cycle fluctuations affect the aggregate quantity purchased and the bias in the aggregate price index when this is calculated as a conventional purchase-based chained index.

To do so, we take the following steps. First, to examine the role of business cycles, we consider two types of exogenous shock: a shock to hours worked and a shock to the real interest rate. For hours worked, we assume a one-time positive shock of two standard deviations of the log difference of hours worked, which is 0.027 (2.7%). For the real interest rate, we assume a one-time positive shock of one percentage point, which is 1.00.

Second, we use the estimation results reported in columns (2) to (5) in Table 8 to examine the effects on $\overline{q}$, $\underline{q}$, $\log\left((P_H - P_L)/P_H\right)$, and m, respectively, as a result of these shocks. The effects are examined in terms of the sum of the two coefficients at t and $t-1$ multiplied by the size of the shocks.

Third, using the approach explained in Section 4.2, we simulate the time-series path of the quantities purchased and prices. The parameter values for $\overline{q}$, $\underline{q}$, $\log\left((P_H - P_L)/P_H\right)$, and m are initially at their cross-sectional and time-series means. We then change them by the amount obtained in the second step. We set σ to four.

Fourth, we calculate the mean of the amount of sales (price times quantity purchased), the quantity purchased, and the unit price (amount of sales divided by quantities purchased) for 365 days, 100 products, and 100 iterations. Moreover, we calculate the change in the price index over the 365 days based on the conventional purchase-based chained index.

The simulation results are shown in Table 9. They show that in response to a shock to hours worked, m decreases from 2.16 to 2.13, while in the case of an interest rate shock, it decreases to 2.12. Further, combined with the changes in firms' pricing, the decrease in m decreases the amount of sales by 0.11% and 0.08%, respectively, and the quantity purchased by 0.05% and 0.12%. In other words, when the economy is in good shape, households stockpile less during a sale, which decreases the average amount of sales and the quantity purchased. The size of these effects is by no means small, particularly when we consider that the size of the shock is relatively small, i.e., 2.7% for hours worked and 1 percentage point for the interest rate.

The size of the chain drift is also affected by these shocks. The conventional chained Törnqvist index based on purchase data exhibits a considerable downward bias, decreasing by 80.2% over the 365 days in the case of no shock. In the case of the hours-worked shock, this changes to 80.6%, while in the case of the interest rate shock, it changes to 79.6%. Thus, the size of the chain drift increases by 0.4 percentage points annually in the case of a hours-worked shock, while it decreases by 0.6 percentage points annually in the case of a interest rate shock. This difference stems from the different responses of the size of sale discounts: a positive hours-worked shock increases the size of sale discounts, while a positive interest rate shock decreases it. However, we can conclude that the effects of these macroeconomic shocks on the chain drift are quantitatively small compared with the size of the chain drift.

6 Conclusion

We investigated how consumer inventories influence the price index and the macroeconomy. Goods storability causes a tremendous degree of chain drift when the price index is based on purchases, since consumers tend to stockpile when prices are low (i.e., during a sale) and purchases exceed consumption. We proposed a tractable approach to infer consumption using data on purchases and prices and applied it to Japanese data. We showed that consumers' stockpiling behavior can be conveniently summarized by a single variable: the degree of stockpiling during a sale, which expresses how long inventories last after a sale ends. Applying the approach to POS data for Japan, we found that our approach of using a consumption-based index succeeds in explaining the chain drift in the Törnqvist index based on purchase data and in mitigating the chain drift. Furthermore, we showed that the degree of stockpiling not only depends on the macroeconomy but also has non-negligible effects on the macroeconomy.

Tasks for the future include, first, a more careful consideration of heterogeneity at the product and household levels. We found that there exists sizable heterogeneity in the degree of stockpiling across products. A more detailed investigation might shed new light on consumer inventory behavior. Equally important is the heterogeneity at the household level. Considering the possibility that stockpiling behaviour depends on the size of the family and home, income, age, etc., could provide new insights.

Second, we should apply our approach to a wider range of product categories than those covered in our data, processed food and daily necessities, which make up only about 20 percent of households' expenditure. For instance, prices for some storable goods (e.g., gasoline and fresh food), durable goods (e.g., clothing and personal computers), and services (e.g., travel) occasionally change substantially just like in a temporary sale, which seems to cause demand fluctuations similar to stockpiling. It is worth testing whether our approach is useful for the analysis of these product categories.

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Table 1: Reasons for the Chain Drift

		$t=1$	$t=2$	$t=3$
Product A	Price	p_A	$(1-r)p_A$	p_A
	Share	W_1	W_2	W_3
Product B	Price	p_B	p_B	p_B
	Share	$1-W_1$	$1-W_2$	$1-W_3$

Note: $0 < r < 1$. If the elasticity of substitution is greater than one, we would expect to observe $W_3 < W_1 < W_2$.

Table 2: Changes in the Quantity Purchased before, during, and after a Sale

	$\log(X_H^2/X_H^1)$		$\log(X_L^2/X_L^1)$		
			$\lvert P_L^1 - P_L^2\rvert < 1.0$		
Constant	-0.0107		-0.0067	0.0440	
	(0.0001)		(0.0001)	(0.0001)	
WeekendDays	0.2468	0.2471	0.0959	0.1190	0.1265
	(0.0002)	(0.0002)	(0.0002)	(0.0001)	(0.0001)
$\log(T)$		-0.0177			-0.0653
		(0.0002)			(0.0001)
$\log((P_L^1 + P_L^2)/2P_H)$		0.0616			
		(0.0011)			
$\log(P_L^2/P_L^1)$				-0.7761	-0.7698
				(0.0006)	(0.0005)
Category fixed effects	no	yes	no	no	yes
R^2	0.008	0.009	0.007	0.020	0.029
Observations	155,922,267	155,922,267	59,935,646	155,922,267	155,922,267
# of categories	214	214	214	214	214

Note: Figures in parentheses represent standard errors. All coefficients are significant at the 1% level.

Table 3: State-dependent Consumption

Dependent variable	1 if household uses product	Number of times a product is used
Inventory	0.0006895***	0.00410***
	(0.0000171)	(0.000901)
Observations	90,545,020	90,545,020
No. of HH	3,602	3,602
R^2	0.12	0.12
Fixed effects	HH/category	HH/category

Note: Figures in parentheses represent robust standard errors. ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

Table 4: Top and Bottom Five Categories for the Degree of Stockpiling

	Product category	m
Top 5	Instant cup noodles	5.65
	Diluted beverages	4.32
	Frozen meals	3.92
	Packaged instant noodles	3.65
	Packaged instant raw noodles	3.37
Bottom 5	Cake and bread ingredients	1.33
	Home medical supplies	1.31
	Cosmetic accessories	1.29
	Chilled condiments	1.27
	Razors	1.18

Note: The degree of stockpiling m is inferred using POS data.

Table 5: Simulation of the Chain Drift

		COLI	Order r superlative	Törnqvist (C)	Törnqvist (Q)	Laspeyres (Q)	Paasche (Q)
(1)	Linear r	1.000	1.000	0.991	0.163	2.71e+01	1.01e-03
	increase	(4.60e-03)	(4.60e-03)	(4.58e-03)	(1.09e-02)	(3.80e+00)	(2.10e-04)
(2)	Linear C	1.000	1.000	0.991	0.168	2.45e+01	1.18e-03
	decrease	(4.60e-03)	(4.60e-03)	(4.58e-03)	(1.09e-02)	(3.26e+00)	(2.35e-04)
(3)	Highly concave r	1.000	1.000	0.992	0.172	2.21e+01	1.37e-03
	increase	(4.00e-03)	(4.00e-03)	(4.00e-03)	(1.09e-02)	(2.83e+00)	(2.62e-04)
(4)	Concave r	1.000	1.000	0.991	0.168	2.45e+01	1.18e-03
	increase	(4.37e-03)	(4.37e-03)	(4.36e-03)	(1.09e-02)	(3.26e+00)	(2.35e-04)
(5)	Convex r	1.000	1.000	0.991	0.159	3.01e+01	8.61e-04
	increase	(4.76e-03)	(4.76e-03)	(4.75e-03)	(1.09e-02)	(4.44e+00)	(1.88e-04)
(6)	Highly convex r	1.000	1.000	0.991	0.154	3.34e+01	7.34e-04
	increase	(4.87e-03)	(4.87e-03)	(4.85e-03)	(1.09e-02)	(5.14e+00)	(1.67e-04)

Note: The table shows the means of the price levels after 365 days, where the initial price level is set to one (so that a value of one indicates no change). Standard deviations in parentheses. Row (1) represents the benchmark in which we assume a linear consumption-price (r) increase after a sale ends and $m = 5$. Row (2) represents the case in which consumption (c) decreases linearly after a sale ends. Rows (3) to (6) represent the cases in which $m = 3$, 4, 6, and 7, respectively, and r increases in a concave or convex manner.

Table 6: Comparison of Inflation Rates

	Annualized inflation rate	Daily log inflation	Daily log inflation
	Mean	Mean	S.D.
All products			
Törnqvist (purchase-weighted)	-50.32	-0.19	0.72
Törnqvist (consumption-weighted)	-12.78	-0.04	1.37
Order r superlative (consumption-weighted)	-10.27	-0.03	0.99
Long-selling products			
Törnqvist (purchase-weighted)	-37.25	-0.13	1.42
Törnqvist (consumption-weighted)	-3.90	-0.01	1.05
Order r superlative (consumption-weighted)	1.81	0.00	1.05

Note: Denoting daily log inflation from date $t-1$ to t and the number of observations for the inflation rate by x_t and n, respectively, we calculate the mean of daily log inflation from 1990 to 2012 as $\bar{x} = \Sigma x_t/n$. The mean of the annualized inflation rate is calculated as $\exp(365 * \bar{x}) - 1$. All figures are then multiplied by 100 (i.e., the unit is percent).

Table 7: Regression of the Degree of Stockpiling

	(1)	(2)		(3)
	$\log(m)$	$\log(m)$		$\Delta\log(m)$
$\log(1-\overline{q})$	1.7463***	-0.901	$\Delta\log(1-\overline{q})$	-1.5348**
	(0.542)	(0.741)		(0.622)
$\log(1-\underline{q})$	-0.6306***	-0.8058***	$\Delta\log(1-\underline{q})$	-0.5160***
	(0.041)	(0.038)		(0.036)
$\log(1-P_L/P_H)$	0.3897***	0.3349***	$\Delta\log(1-P_L/P_H)$	0.2623***
	(0.047)	(0.045)		(0.032)
Fixed effects	category	category/month		category
Observations	40,296	40,296		40,150
Within R^2	0.588	0.719		0.277
# of categories	146	146		146

Note: ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

Table 8: Effects of Macroeconomic Variables on Stockpiling Behavior

	(1)	(2)	(3)	(4)	(5)
	$\Delta\log(m)$	$\Delta\log(1-\overline{q})$	$\Delta\log(1-\underline{q})$	$\Delta\log(1-P_L/P_H)$	$\Delta\log(m)$
Δ(unemp rate)	0.0035	-0.0004***	-0.0347***	0.0417***	0.0333***
	(0.005)	(0.000)	(0.004)	(0.004)	(0.005)
Δ(unemp rate(-1))	-0.0098**	-0.0004***	0.0298***	-0.0334***	-0.0336***
	(0.005)	(0.000)	(0.004)	(0.003)	(0.005)
Δlog(hours worked)	-0.2746***	0.0128***	0.1253***	0.2607***	-0.2882***
	(0.057)	(0.001)	(0.038)	(0.036)	(0.057)
Δlog(hours worked(-1))	-0.8088***	0.0091***	-0.7862***	0.6977***	-0.2270***
	(0.068)	(0.001)	(0.056)	(0.050)	(0.060)
Δ(real r)	-0.0093***	0.0005***	0.0115***	-0.0013	-0.0163***
	(0.004)	(0.000)	(0.003)	(0.002)	(0.003)
Δ(real r(-1))	-0.0058*	-0.0002***	-0.0080***	-0.0018	-0.0018
	(0.003)	(0.000)	(0.003)	(0.002)	(0.003)
$\Delta\log(1-\overline{q})$	-1.4929**				-1.4929**
	(0.626)				(0.626)
$\Delta\log(1-\underline{q})$	-0.5179***				-0.5179***
	(0.037)				(0.037)
$\Delta\log(1-P_L/P_H)$	0.2697***				0.2697***
	(0.032)				(0.032)
Fixed effects	category	category	category	category	category
Observations	40,004	40,004	40,004	40,004	40,004
Within R^2	0.283	0.002	0.009	0.008	0.283
# of categories	146	146	146	146	146

Note: In column (5), the explanatory variables corresponding to $\Delta\log(1-\overline{q})$, $\Delta\log(1-\underline{q})$, and $\Delta\log(1-P_L/P_H)$ are the residuals of the estimation for columns (2), (3), and (4), respectively. ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

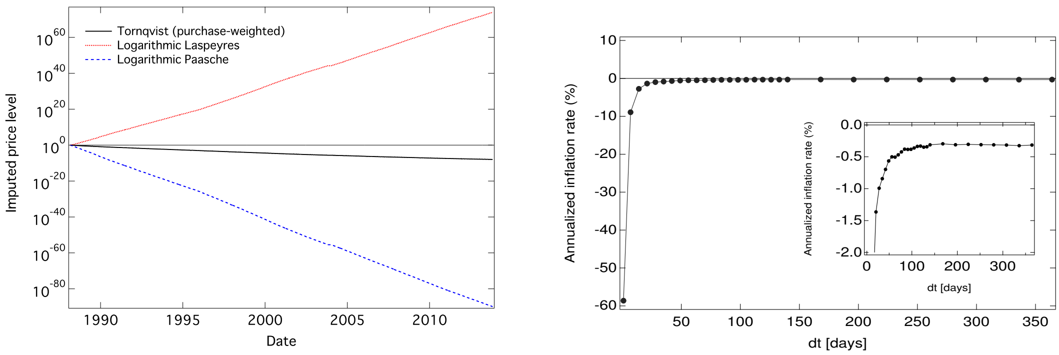
Table 9: Effects of Endogenous Changes in Stockpiling and Pricing on the Macroeconomy and Price Indices

	m	$\overline{q}$	$\underline{q}$	$1 - P_L/P_H$
Data	2.156	0.032	0.567	0.873
Hours worked ↑ by 0.027 (2.7%)				
	2.126	0.032	0.575	0.870
Real interest rate ↑ by 0.010 (1.0%)				
	2.117	0.032	0.569	0.874

	Sales (PX)	Quantity (X)	Unit price (P)	COLI	Törnqvist (PX)	Laspeyres (PX)	Paasche (PX)
Data	0	0	0	1.000	0.198	9.659	0.004
Hours worked ↑ by 0.027 (2.7%)							
	-1.1E-03	-5.4E-04	-5.9E-04	1.000	0.194	9.716	0.004
Real interest rate ↑ by 0.010 (1.0%)							
	-7.9E-04	-1.2E-03	4.0E-04	1.000	0.204	9.114	0.005

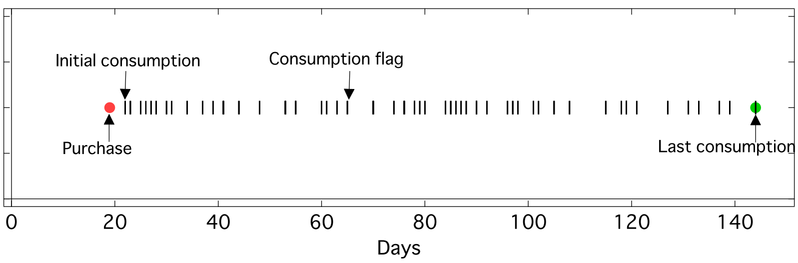
Note: For the amount of sales, the quantity purchased, and the unit price, the figures show changes in the values from their means calculated from the POS data. The price indices are expressed as the changes over 365 days (1 indicates no change).

Figure 1: Chain Drift



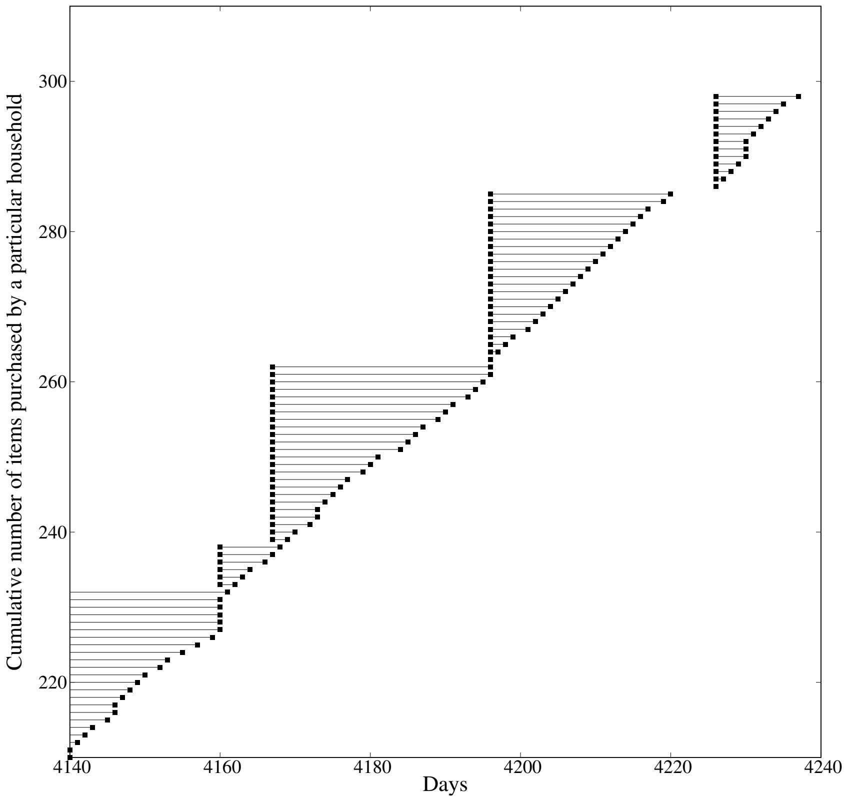
Note: The left-hand figure shows the time-series of price levels based on the purchase-weighted Laspeyres, Paasche, and Törnqvist indices using the POS data. The initial price level is normalized to one. The right-hand figure shows the average price change based on the Törnqvist index, where we employ different time intervals dt from 1 day to 365 days.

Figure 2: Consumption Pattern of Salt



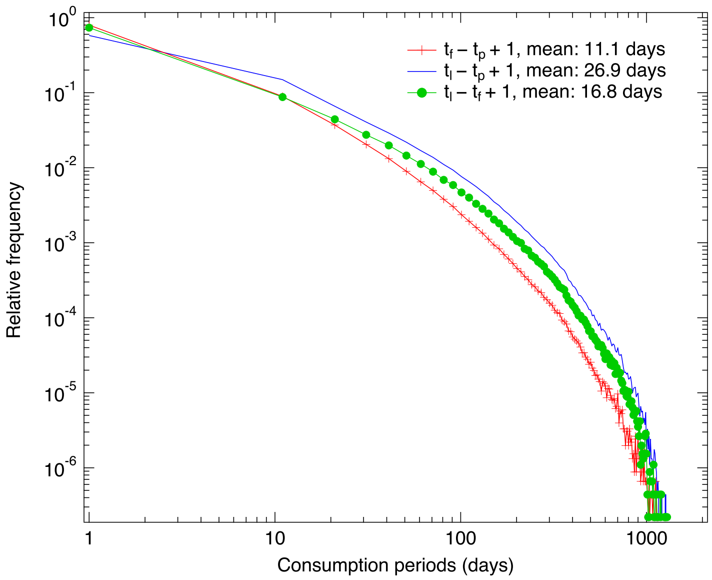
Note: The figure shows the consumption pattern for salt of a particular household in the *Shoku-map* data. Each vertical line represents a consumption flag.

Figure 3: Purchase and Consumption Pattern of Beer



Note: The figure shows the purchase and consumption pattern for beer of a particular household in the *Shoku-map* data. The left and right ends of each horizontal line show the days when products are purchased and consumed, respectively.

Figure 4: Density of Consumption Periods



Note: The figure shows the density of consumption periods in the *Shoku-map* data. For each product i and household j, we look at three dates: the date of purchase (t_p), the date a household starts consuming the product (t_f), and the date the household finishes the product (t_l). We then calculate the periods between the various dates.

Figure 5: Changes in the Price Index ($dt = 365$)

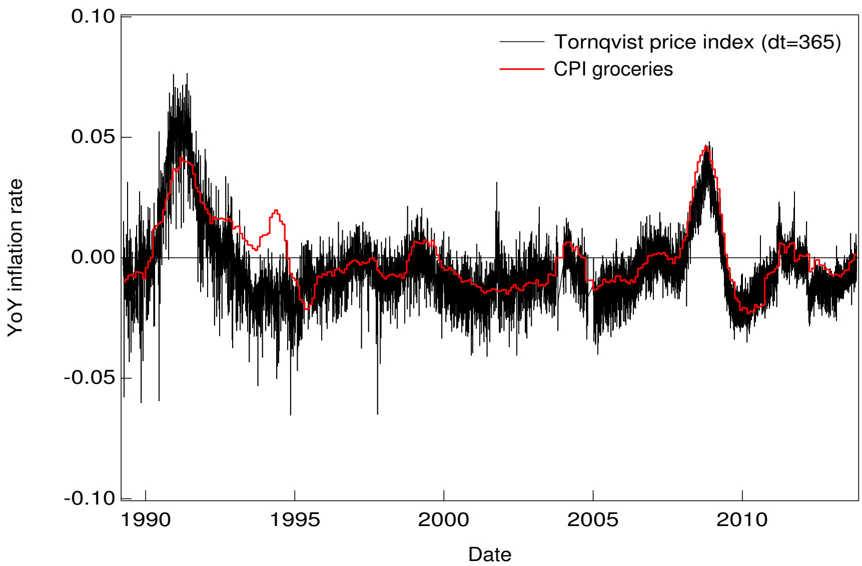
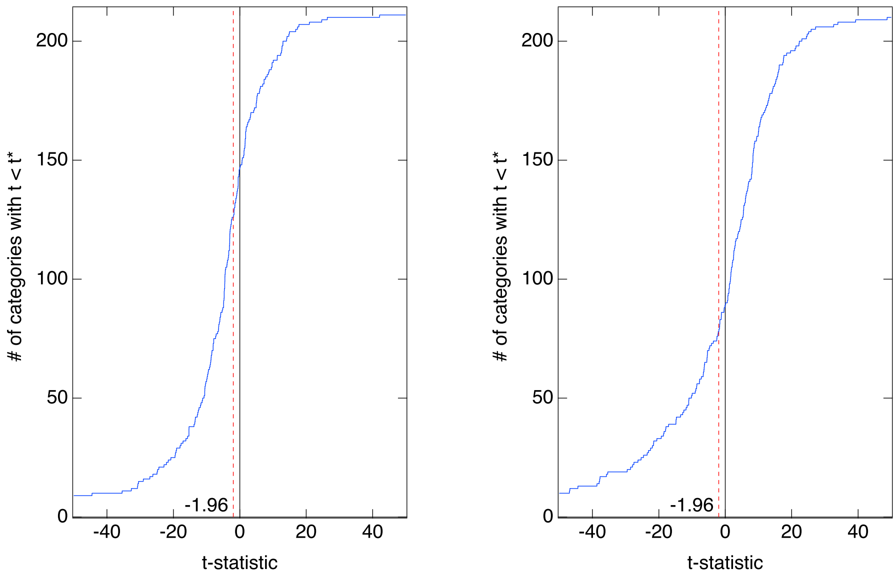
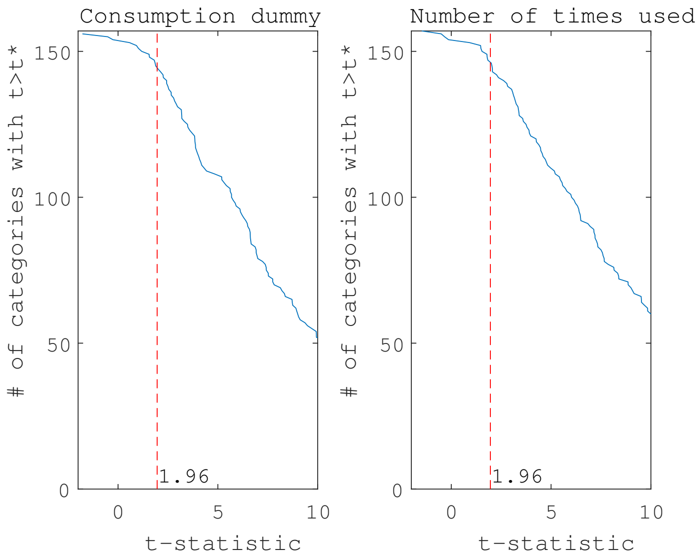


Figure 6: Distributions of the Difference in the Quantity Purchased before, during, and after a Sale



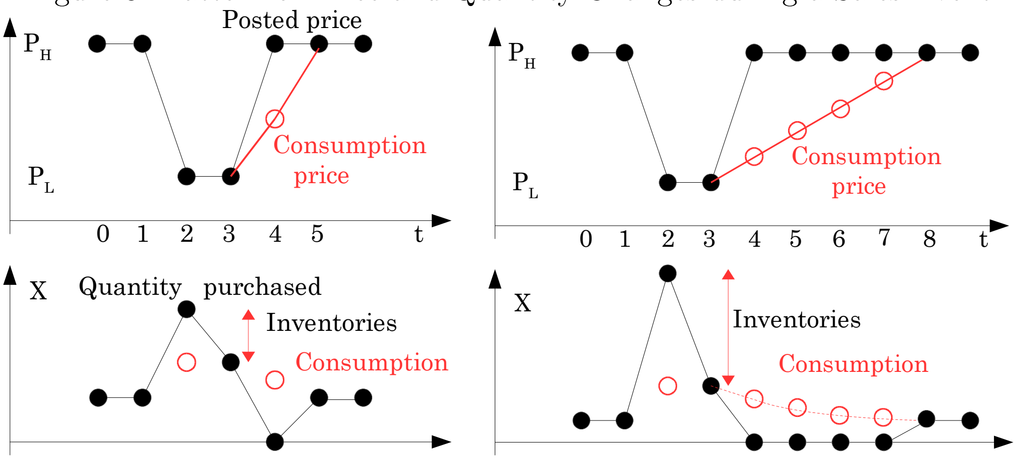
Note: We regress the equations for the difference in the quantity purchased before, during, and after a sale for each 3-digit category level and obtain the t-statistic for the intercept. The vertical axis shows the number of categories whose t-statistic is larger than t^*, while the horizontal axis represents t^*. For the left-hand panel, the dependent variable is the difference in the quantity purchased just after a sale from that just before a sale, while for the right-hand panel it is the difference in the quantity purchased in the second half of a sale from the first half of a sale.

Figure 7: Distribution of the Estimates of the Inventory Elasticity of Consumption



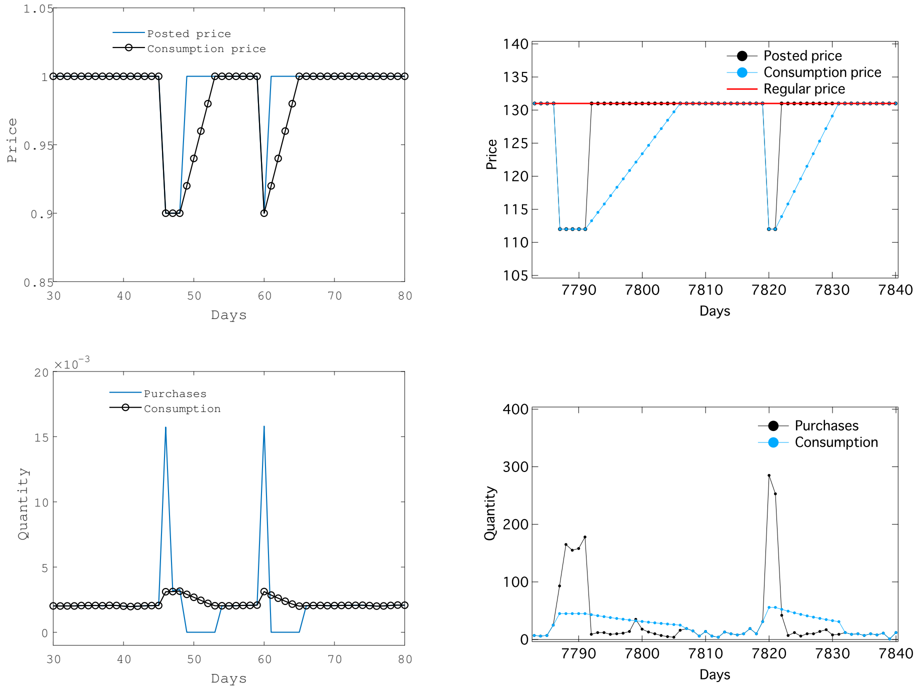
Note: We regress consumption for each 3-digit category level and obtain the t-statistic for the coefficient on inventories. The vertical axis shows the number of categories whose t-statistic for the inventory elasticity of consumption is larger than t^*, while the horizontal axis represents t^*. For the left-hand panel, the dependent variable is the consumption dummy which takes a value of one if a product is used, while for the right-hand panel it is the number of times a product is used.

Figure 8: Pattern of Price and Quantity Changes during a Sales Event



Note: The solid dots represent observable posted prices (top) and quantities purchased (bottom). The circles represent unobservable consumption prices (top) and quantities consumed (bottom).

Figure 9: Simulated, Actual, and Inferred Paths of Price and Quantity



Note: The left-hand panel shows the simulated paths of the price and the quantity purchased as well as those of the consumption price and consumption. The right-hand panel shows the actual paths of these variables for a particular brand of cup noodles purchased at a particular retailer.

Figure 10: Histogram of the Degree of Stockpiling

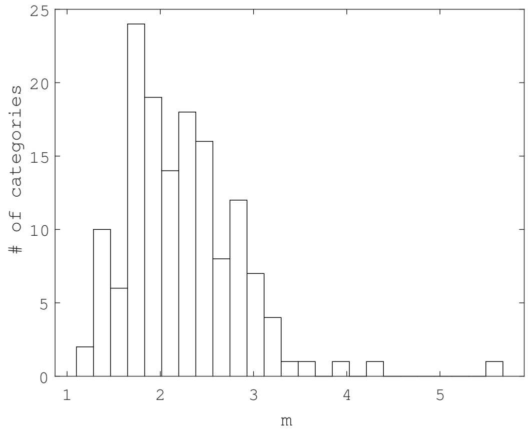
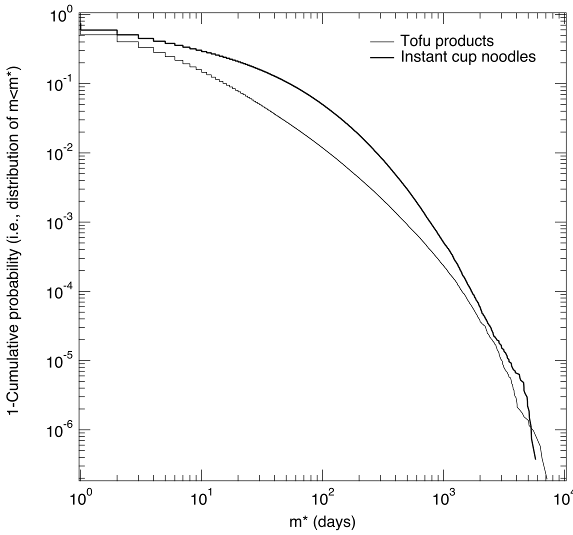
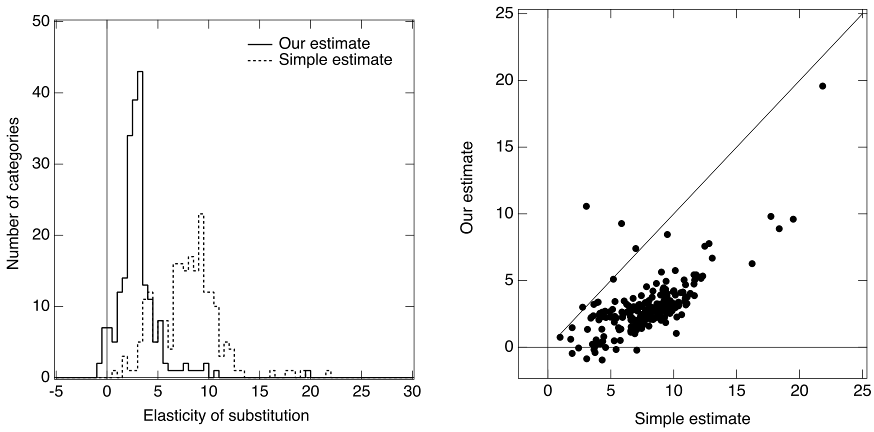


Figure 11: Distribution of the Degree of Stockpiling



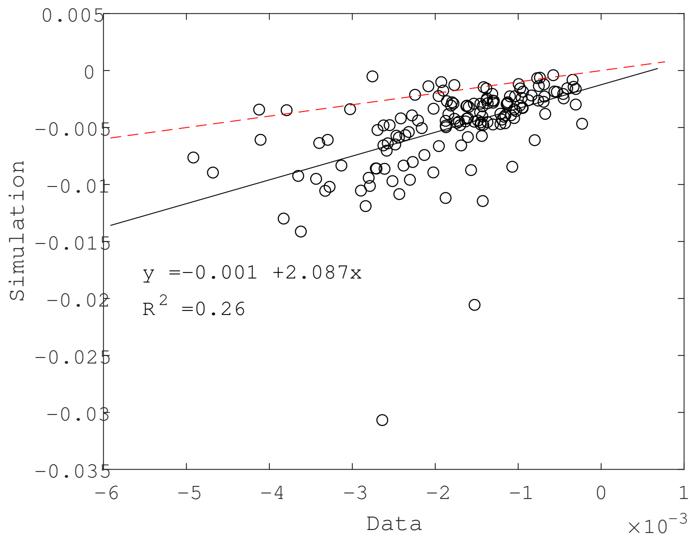
Note: The figure shows the distribution of the inferred degree of stockpiling ($m < m^*$) for "instant cup noodles" (thick upper line) and "tofu products" (thin lower line). In each category, we collect m for each retailer, product, and sales event.

Figure 12: Elasticity of Substitution



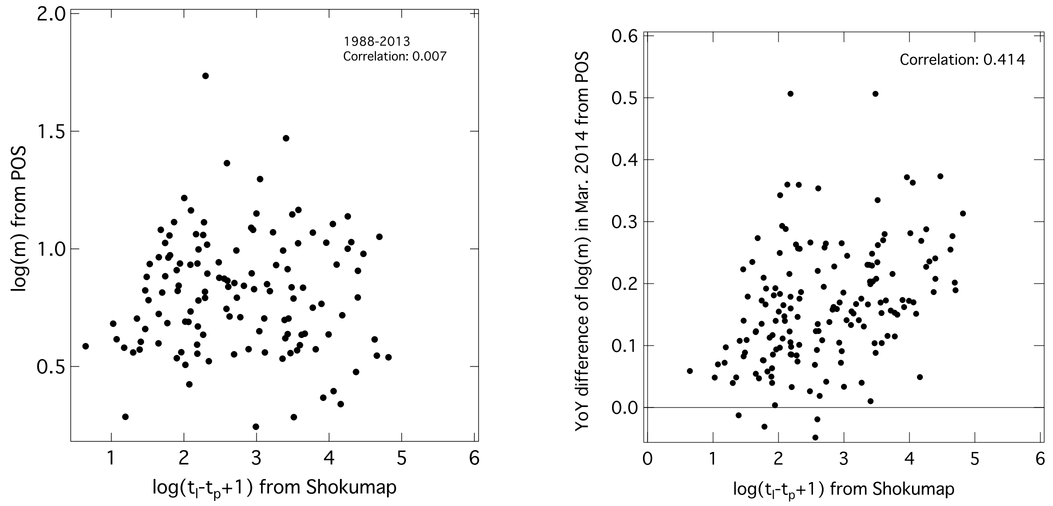
Note: "Our estimate" represents our calculation of the elasticity of substitution σ from $\Gamma \equiv -\log(c_L/c_H)/\log(r_L/r_H)$ using the inferred series of consumption c and consumption price r. "Simple estimate" represents the calculation of σ from $-\Delta\log X_t/\Delta\log p_t$, where X and p represent the quantity purchased and posted price, respectively. The left-hand panel shows the histogram of the values of σ for 3-digit product categories, while the right-hand panel shows a scatter plot where each dot represents a 3-digit product category.

Figure 13: Inflation Rates Based on the Törnqvist Index: Data and Simulation



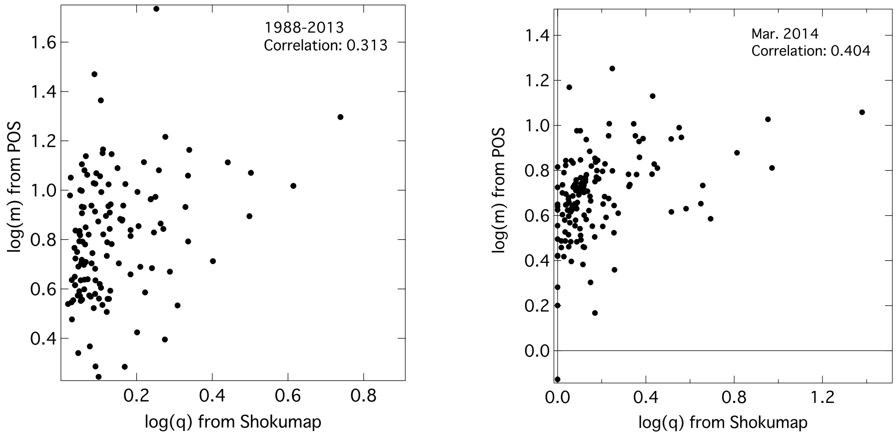
Note: Each circle represents a 3-digit product category. The inflation rates are the daily averages and are based on the purchase-weighted Törnqvist index. The red dashed line represents the 45 degree line.

Figure 14: Relationship between the Consumption Period (*Shoku-map*) and the Degree of Stockpiling (POS)



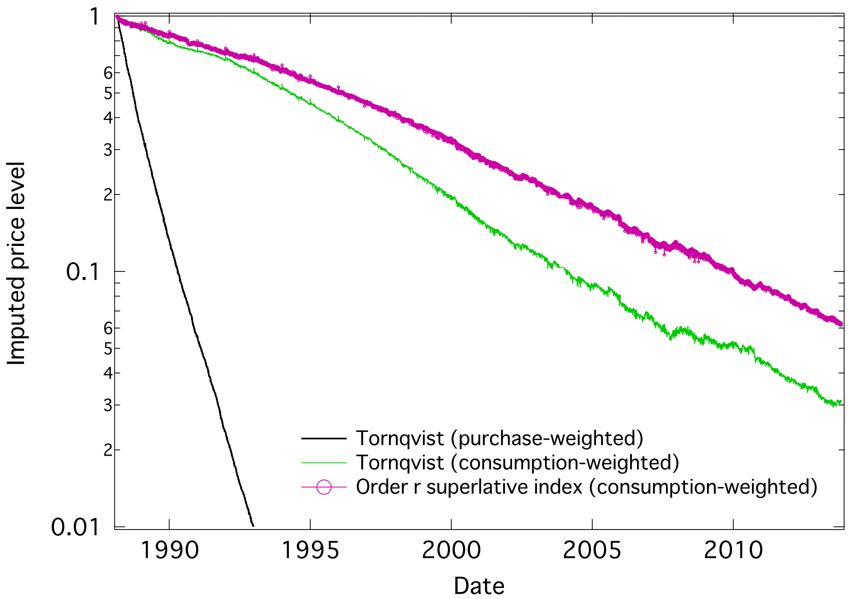
Note: Each dot represents a 3-digit product category. The horizontal axis represents the log consumption period, where the consumption period is defined as the difference between the date of purchase (t_p) and the date the household finishes the product (t_l) plus one.

Figure 15: Relationship between the Quantity Purchased (*Shoku-map*) and the Degree of Stockpiling (POS)



Note: Each dot represents a 3-digit product category. The horizontal axis represents the log quantity purchased.

Figure 16: Price Indices



Note: The figure shows the time-series of price levels based on the purchase-weighted Törnqvist, consumption-weighted Törnqvist, and consumption-weighted order r superlative indices using the POS data. The initial price level is normalized to one.

Figure 17: Price Indices for Long-Selling Products

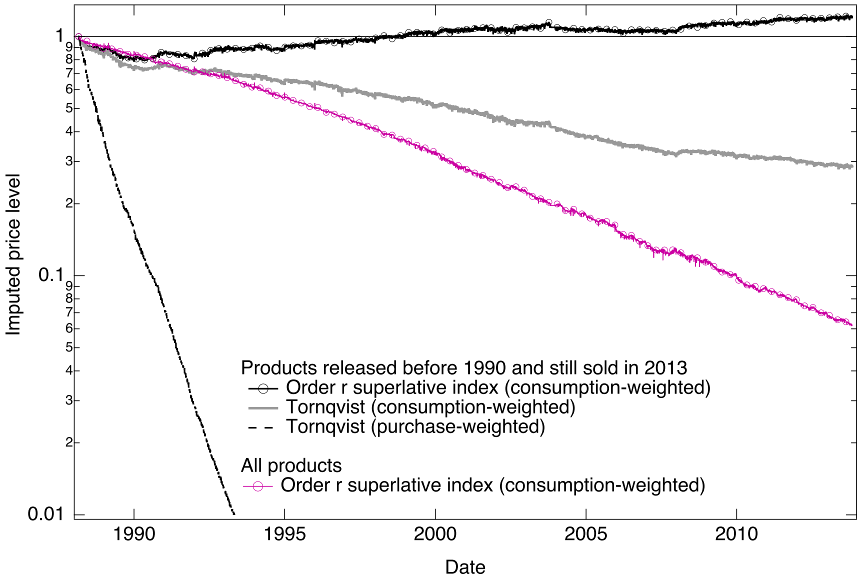


Figure 18: Annualized Inflation Rates

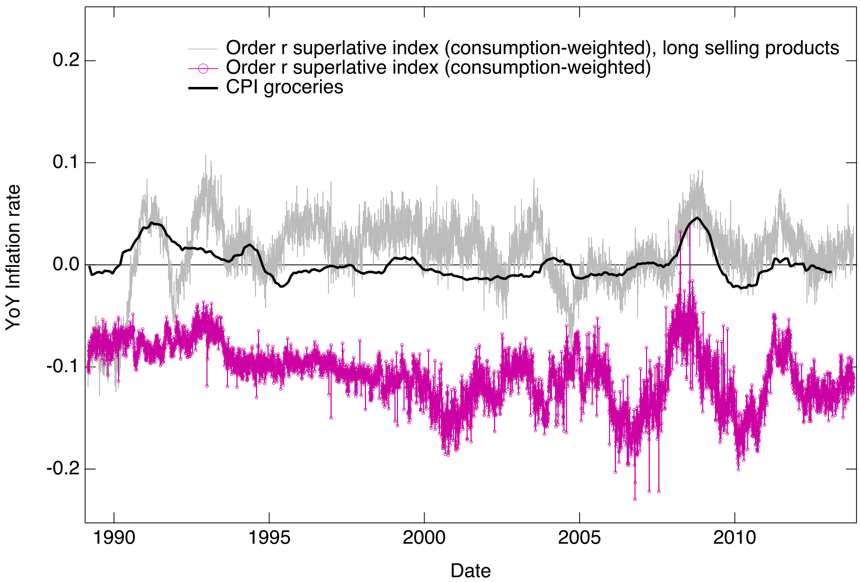
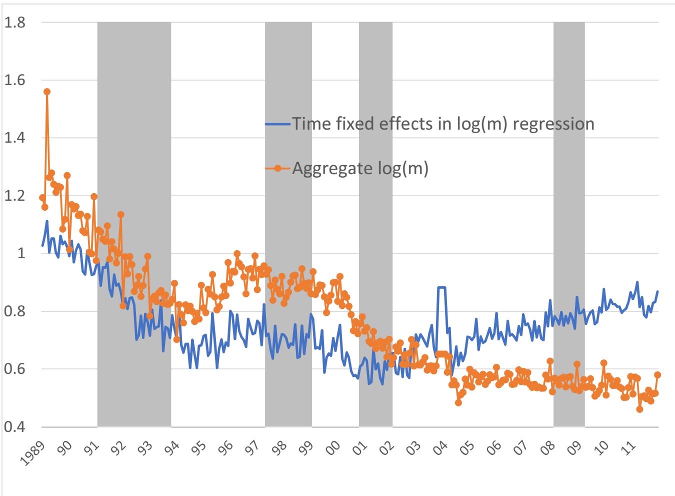


Figure 19: Aggregate Time-Series of the Degree of Stockpiling



Note: The shaded areas indicate recession periods determined by the Cabinet Office.